SOLUTIONS

PROGRESS TEST-6

GR, GRK & GRS

(JEE ADVANCED PATTERN)

Test Date: 03-09-2017

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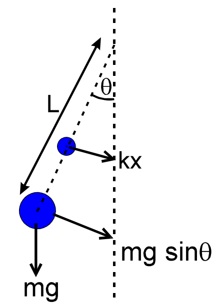
PHYSICS

1. (D)

The extension developed in the string due to small values of 'θ' is :



$$x = h \sin\theta \cong h\theta$$

Torque about 'O' :

$$\tau_0 = (Mg \sin\theta) L + (kx)h$$

or, $\tau_0 \cong mg\,\theta L + kh^2\theta = (mgL + kh^2)\theta$ ⋯ (1)

Also;

$\tau_0 = I_0\alpha = mL^2\alpha$ ⋯ (2)

From (1) and (2) :

$$mL^2\alpha = (mg L + kh^2)\theta$$

or $\alpha = \dfrac{1}{L^2}\left(gL + \dfrac{kh^2}{m}\right)\theta$

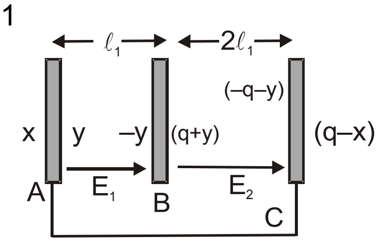
Now : $T = 2\pi\sqrt{\dfrac{\theta}{\alpha}} = 2\pi\sqrt{\dfrac{\theta}{\dfrac{1}{L^2}\left(gL + \dfrac{kh^2}{m}\right)\theta}}$

$\Rightarrow \upsilon = \dfrac{1}{T} = \dfrac{1}{2\pi L}\sqrt{\left(gL + \dfrac{kh^2}{m}\right)}$ Hence (D).

2. (B)

Let the charge on outer surface of plates (1) be x and on the inner (interfacing) surface of plate (1) be y, then charge distribution on other plates is shown in figure (by using charge conservation and E = 0 inside the metallic plates).

For electric field to be zero inside the plate 1



$$\frac{x}{2A\varepsilon_0} = \frac{(q-x)}{2A\varepsilon_0}$$

$\Rightarrow x = \dfrac{q}{2}$ & $q - x = \dfrac{q}{2}$

Since $V_A = V_C$ $\Rightarrow V_A - V_B = V_C - V_B$

$\Rightarrow E_1\,\ell_1 = E_2\,\ell_2$

$\Rightarrow \frac{y}{A\varepsilon_0}\ell_1 = \frac{(-q-y)}{A\varepsilon_0}2\ell_1$

$y\ell_1 = -2q\ell_1 - 2y\ell_1$

$3y\ell_1 = -2q\ell_1$

$y = \frac{-2q}{3}$

So, $-q - y = -q + \frac{2q}{3} = \frac{-q}{3}$

So, Charge on outer surface of plate 1 is $x = \frac{q}{2}$

Charge on inner (interfacing) surface of plate 1 is $y = \frac{-2q}{3}$

Charge on outer surface of plate 2 is $q - x = \frac{q}{2}$

Charge on inner (interfacing) surface of plate 2 is $(-q - y) = \frac{-q}{3}$

3. **(A)**

4. **(C)**

$$\int_5^V dV = -\int \vec{E}\cdot\overline{dr} = -\int_0^x E_x dx - \int_0^y E_y dy$$

$$\Rightarrow V - 5 = -\frac{2x^2}{2} - \frac{3y^2}{2}$$

$$\Rightarrow V = -\frac{2x^2}{2} - \frac{3y^2}{2} + 5.$$

5. **(C)**

6. **(A)**

7. **(C)**

8. **(D)**

$B = \frac{\mu_0 i}{2\pi r}$ $r = \sqrt{2^2 + 3^2 + 6^2} = 7$

$$\therefore B = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{7}{7} = 2 \times 10^{-7} T.$$

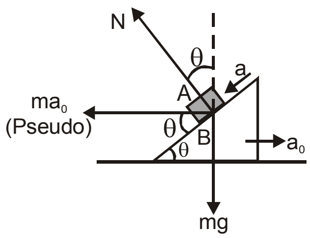
9. **(D)**

10. (A) **11. (C)**

12. (A)

$$E = \frac{KQ}{r^2}, V = \frac{Kv}{r}$$

13. (A,D)



$ma_0 \sin\theta + N = mg \cos\theta \Rightarrow mg \cos\theta \Rightarrow N = mg\cos\theta - ma_0\sin\theta$

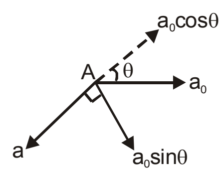
$\Rightarrow N < mg \cos\theta$

Hence, (D) is true.

$ma_0 \cos\theta + mg \sin\theta = ma$

$\Rightarrow a = g \sin\theta + a_0 \cos\theta$

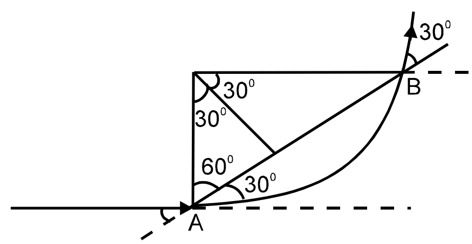
Hence acceleration of A



$= \sqrt{(a - a_0 \cos\theta)^2 + (a_0 \sin\theta)^2} > g\sin\theta$.

14. (B,C)

15. (B,C)



Arc AB $= \frac{\pi}{3} r = \frac{\pi m V}{3qB}$

Time 't' $= \left(\frac{I}{2\pi}\right)\cdot\left(\frac{\pi}{3}\right) = \frac{T}{6} = \frac{\pi m}{3qB}$.

16. (3)

$$\tau_{max} = \sqrt{\frac{5}{2}} mg\ell$$

m : mass of one rod

$$I = \frac{m\ell^2}{12} + m\left(\frac{5\ell^2}{4}\right) + \frac{m\ell^2}{3} = \frac{5}{3} m\ell^2$$

$$\alpha = \frac{3}{\sqrt{10}}\left(\frac{g}{\ell}\right)$$

17. (4)

$$\frac{\mu_0 . i_1}{2\pi(d+r)} = \frac{N \times \mu_0 i_2}{2r}$$

$$\Rightarrow \quad N = \frac{i_1}{i_2} \cdot \frac{r}{\pi(d+r)} = \frac{54}{3.5} \cdot \frac{0.22 \times 7}{22(0.27)} = 4$$

18. (1)

The particle will move along a circle of radius equal to the radius of circular region in which magnetic field is present. (See in fig.)

From $R = \frac{mv}{qB}$; $v = \frac{qBR}{m}$

$$= \frac{5 \times 10^{-6} \times 4 \times (0.1)}{2 \times 10^{-3}} = 10^{-3} \text{ m/s} = 1 \text{ mm/s.}$$

19. (2)

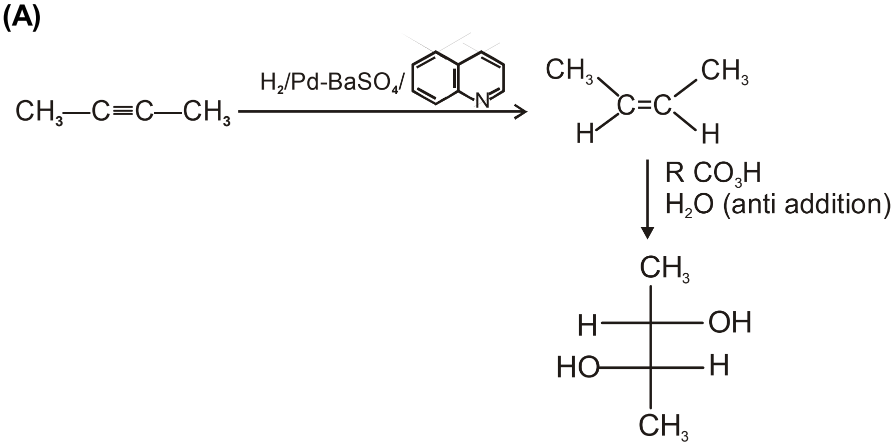
$q = \frac{\Delta\phi}{R}$ $\quad\quad \therefore \quad \Delta\phi = qR$ = area of i-t graph × R

20. (0)

Due to the motion of the loop, there will be an induced current flowing in the circuit, resulting in a force acting on each element of the loop equally & radially. Therefore the net force on the loop is zero.

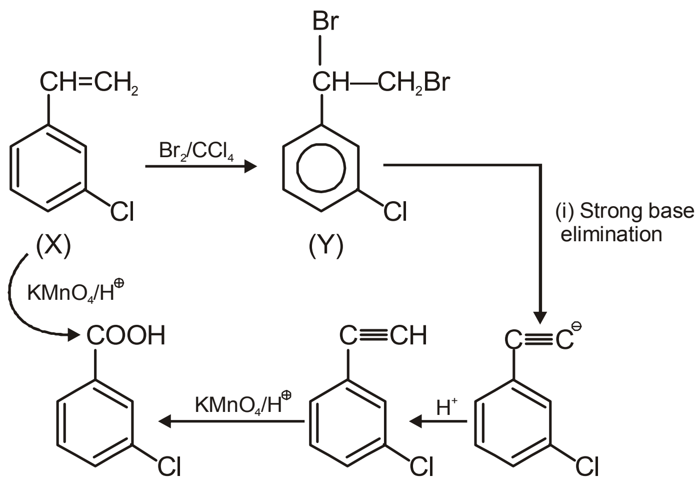
CHEMISTRY

21. (A)



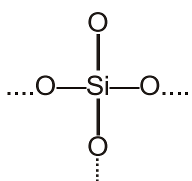
22. (B)

For C_8H_7Cl DU is 5 ie



23. (A)

each silicon atom is surrounded by four oxygen atoms and each oxygen atom is bonded to two silicon atoms



24. (C)

$\Delta T_f = 8^o C = 8k$ in final solution

$\Delta T_f = k_f m$

$$8 = 2 \times \frac{50}{100\, w_1(kg)}$$

or, $w_1(kg) = \frac{1}{8} kg$

or, $W_1 = \frac{1}{8} \times 1000g = 125g$

ice separated = 375 g

25. (C)

$$\xrightarrow[\text{Back bonding}\uparrow \quad \text{Lewis acid}\downarrow]{BI_3 > BBr_3 > BCl_3 > BF_3 = \text{Lewis acidic nature}}$$

26. (B)

$\Delta T_f = 3 = ik_f m$

$3 = i \times 1.86 \times \frac{1}{1.86}$

or i = 3

So, complex is $[Co(NH_3)_5Cl]Cl_2$

27. (C)

$NF_3 < NCl_3 < NBr_3 < NI_3$ Dipole Moment.

28. (D)

$P_{ext} = \pi_{final}$

$\pi = CRT$

3 × 82.1 = C × 0.0821 × 300

or C = 10 M

$M_1V_1 = M_2V_2$

1 × 1 = 10 × V_2

or V_2 = 0.1 litre = 100 ml

water purified = 900 ml

29. (B)

30. (B)

$W = \frac{i \times t}{F} E$

$0.01 \times 2 = \frac{10}{1000} \times \frac{t}{96500} \times 1$

t = 19.3 × 10^4s

31. (B), (C), (D)

32. (A), (B)

33. (B,D)

$$CH_2 = CH - Br \xrightarrow{H^+Br} \underset{\text{More Stable}}{CH_3 - \overset{\oplus}{C}H - Br} \xrightarrow{Br^-} CH_3 - \underset{}{\overset{Br}{\overset{|}{C}H}} - Br$$

$$CH_2=CH-Br \xrightarrow{H^+} CH_3-\overset{\oplus}{C}H-Br \xrightarrow{Br^-} CH_3-CH(Br)-Br$$

More Stable

$$CH_2=CH-COOH \xrightarrow{H^+} \overset{\oplus}{C}H_2-CH_2-COOH \xrightarrow{Br^-} CH_2-CH_2(Br)-COOH$$

34. (A), (B), (C)

$q=0, \quad w=0$

$\Delta U = q + w = 0$

$T_1 = T_2$

$P_1V_1 = P_2V_2$

Note:- $P_2V_2^\gamma = P_1V_1^\gamma$ is for reversible adiabatic process

35. (C), (D)

No of particle = i × c

no of particle are same solution are isotonic

(A) $.1\times1 \neq .1\times2$

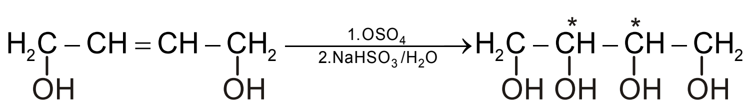
(B) $.1\times1 \neq .1\times3$

(C) $.1\times3 = .1\times3$

(D) $.1\times3 = .1\times3$

C,D have example of isotonic solution

36. (2)



$\overset{*}{C}$=2 (similar)

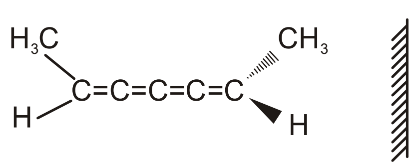
So, optically active compounds = 2

37. (1)

38. (4)

Total number of species are : $N_2^+, O_2, B_2, N_2^{2-}$

39. (2)



40. (3)

Let $\lambda_{HX}^{0.01} = a,\ \lambda_{HY}^{0.1} = 10a$

$\lambda_{HX}^{\infty} = \lambda_{HY}^{\infty} = \lambda^{\infty}$

$\alpha_{HX} = \dfrac{a}{\lambda^{\infty}}$, $\quad\alpha_{HY} = \dfrac{10a}{\lambda^{\infty}}$

$$K_a(HX) = 0.01\times\left(\frac{a}{\lambda^{\infty}}\right)^2$$

$$K_a(HY) = 0.1\left(\frac{10a}{\lambda^{\infty}}\right)^2$$

$$\frac{K_a(HX)}{K_a(HY)} = \frac{1}{1000}$$

$pK_a(HX) - pK_a(HY) = 3$

MATHEMATICS

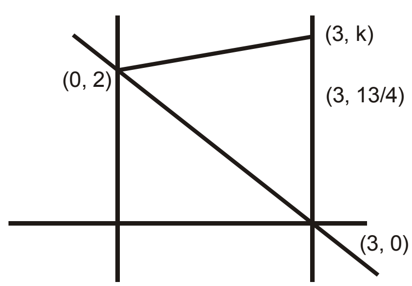
41. (C)

$$\int_0^1 \cos^{-1}\cos[x]dx + \int_1^2 \cos^{-1}(\cos[x])dx + \int_2^3 \cos^{-1}\cos[x]dx + \int_3^4 \cos^{-1}\cos[x]dx + \int_4^5 \cos^{-1}\cos[x]dx$$

$$= \int_0^1 \cos^{-1}(1)dx + \int_1^2 1.dx + \int_2^3 2dx + 3\int_3^4 dx + \int_4^5 (2\pi - 4)dx$$

$= 0 + 1 + 2 + 3 + 2\pi - 4 = 2\pi + 2$

42. (B)



$$3^2 + (2-k)^2 = k^2 \Rightarrow 9 + 4 - 4k + k^2$$

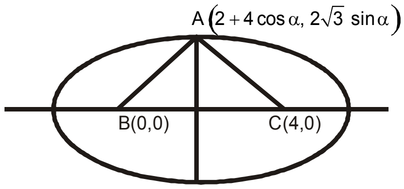
$$\Rightarrow k = \frac{13}{4}$$

$\Rightarrow y-2=\dfrac{\frac{13}{4}-2}{3-0}(x-0)$,

$\Rightarrow$ 5x – 12y = –24

$-\dfrac{5}{24}x+\dfrac{y}{2}=1$

43. **(C)**



On solving equation of an ellipse

$\dfrac{(x-2)^2}{4}+\dfrac{3}{4}y^2=1 \quad \Rightarrow \quad e=\sqrt{\dfrac{2}{3}}$

44. **(A)**

$\dfrac{a^2+b^2}{2}>\sqrt{a^2b^2}$

$a^2+b^2>2ab \Rightarrow b^2+c^2>2bc \Rightarrow c^2+a^2>2ca$

$\Rightarrow a^2+b^2+c^2>ab+bc+ca$

$\Rightarrow ab+bc+ca<1$

45. **(C)**

46. **(B)**

$1-2\sin^2 x+k\sin x=2k-7 \quad \Rightarrow 2\sin^2 x-k\sin x+(2k-8)=0$

$\sin x=\dfrac{k-4}{2}$ or 2

$2\le k\le 6$

47. **(D)**

$\Rightarrow \cos^7 x-\cos^2 x(1+\sin^2 x)=0$

$\Rightarrow \cos^2 x(\cos^5 x+\cos^2 x-2)=0 \quad \Rightarrow \cos x=0$ and $\cos x=1$

48. (C)

$$\sqrt{3h}-\sqrt{3k+2}-1=0$$

On solving we get

$$9x^2+9y^2-18xy-18x+6y+1=0 \qquad \because h^2=ab$$

So locus is parabola

49. (A)

$$I_n=-n\left[I_n-(n-1)I_{n-2}+(n-1)I_n\right]$$

$$\frac{I_3}{I_1}=\frac{3}{5}$$

50. (D)

$$\int_0^{\tan 1}\left[\tan^{-1}x\right]dx+\int_{\tan 1}^{1000}\left[\tan^{-1}x\right]dx=\int_0^{\tan 1}0\cdot dx+\int_{\tan 1}^{1000}1\cdot dx\Rightarrow 1000-\tan 1$$

51. (A, C)

$$\frac{1}{1+x^2}>\frac{1}{1+x^{\frac{\pi}{2}}}>\frac{1}{1+x}$$

52. (A, B)

Put $\sin^2 x=t$

53. (A, D)

54. (A, B, C)

55. (A, D)

56. (0)

$$I=\int_0^{46}(x-1)(x-2).....(x-45)dx$$

$$I=\int_0^{46}(46-x-1)(46-x-2)......(46-x-45)dx$$

$$=\int_0^{46}(45-x)(43-x)......(1-x)dx \quad\Rightarrow I=0$$

57. (1)

$$P = \lim_{n\to\infty}\left(\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdot.....\frac{n}{n}\right)^{\frac{1}{n}}$$

$$\ln p = \lim_{n\to\infty}\frac{1}{n}\sum \ln\frac{r}{n}$$

$$\ln p = \int_0^1 \ln x dx = \left[x(\ln x - 1)\right]_0^1$$

$$\Rightarrow p = e^{-1}; k = -2 \qquad \therefore 2k + 5 = 1$$

58. (2)

$$\int \tan^3 x\left(\sec^2 x - 1\right)dx$$

$$= \quad \int \tan^3 x \cdot \sec^2 x dx - \int \tan^3 x dx \quad \Rightarrow \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \ln\sec x + c$$

59. (1)

$$-1 \le [x] \le 1$$

$$-1 \le x < 2$$

$$-1 \le \cos^{-1}[x] \le 1$$

$$\cos 1 \le x < 2$$

$\therefore$ [x] = 1

Domain [1, 2)

60. (3)

On solving we get

B = 12