

# **SOLUTIONS**

## **PROGRESS TEST-1**

**RBPA**

**JEE MAIN PATTERN**

**Test Date: 03-09-2017**



Corporate Office: Paruslok, Boring Road Crossing, Patna-01  
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20  
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd.,  
Bazar Samiti Patna-06  
Call : 9569668800 | 7544015993/4/6/7

## PHYSICS

1.  $u = 7 \text{ m/s}$  and  $a = 4 \text{ m/s}^2$

$$\text{Distance traveled in } n^{\text{th}} \text{ second} = u + \frac{a}{2}(2n - 1)$$

$$\therefore \text{Distance traveled in } 5^{\text{th}} \text{ second} = 7 + \frac{4}{2}[2(5) - 1] = 25\text{m}$$

$\therefore$  (A)

2. Let after  $t$  second particle will reach at P again,

$$\therefore \text{ area of } v - t \text{ curve} = 0$$

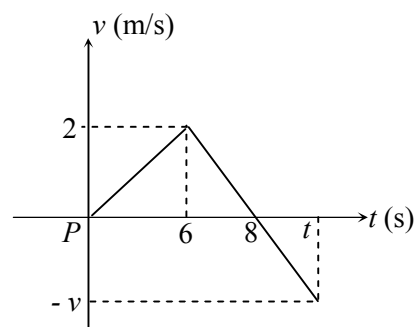
$$\frac{1}{2} \times 2 \times 8 - \frac{1}{2} \times (t - 8) \times (t - 8) \times 1 = 0$$

$$(t - 8)^2 = 16$$

$$t - 8 = 4$$

$$t = 12\text{s}$$

$\therefore$  (C)



3. The stopping distance  $S \propto u^2$

$\therefore$  (D)

4.  $x^2 + y^2 = l^2 \Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow -xv_A + yv_B = 0$

$$\therefore v_B = \frac{v_A}{\tan \alpha} = 10\sqrt{3} = 17.3 \text{ m/s}$$

$\therefore$  (D)

5.  $\frac{v_T + v_B}{2} = \frac{3}{0.5} = 6$  or  $v_T + v_B = 12 \text{ ms}^{-1}$

$\therefore$  (A)

6. Block B again comes to rest when speed of A = speed of C

$$v_A = 6t^2, v_C = 3t, 6t^2 = 3t, t = \frac{1}{2} \text{ s}$$

$\therefore$  (D)

7.  $\frac{ds}{dt} = 4\sqrt{1+s}$

$$\Rightarrow \int_0^s \frac{ds}{\sqrt{1+s}} = \int_0^t 4dt \Rightarrow 2\sqrt{1+s} = 4t \Rightarrow s = 4t^2 - 1$$

$$\Rightarrow v = 8t \text{ at } t=0, v=0$$

$\therefore$  (A)

8. For first projectile,  $h_1 = ut - \frac{1}{2}gt^2$

For second projectile,  $h_2 = u(t-T) - \frac{1}{2}g(t-T)^2$

When both meet i.e.  $h_1 = h_2$

$$ut - \frac{1}{2}gt^2 = u(t-T) - \frac{1}{2}g(t-T)^2 \Rightarrow uT + \frac{1}{2}gT^2 = gtT \Rightarrow t = \frac{u}{g} + \frac{T}{2}$$

$\therefore$  (B)

9.  $a = v \frac{dv}{dx} = \frac{25}{(x+2)^3}$ ,  $\frac{v^2}{2} = 25 \times \left[ -\frac{1}{2(x+2)^2} \right]_0^x$ ,  $v^2 = 25 \left[ \frac{1}{4} - \frac{1}{(x+2)^2} \right]$

$$v = \sqrt{25 \left[ \frac{1}{4} - \frac{1}{(x+2)^2} \right]}, v_{\max} = \frac{5}{2} = 2.5 \text{ m/s (at } x = \infty)$$

$\therefore$  (A)

10. The graph will be parabolic and in downward motion velocity will be negative and upward motion velocity will be positive

$\therefore$  (A)

11. For train B,  $-\frac{dv}{dt} = 0.3t$ ,  $-\int_{15}^0 dv = 0.3 \int_0^t t dt \Rightarrow t = 10 \text{ s}$

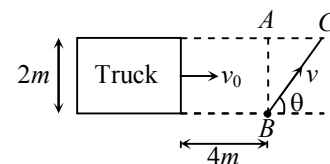
In this 10 s, the train B travels a distance of 100 m.

$\therefore$  Train A can travel a distance of 125 m before coming to rest.

$$v^2 = u^2 + 2as, a = -2.5 \text{ m/s}^2$$

$\therefore$  (B)

12. For safe crossing, the condition is that the man must cross the road by the time the truck covers the distance  $4 + AC$  or  $4 + 2\cot \theta$



$$\therefore \frac{4 + 2 \cot \theta}{8} = \frac{2 / \sin \theta}{v}$$

$$\text{or } v = \frac{8}{2 \sin \theta + \cos \theta} \quad \dots(i)$$

$$\text{For minimum } v, \frac{dv}{d\theta} = 0 \Rightarrow \tan \theta = 2$$

$$\text{From equation (i), } v_{\min} = \frac{8}{\sqrt{5}} = 3.57 \text{ m/s}$$

$\therefore$  (C)

$$13. A_1 = \frac{1}{2}(2 + 4) \times 1 = 3\text{m}$$

$$A_2 = \frac{1}{2}(2 + 1) \times 4 = 6\text{m}$$

$$A_3 = \frac{1}{2}(2 \times 4) = 4\text{m}$$

$$A_4 = \frac{1}{2}(2 \times 6) = 6\text{m}$$

$$\text{Distance travelled in 7 s} = A_1 + A_2 + A_3 + A_4 = 19 \text{ m}$$

$$\text{Average speed} = \frac{19}{7} \text{ m/s}$$

$\therefore$  (D)

14.  $0 - t_1 \rightarrow$  uniformly retarded motion

$t_1 - t_2 \rightarrow$  particle at rest

$t_2 - t_3 \rightarrow$  uniform negative velocity

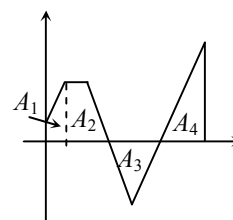
$t_3 - t_4 \rightarrow$  particle at rest

$t_4 - t_5 \rightarrow$  uniform negative velocity

$\therefore$  (C)

15. (C)

Fringe visibility gives the contrast of the fringes given by



$$V = \frac{2\sqrt{I_1/I_2}}{1+I_1/I_2}$$

16. (C)

17. (C)

After immersing, no change in central maxima in air, separation between central maxima & 10<sup>th</sup> maxima = 5cm - 2cm = 3cm =  $10\frac{D\lambda}{d}$  in liquid, separation between central maxima & 10<sup>th</sup> maxima =

$$10\frac{D\lambda'}{d} = 10\frac{D\lambda}{d\mu} = \left(\frac{10D\lambda}{d}\right)/1.5 = \frac{3\text{cm}}{1.5} = 2\text{cm} . \text{ So new co-ordinate of } 10^{\text{th}} \text{ maxima} = 2\text{cm} + 2\text{cm} = 4\text{cm}$$

18. (A)

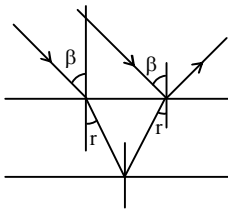
Phase difference corresponding to  $y_1 = \frac{-\pi}{2}$  and that for  $y_2 = +\frac{\pi}{2}$

∴ Average intensity between  $y_1$  and  $y_2$

$$\begin{aligned} &= \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} I_{\max} \cos^2\left(\frac{\phi}{2}\right) d\phi \\ &= I_{\max} \frac{(\pi + 2)}{2\pi} \end{aligned}$$

$$\text{Hence required ratio} = \frac{1}{2} \left(1 + \frac{2}{\pi}\right)$$

19. (C)



$$2\mu t \cos r = (2n + 1) \frac{\lambda}{2}$$

$$t = \frac{(2n+1)\lambda}{4\mu \cos r} \quad (\text{putting } n = 0) = \frac{\lambda}{4\mu \cos r}$$

$$\cos r = \sqrt{1 - \sin^2 r} = \frac{1}{\mu} \sqrt{\mu^2 - \sin^2 \beta}$$

$$\text{Substituting all value} \quad t = 1.01 \times 10^{-7} \text{m}$$

$$\text{mass of soap} = \rho \times \ell \times h \times t = 6.06 \times 10^{-2} \text{mg}.$$

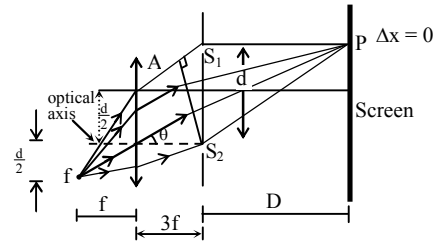
20. (A)

$$\Delta x = AS_1 + S_1P - S_2P$$

$$= d \sin \theta - \frac{dx}{D} = 0$$

$$\Delta x = \frac{d \times d}{2f} - \frac{dx}{D} = 0 \quad d \ll \ll D$$

$$x = \frac{dD}{2f}$$



21. (B)

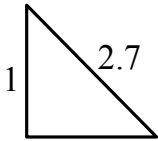
Condition for constructive interference

$$2\mu t \cos r = n\lambda$$

$$\sin 30^\circ = 1.34 \sin r$$

$$\sin r = \frac{1}{2.68} \quad \text{or} \quad \frac{1}{3}$$

$$\cos r = \sqrt{1 - \sin^2 r}$$



$$2\mu t \cos r = n\lambda \quad \dots (1)$$

$$2\mu(t - \Delta t) \cos r = (n - 1)\lambda \quad \dots (2)$$

Equation (1) and equation (2)

By eq.(1) and eq.(2)

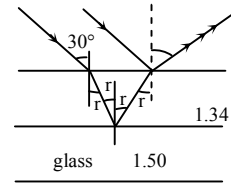
$$2\mu \Delta t \cos r = \lambda$$

$$\Delta t = \frac{\lambda}{2\mu \cos r}$$

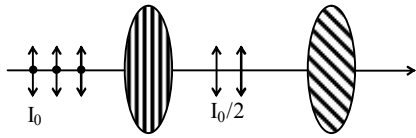
$$\text{Rate of decrease} = \frac{\Delta t}{\text{time}} = 1.01 \mu\text{m/hr}$$

$$[\text{Time} = 15 \text{ min} = \frac{15}{60}]$$

22. (C)



23. (D)



$$\left(\frac{I_0}{2}\right) \cos^2 60^\circ = \frac{I_0}{8}$$

24. (A)

25. (C)

$$\mu = \tan i_p$$

$$\therefore \frac{c}{v} = \tan i_p$$

$$\therefore v = \frac{3 \times 10^8}{\sqrt{3}} = \sqrt{3} \times 10^8 \text{ m/s}$$

26. For charge + q at A to come down,  $F_e < mg$ 

$$\therefore \frac{q^2}{4\pi\epsilon_0 h^2} < mg$$

$$\therefore \text{(C)}$$

$$27. F = \frac{Q^2}{4\pi\epsilon_0 r^2}$$

$$F_C = \frac{Q^2/2}{4\pi\epsilon_0 \left(\frac{r}{2}\right)^2} - \frac{Q^2/4}{4\pi\epsilon_0 \left(\frac{r}{2}\right)^2} = \frac{Q^2}{4\pi\epsilon_0 r^2} = F$$

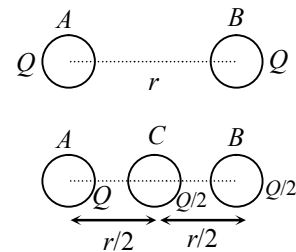
$$\therefore \text{(C)}$$

28. (A)

$$29. \frac{K(4e)q}{(x-y)^2} = \frac{Kqe}{y^2}, \quad y = \frac{x}{3}$$

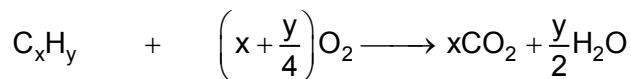
$$\therefore \text{(C)}$$

$$30. -\frac{2q}{4\pi\epsilon_0 (x-L)^2} + \frac{8q}{4\pi\epsilon_0 x^2} = 0 \quad \text{or } x = 2L$$

$$\therefore \text{(A)}$$


## CHEMISTRY

31. (D)



$$A \text{ ml} \quad A\left(x + \frac{y}{4}\right) \text{ ml} \quad 0 \quad 0$$

$$0 \quad 0 \quad Ax \quad \frac{Ay}{2}$$

$$A + Ax + \frac{Ay}{4} = 600 \quad \dots\dots\dots (1)$$

$$Ax + \frac{Ay}{2} = 700 \quad \dots\dots\dots (2)$$

$$\Rightarrow \frac{x}{y} = \frac{3}{8}$$

Since,  $x < 5$

$$\Rightarrow x = 3, y = 8$$

$$\Rightarrow A = 100 \text{ ml}$$

So, the hydrocarbon is  $C_3H_8$  and the volume taken was 100 ml

32. (C)

$$\left(P + \frac{a}{V^2}\right)(V) = RT$$

$$PV^2 - RTV + a = 0$$

$$V = \frac{+RT \pm \sqrt{(RT)^2 - 4aP}}{2P}$$

$$V \text{ is single valued, } (RT)^2 - 4aP = 0 ; P = \frac{R^2T^2}{4a}$$

33. (B)

Permanent gas have extremely low critical temperature & hence to liquify them, temp must be extremely low below critical temp.



34. (C)

Liquefaction of the gas depends upon pressure correction. More is the pressure correction more easily the gas can be liquefied.

35. (D)

36. (A)

High temp. has less deviation from ideal behaviour

37. (C)

At critical condition, inflection point exist & hence  $\frac{\partial P}{\partial V_m}$  &  $\frac{\partial^2 P}{\partial V_m^2} = 0$

38. (B)

A gas having higher value of 'a' can be easily liquefied due to strong intermolecular force of attraction.

39. (A)

Below critical temp.  $H_2$  gas can be liquified

40. (A)

Vander Waal's equation for one mole of gas is given by

$$\left( \frac{P + a}{V^2} \right) [V - b] = RT$$

at low P, volume V is high

$$V - b \approx V$$

$$\therefore \left[ P + \frac{a}{V} \right] V = RT$$

$$PV = RT - \frac{a}{V} ; Z = 1 - \frac{a}{RTV}$$

41. (B)

42. (B)

43. (A)

44. (A)

45. (D)

46. (B)

47. (C)

48. (D)

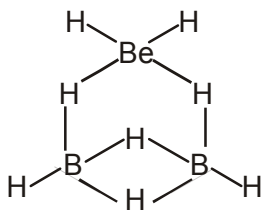
49. (B)

50. (B)

51. (C)

52. (B)

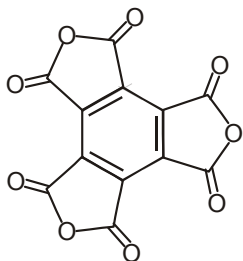
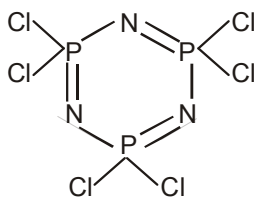
53. (C)



54. (A)



55. (A)

(iii)  $\text{C}_{12}\text{O}_9 - \text{sp}^2$  ;(iv)  $\text{N}_3\text{P}_3\text{Cl}_6 - \text{sp}^2 \text{ \& \; } \text{sp}^3$  ;

56. (A)

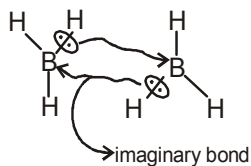
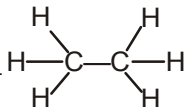
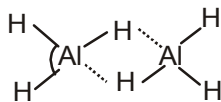
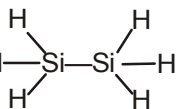
(i)  $\text{LiF} > \text{NaF} > \text{KF} > \text{RbF}$  : Lattice energy(iii)  $\text{Li}^+ < \text{Mg}^{2+} < \text{Al}^{3+}$  : Hydration energy

57. (A)

58. (C)

Due to size of Nitrogen is smaller than another.

59. (D) 1 &amp; 3 have x - x bond absent.

(1)  $\text{B}_2\text{H}_6$ (2)  $\text{C}_2\text{H}_6$  .....(3)  $\text{Al}_2\text{H}_6$ (4)  $\text{H}-\text{Si}-\text{Si}-\text{H}$  x-x bond are present

60. (C)

## MATHEMATICS

61. (C)

$(\sin^{-1} x + \sin^{-1} y)(\sin^{-1} z + \sin^{-1} w) = \pi^2$  is possible only when  $x = y = z = w = 1$

or  $x = y = z = w = -1$ , then

$$\begin{vmatrix} x^{n_1} & y^{n_2} \\ z^{n_3} & w^{n_2} \end{vmatrix} = 0, 2 \text{ or } -2$$

62. (C)

$$\begin{aligned} \sin^{-1}(\sin x) &= x \text{ in } \left(0, \frac{\pi}{2}\right] = \pi - x \text{ in } \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] = x - 2\pi \text{ in } \left[\frac{3\pi}{2}, 2\pi\right] \\ &= x - 2\pi \text{ in } \left[\frac{3\pi}{2}, 2\pi\right] \end{aligned}$$

$$\cos^{-1}(\cos x) = x \text{ in } [0, \pi] = 2\pi - x \text{ in } [\pi, 2\pi]$$

$$\begin{aligned} \therefore f(x) &= 2x \text{ in } \left[0, \frac{\pi}{2}\right] \\ &= \pi \text{ in } \left[\frac{\pi}{2}, \pi\right] \\ &= 3\pi - 2x \text{ in } \left(\pi, \frac{3\pi}{2}\right) \\ &= 0 \text{ in } \left[\frac{3\pi}{2}, 2\pi\right] \end{aligned}$$

$\therefore (15)f(x)$  increases in  $\left(0, \frac{\pi}{2}\right)$

63. (D)

$$\frac{1+x^2}{x} \geq 2 \quad (\because x > 0)$$

$$\sin^{-1}\left(\frac{x}{1+x^2}\right) \in \left(0, \frac{\pi}{6}\right]$$

$$\therefore \sin^{-1}\left(\frac{2y}{1+y^2}\right) \in \left(0, \frac{\pi}{2}\right]$$

$$\text{Range} = \left(0, \frac{2\pi}{3}\right]$$

64. (D)

$$\sin^{-1}(x^2 - 2x + 2) + \cos^{-1}(4x^2 - 4x + 2) = \frac{\pi}{2}$$

$$\Rightarrow x^2 - 2x + 2 = 4x^2 - 4x + 2$$

$$\Rightarrow 3x^2 - 2x = 0 \Rightarrow x = 0, \frac{2}{3}$$

But for  $x = 0$  and  $\frac{2}{3}$

$$x^2 - 2x + 2 > 1 \text{ and } 4x^2 - 4x + 2 > 1$$

Hence No solution

65. (D)

$$x = \sin 2\theta = 2\sin\theta \cos\theta = \frac{4}{5} \quad (\theta = \tan^{-1}2)$$

$$y = \sin \frac{\phi}{2}; y > 0, \tan \phi = \frac{4}{3}$$

$$y^2 = \sin^2 \frac{\phi}{2} = \frac{1 - \cos \phi}{2} = \frac{1}{5}$$

66. (C)

$$\sin^{-1}(x - 1) \Rightarrow -1 \leq x - 1 \leq 1 \Rightarrow 0 \leq x \leq 2$$

$$\cos^{-1}(x - 3) \Rightarrow -1 \leq x - 3 \leq 1 \Rightarrow 2 \leq x \leq 4$$

$$\tan^{-1}\left(\frac{x}{2 - x^2}\right) \Rightarrow x \in \mathbb{R}, x \neq \sqrt{2}, -\sqrt{2}$$

hence  $x = 2$ .

$$\sin^{-1}(2 - 1) + \cos^{-1}(2 - 3) + \tan^{-1} \frac{2}{2 - 4} = \cos^{-1} k + \pi$$

$$\Rightarrow \sin^{-1} 1 + \cos^{-1}(-1) + \tan^{-1}(-1) = \cos^{-1} k + \pi$$

$$\frac{\pi}{2} + \pi - \frac{\pi}{4} = \cos^{-1} k + \pi$$

$$\Rightarrow \cos^{-1} k = \frac{\pi}{4}$$

$$\Rightarrow k = \frac{1}{\sqrt{2}}$$

67. (B)

$$\text{Given equation reduces to } (x-2)^4 + \left(\tan^{-1}(y-1) - \frac{\pi}{4}\right)^2 = 0$$

$$\Rightarrow x = 2, y = 2 \Rightarrow x + y = 4.$$

68. (D)

$$\cos^{-1}\sqrt{x} + \cos^{-1}\sqrt{1-x} = \cos^{-1}\sqrt{x} + \sin^{-1}\sqrt{x} = \frac{\pi}{2}$$

$$\text{So given equation, } \frac{\pi}{2} + \cos^{-1}\sqrt{1-y} = \frac{3\pi}{4} \Rightarrow \cos^{-1}\sqrt{1-y} = \frac{\pi}{4} \Rightarrow y = \frac{1}{2}.$$

Clearly  $x \in [0, 1]$  (for domain)

69. (B)

$$\cos^{-1}\left(\frac{1+x^2}{2x}\right) = \frac{\pi}{2} + (\sin^{-1}x + \cos^{-1}x)$$

$$\cos^{-1}\left(\frac{1+x^2}{2x}\right) = \pi, \text{ hence } x = -1 \text{ is the only solution}$$

70. (A)

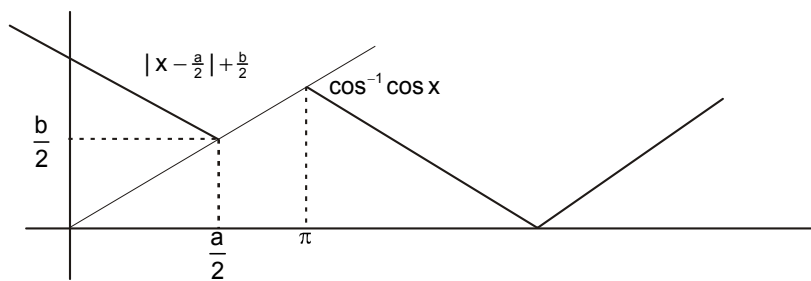
$$\text{The range of } \cos^{-1}x + \cot^{-1}x - \sin^{-1}(\sin x) \text{ is } \left[\frac{\pi}{4} - 1, \frac{7\pi}{4} + 1\right]$$

$$\text{Then } \frac{\pi}{4} - 1 \leq 2p - 1 \leq \frac{7\pi}{4} + 1 \Rightarrow \frac{\pi}{8} \leq p \leq \frac{7\pi}{8} + 1$$

Hence  $p = 1, 2, 3$ , four values.

71. (A)

$$\frac{a}{2} \in [0, 3]$$



$$a = 0, 1, 2, 3, 4, 5, 6$$

Hence, 7 Pairs.

72. (B)

$$\begin{aligned} & \cot\{\cot^{-1}3 + \cot^{-1}7 + \cot^{-1}13 + \cot^{-1}21\} \\ &= \cot\left\{\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \tan^{-1}\frac{1}{21}\right\} \\ &= \cot\{\tan^{-1}2 - \tan^{-1}1 + \tan^{-1}3 - \tan^{-1}2 + \dots + \tan^{-1}5 - \tan^{-1}4\} \\ &= \cot[\tan^{-1}5 - \tan^{-1}1] = \frac{3}{2} \end{aligned}$$

73. (D)

$$\cos^{-1}x \in [0, \pi]$$

$$\text{Thus } \cos^{-1}x + \cos^{-1}y + \cos^{-1}z \leq 3\pi$$

$$\Rightarrow \cos^{-1}x = \cos^{-1}y = \cos^{-1}z = \pi \Rightarrow x = y = z = -1$$

$$\Rightarrow \frac{100x^2 + 182y^2}{47z^2} = \frac{100 + 182}{47} = 6$$

74. (B)

$$\tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\Rightarrow \cos(\tan^{-1}x) \in (0, 1] \Rightarrow \cos^{-1}(\cos(\tan^{-1}x)) \in \left[0, \frac{\pi}{2}\right)$$

$$\Rightarrow \sin(\cos^{-1}(\cos(\tan^{-1}x))) \in [0, 1)$$

75. (D)

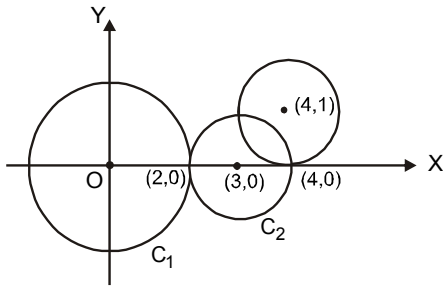
$$x = 2\pi - 4; y = \pi - 3 \Rightarrow x + y = 3\pi - 7$$

76. (C)

The points on the circle with integer co-ordinates are  $(\pm 5, 0)$ ,  $(\pm 4, \pm 3)$ ,  $(\pm 3, \pm 4)$ ,  $(0, \pm 5)$ , which are 12 in number. Joining any two of them will form a chord with extremities having integral co-ordinates.

$$\text{The number of the chords} = {}^{12}C_2 = 66.$$

77. (C)



Circles  $C_1$  and  $C_2$  touch each other externally, so they have three common tangents.

Circles  $C_2$  and  $C_3$  cut each other at two points, so they have two common tangents.

Circles  $C_1$  and  $C_3$  are external to each other, so they have four common tangents.

No common tangent can be drawn to touch all the three circles.

So, total no. of common tangents =  $3 + 2 + 4 = 9$ .

78. (A)

Let equation to the circle be  $(x - r)^2 + (y - r)^2 = r^2$

If it passes through  $(a, b)$ , then  $a^2 + b^2 - 2ra - 2rb + r^2 = 0 \Rightarrow r^2 - 2r(a + b) + a^2 + b^2 = 0$

$$\therefore r_1 + r_2 = 2(a + b) \text{ \& } r_1 r_2 = a^2 + b^2$$

According to the given condition

$$r_1^2 + r_2^2 = 4r_1 r_2 \Rightarrow a^2 + b^2 = 4ab$$

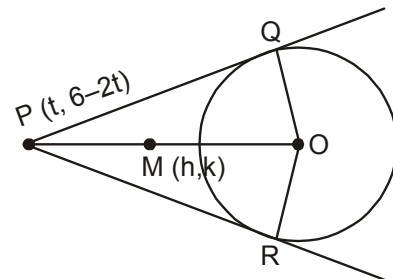
79. (A)

$\therefore$  PQOR is a cyclic quadrilateral with OP as diameter

Mid point of OP will be circumcentre of  $\Delta PQR$

$$\Rightarrow h = \frac{t}{2}, k = 3 - t$$

$$\Rightarrow \text{Locus is } y = 3 - 2x$$



80. (C)

Given  $QT = QA = 1$

Let  $PQ = x$ , then  $PT = \sqrt{x^2 - 1}$ ,

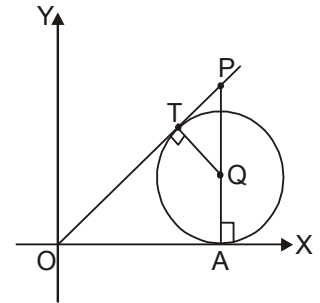
$$\therefore \triangle TQP \sim \triangle AOP \therefore \frac{OA}{AP} = \frac{QT}{TP}$$

$$\Rightarrow OA = OT = \frac{x+1}{\sqrt{x^2-1}}$$

perimeter of  $\triangle OAP = 8$

$$\Rightarrow 1 + x + \frac{2(x+1)}{\sqrt{x^2-1}} + \sqrt{x^2-1} = 8$$

$$\Rightarrow x = \frac{5}{3}$$



81. (B)

Let the equation be  $x^2 + y^2 - 9 + \lambda(x + y - 1) = 0$

For the circle to be smallest the centre  $\left(-\frac{\lambda}{2}, -\frac{\lambda}{2}\right)$  must lie on  $x + y = 1$ .

$$\therefore \lambda = -1$$

$$\therefore \text{Equation is } x^2 + y^2 - x - y - 8 = 0$$

82. (B)

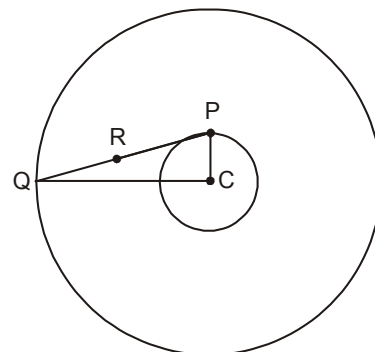
Using Appollonius theorem,

$$CP^2 + CQ^2 = 2(CR^2 + RQ^2)$$

$$16 + 36 = 2(CR^2 + 4)$$

$$26 = CR^2 + 4$$

$$CR = \sqrt{22}$$





83. (A)

$$a \leq \sin A \Rightarrow \frac{a}{\sin A} \leq 1 \Rightarrow 2R \leq 1 \Rightarrow R \leq \frac{1}{2}$$

For any point (x, y) inside the circumcircle,  $x^2 + y^2 < \frac{1}{4}$

$$\frac{x^2 + y^2}{2} \geq |xy| \Rightarrow |xy| < \frac{1}{8}$$

84. (B)

Let the equation of chord be  $y = mx + c$ ; Joint equation of OA & OB is

$$4x^2 + y^2 - x\left(\frac{y - mx}{c}\right) + 4y\left(\frac{y - mx}{c}\right) = 0$$

$$\therefore OA \perp OB \Rightarrow \left(4 + \frac{m}{c}\right) + \left(1 + \frac{4}{c}\right) = 0$$

$$\Rightarrow 5c + m + 4 = 0$$

$$\therefore y = mx + c \Rightarrow y + 4x + c(5x - 1) = 0$$

$\Rightarrow$  passing through the intersection of

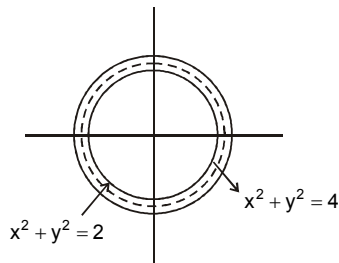
$$y + 4x = 0 \text{ and } 5x - 1 = 0$$

85. (B)

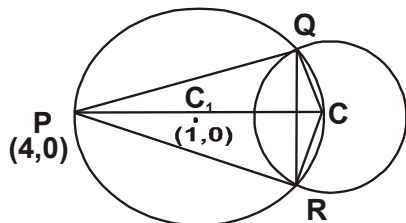
Required Area

$$= \pi(4 - 2)$$

$$= 2\pi$$



86. (C)



Let 'C' be the centre of the circle  $S = 0$ , then circumcircle of the  $\Delta PQR$  will pass through C.

Hence, (1,0) centre of the circumcircle of PQR is mid-point of PC. Hence, C is (-2, 0) so equation of  $S = 0$  is  $(x+2)^2 + y^2 = (2\sqrt{3})^2$ ,

Hence,  $(-5, \sqrt{3})$  will be on the circle  $S = 0$

87. (D)

Here  $ax + by = 20$  is a chord with (2, 3) as its mid-point.

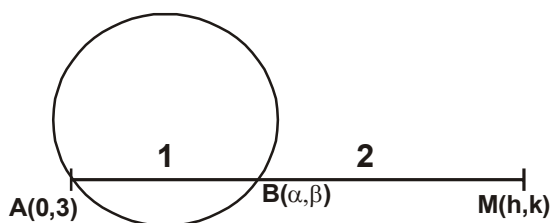
$$\Rightarrow -\frac{a}{b} = -1 \quad \Rightarrow a = b$$

Now,  $2a + 3b = 20$

$$\Rightarrow 5a = 20 \Rightarrow a = b = 4$$

Hence  $a^{103} + b^{103} = 2^{207}$

88. (D)



$$\alpha = \frac{h}{3} \quad \beta = \frac{k+6}{3}$$

Hence  $\frac{h^2}{9} + \frac{(k+6)^2}{9} + 4 \times \frac{h}{3} - 6 \times \frac{k+6}{3} + 9 = 0 \Rightarrow h^2 + k^2 + 12h - 6k + 9 = 0$

$$\Rightarrow x^2 + y^2 + 12x - 6y + 9 = 0$$

89. (C)

$$P \equiv \frac{x}{\cos \frac{\pi}{4}} = \frac{y}{\sin \frac{\pi}{4}} = 6\sqrt{2} \Rightarrow x = 6, y = 6$$

Since P(6,6) lie on circle

$$72 + 12(g + f) + c = 0 \quad \dots(i)$$

Since  $y = x$  touches the circle, then

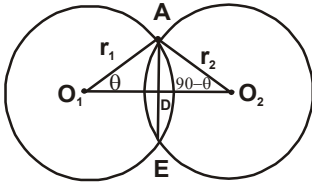
$$2x^2 + 2x(g + f) + c = 0 \text{ has equal roots } D = 0$$

$$4(g + f)^2 = 8c \Rightarrow (g + f)^2 = 2c \quad \dots\dots(ii)$$

From, we get

$$(12(g + f))^2 = [-(c + 72)]^2 \Rightarrow 144(2c) = (c + 72)^2 \Rightarrow (c - 72)^2 = 0 \Rightarrow c = 72$$

90. (A)



Let  $O_1$  and  $O_2$  are the centre of circle with radii  $r_1$  and  $r_2$  respectively and  $\angle AO_1O_2 = \theta$

$$AD = r_1 \sin \theta; \quad AD = r_2 \cos \theta$$

$$AD^2 \left( \frac{1}{r_1^2} + \frac{1}{r_2^2} \right) = 1 \Rightarrow AD = \frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}} \quad \text{so hence } L = 2AD$$