SOLUTIONS

PHASE TEST-2

GRA

JEE ADVANCED PATTERN

Test Date: 24-09-2017

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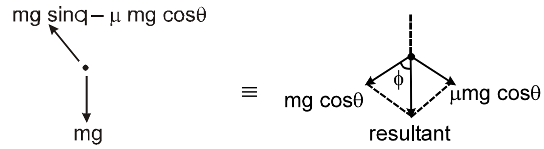
PHYSICS

1. (A)

U_1 will be positive and greatest since all forces among dipoles are repulsive. U_2 is negative as potential energy of first & second dipole pair cancels out potential energy of second and third pair, leaving only potential energy of interaction of first and third, that is negative. In (c), effect of attraction is greatest.

2. (A)

figure shows forces acting on a 'particle' on the surface, with respect to vessel.



(mg sin θ – μ mg cos θ is pseudo force).

$\tan\phi = \mu \qquad \therefore \quad \phi = \tan^{-1}\mu$.

ϕ is angle between normal to the inclined surface and the resultant force. The same angle will be formed between the surface of water & the inclined surface.

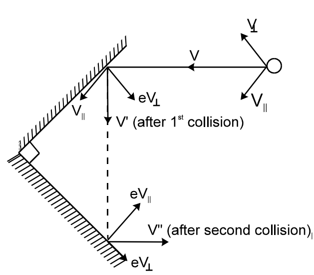
{$\because$ free surface is $\perp$ to the resultant force acting on it.}

3. (B)

$\therefore \quad C_{eq} = 3/2\ \mu F$

Charge flow $\Delta q = C_{eq}\left(10 - \frac{15}{3}\right) = \frac{3}{2} \times 5 = 7.5\ \mu C$.

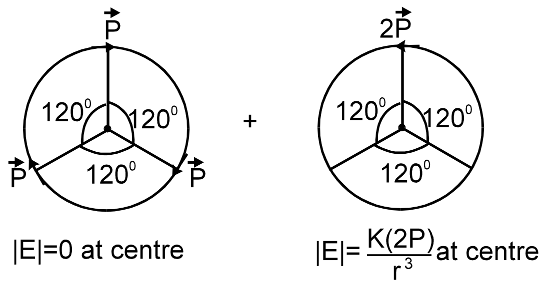
4. (C)



During 1st collision perpendicular component of V, $V_\perp$ becomes e times, while IInd component V_{II} remains unchanged and similarly for second collision. The end result is that both V_{II} and $V_\perp$ becomes e times their initial value and hence V'' = – eV (the (–) sign indicates the reversal of direction).

5. (B)

Given system is equivalent to

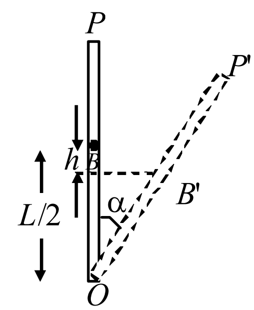


$\therefore \frac{2kp}{R^3}$.

6. (B)

When the rod falls through an angle α the C.G. falls through a height *h*.

In $\Delta OB'B$,



$$\cos\alpha = \frac{\left(\frac{L}{2} - h\right)}{L/2}$$

i.e. $h = \frac{1}{2}(1 - \cos\alpha)$

K.E. rotation = Decrease in P.E.

i.e. $\frac{1}{2}I\omega^2 = mgh$

i.e. $\frac{1}{2}\left(\frac{mL^2}{3}\right)\omega^2 = mg\frac{L}{2}(1 - \cos\alpha)$ or $\omega = \sqrt{\frac{6g}{L}}\sin\frac{\alpha}{2}$

7. (A,D)

(A) Charge on capacitor B decreases as dielectric slab is taken out. Charge from positive plate of B flows towards battery.

Charge on A and B can not be different, as being connected in series.

During the process, battery is being charged.

8. (A,B)

9. (A,C,D)

10. (A,B,C)

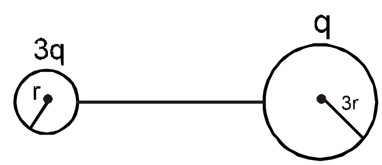
Maximum acceleration block $A = \frac{0.5mg}{m} = \frac{g}{2}$

So, if $M = 2m$, $a_A = a_B = \frac{2mg}{4m} = \frac{g}{2}$ and friction force is $\frac{1}{2}mg$.

11. (B,D)

Let charge on smaller sphere be x and on larger sphere be 4q – x

force between them is given by $F = \frac{kx(4q-x)}{d^2}$



$\frac{dF}{dx} = 0 \Rightarrow 4q - 2x = 0 \Rightarrow x = 2q$

$\frac{d^2F}{dx^2} = \frac{K}{d^2}(-2) < 0$

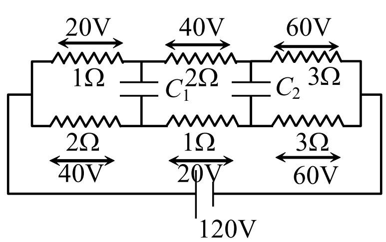
$\therefore$ it represents a maximum.

Final charges on the smaller sphere and the larger sphere are q & 3q respectively as required by the equality of potentials

$\therefore$ force will increase until the charges become equal and after that force will decrease.

12. (B,D)

Now, potential difference across C_1 is 20 V and across C_2 is zero.



$\therefore$ charge stored in C_1 is 40 μC and in C_2 is zero.

13. (B, D)

R_{eq} = 400 Ω, $I = \frac{100}{400} = \frac{1}{4} A$

Potential difference across voltmeter $= \frac{1}{4} \times 200\Omega$ = 50V

14. (A,B,C) **15. (A)** **16. (D)** **17. (A)**

18. (D)

CHEMISTRY

19. (B)

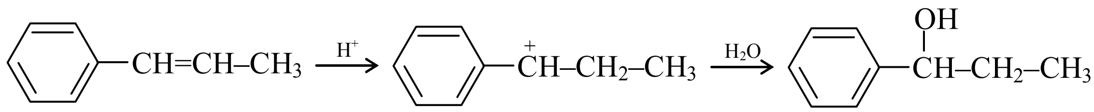
Degree of dissociation

$$\alpha = \frac{(\Lambda_M^c)}{(\Lambda_M^o)} = \frac{3.9}{390} = 0.01$$

$$K_a = \frac{[H^+][A^-]}{[HA]} = \frac{c\alpha.c\alpha}{c - c\alpha} = \frac{c\alpha^2}{1-\alpha} \approx c\alpha^2 = 10^{-6}$$

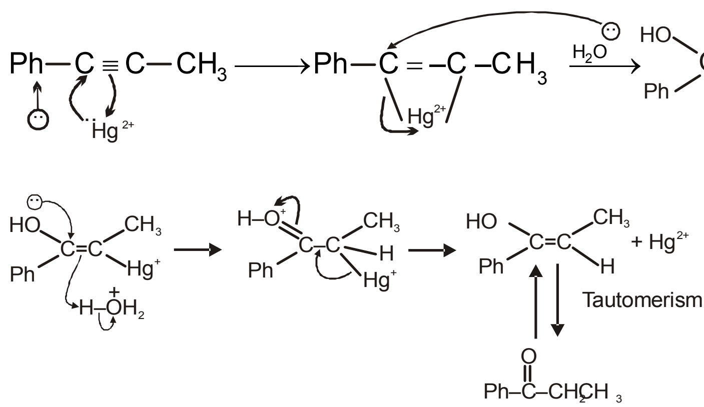
$p^{ka} = 6$

20. (B)



21. (B)

22. (A)



23. (A)

24. (D)

$\underset{A}{C \equiv C}$ $\quad C = \underset{B}{C - C} \equiv C$ $\quad C = \underset{C}{C - C} = C$

25. (A,D)

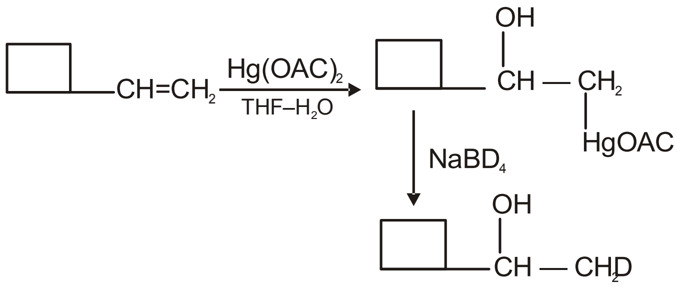
$$P = \frac{A}{Z}$$

When P > 1 experimentally determined value is higher than the predicted value by Arrhenius

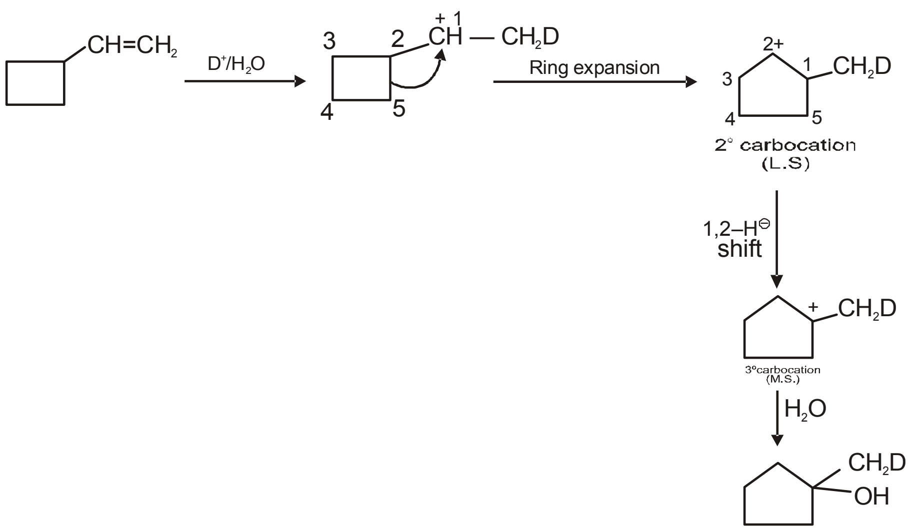
P < < 1, use of catalyst is required.

P > 1. no need to add catalyst. Activation energy can be experimetally calculated by eliminating steric factor.

26. (B,D)



In oxymercuration-demercuration the rearrangement of carbon skeleton doesnot involved.



In acid catalysed-hydration the rearrangement of carbon skeleton involve.

27. (A, B, C, D)

$$\text{RLVP} = \frac{20}{760} = \frac{1}{38} \text{ Ans. (D)}$$

$$\text{Also, } \frac{P^0 - P_s}{P_s} = \frac{n}{N} \Rightarrow \frac{20}{740} = \frac{1}{N} \Rightarrow N = 37 \text{ mol}$$

$\therefore$ No. of moles of ice separated = (200 – 37) = 163 moles Ans. (A)

$$\text{For original solution : } \Delta T_f = 2 \times \frac{1 \times 1000}{200 \times 18} = \left(\frac{10}{18}\right) K = \left(\frac{10}{18}\right) {}^\circ C$$

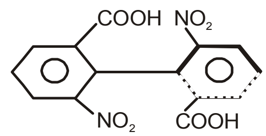
$$\therefore \text{ Freezing point = } 0^\circ C - \left(\frac{10}{18}\right) {}^\circ C = -\left(\frac{10}{18}\right) {}^\circ C \text{ Ans. (C)}$$

28. (B), (C)

29. (A), (C)

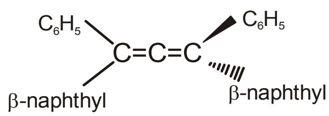
30. (A), (C)

31. (A, C)



Chiral i.e. why it show optical isomerism

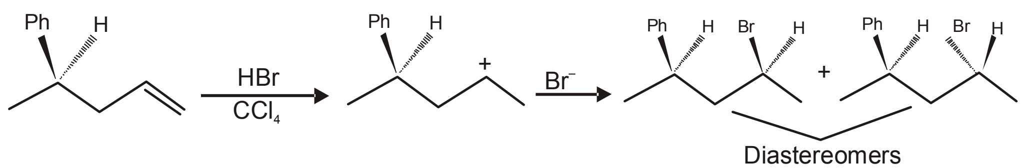
(C)



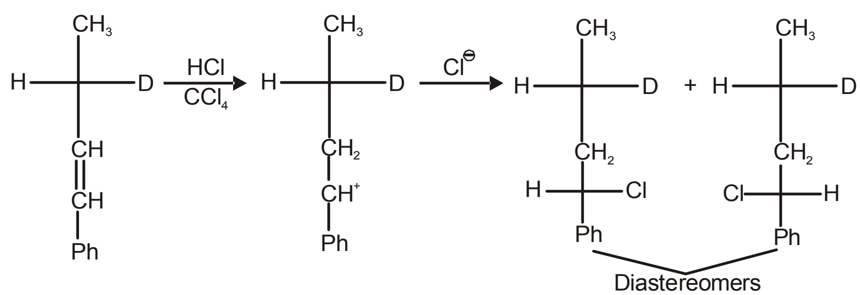
Chiral i.e. why it show optical isomerism.

32. (B), (D)

(B)



(D)



33. (D)

In the given solution 'M', H_2O is solute.

Therefore, molality of H_2O = $\frac{0.1}{0.9 \times 46} \times 1000 = 2.4$

$\Rightarrow \quad \Delta T_f = k_f^{ethanol} \times 2.4 = 2 \times 2.4 = 4.8$

$\Rightarrow \quad T_f = 155.7 - 4.8 = 150.9$ K

34. (B)

Now ethanol is solute.

Molality of solute = $\frac{0.1}{0.9 \times 18} \times 1000 = 6.17$

$\Rightarrow \quad \Delta T_b = 6.17 \times 0.52 = 3.20$

$\Rightarrow \quad T_b = 373 + 3.2 = 376.2$ K

35. (B)

36. (C)

Stability of C^+.

MATHEMATICS

37. (A)

$$\int_0^3 (3x-x^2)dx = \left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^3 = \left[\frac{27}{2}-9\right] = \frac{9}{2}$$

38. (A)

Family of parabolas is $y^2 = \alpha(x-\beta)$

$\Rightarrow 2yy' = \alpha \Rightarrow (y')^2 + yy'' = 0$

order $\rightarrow 2$, degree $\rightarrow 1$

39. (D)

Let $g(x) = f^{-1}(x)$; $f\left(\frac{\pi}{2}\right) = \pi \Rightarrow f^{-1}(\pi) = \frac{\pi}{2}$

$f'(x) = 6(2x-\pi)^2 + 2 + \sin x \Rightarrow f'\left(\frac{\pi}{2}\right) = 3$

Also $g(\pi) = \frac{\pi}{2}$

Now f(g(x)) = x $\Rightarrow f'(g(x)).g'(x) = 1$

$\Rightarrow f'(g(\pi)).g'(\pi) = 1 \Rightarrow f'\left(\frac{\pi}{2}\right)\cdot g'(\pi) = 1 \Rightarrow 3g'(\pi) = 1 \Rightarrow g'(\pi) = \frac{1}{3}$

40 (D)

$$I = \int \frac{x^2+2}{x^4-x^2+4} = \int \frac{1+\frac{2}{x^2}}{x^2+\frac{4}{x^2}-1}dx$$

say $x - \frac{2}{x} = t \Rightarrow \left(1+\frac{2}{x^2}\right)dx = dt$

$$\Rightarrow I = \int \frac{dt}{t^2+3} = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + c = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-2}{\sqrt{3}x}\right) + c.$$

41. (C)

The given expression can be written as 4 sin3x (cos3x – sin3x) + 5

= 2 sin6x + 5 – 4 $\sin^2 3x$ = 2 (sin6x + cos6x) + 4

Hence minimum value = $3-2\sqrt{2}$

42. (C)

We have $\lim\limits_{n\to\infty}\dfrac{3n\cdot 4^{2n}}{3n(x-3)^{2n}+3n\cdot 4^{2n+1}-4^{2n}}=\dfrac{1}{4}$;

So $\lim\limits_{n\to\infty}\dfrac{1}{\left(\dfrac{x-3}{4}\right)^{2n}+4-\dfrac{1}{3n}}=\dfrac{1}{4}$

Clearly $-1<\dfrac{x-3}{4}<1$ $\Rightarrow$ $-1<x<7$

$\therefore$ Possible integers in the range 'x' are 0, 1, 2, 3, 4, 5, 6 $\Rightarrow$ 7 integers

43. (A, B)

Normal is y = mx – 2am – am^3 passes through (5a, 2a)

$\Rightarrow am^3-3am+2a=0 \Rightarrow m^3-3m+2=0, (m-1)(m^2+m-2)=0$

$\Rightarrow m=1,-2 \Rightarrow$ normals are $y=x-3a$ and $y=-2x+12a$

44. (A, B, C, D)

45. (A,B,D)

We have $f(x)=\cos^{-1}(-\{-x\})$

$D_f = R$

As $0 \le \{-x\} < 1 \ \forall x \in R$

$\Rightarrow -1 < -\{-x\} \le 0$

So $R_f = \left[\dfrac{\pi}{2}, \pi\right)$

Clearly, f is neither even nor odd.

But f (x + 1) = f (x) $\Rightarrow$ f is periodic with period 1.

46. (B, C)

From given

$\sum\limits_{i=1}^{2p}\sin^{-1}x_i=-(2p)\dfrac{\pi}{2}$ $p\in N$ $\Rightarrow$ $\sin^{-1}x_i=-\dfrac{\pi}{2}$ $\forall i$ $\Rightarrow$ $x_i=-1\ \forall i$

So, (B) and (C) are true

47. (B, D)

For (A) Put $\sqrt{3}x = y$, we get $\int_0^\infty e^{-3x^2}dx = \frac{\sqrt{\pi}}{2\sqrt{3}}$

For (B) $\int_0^\infty xe^{-x^2}dx = \left|-\frac{1}{2}e^{-x^2}\right|_0^\infty = \frac{1}{2}$

But $\int_0^\infty x^2e^{-x^2}dx = \left|x\left(-\frac{1}{2}e^{-x^2}\right)\right|_0^\infty + \frac{1}{2}\int_0^\infty e^{-x^2}dx \;= \frac{\sqrt{\pi}}{4}$

48. (B, C)

$4a^2 + b^2 = 4c^2 + 4ab \Rightarrow 4a^2 + b^2 - 4ab = 4c^2 \Rightarrow (2a-b)^2 = 4c^2$

$\Rightarrow 2a - b - 2c = 0,\ 2a - b + 2c = 0$

Take 2a – b – 2c = 0 the 2ax + by + 2c = 0

$\Rightarrow\ 2ax + by + (2a - b) = 0$

2a (x + 1) + b(y - 1) = 0

$\Rightarrow\ y - 1 = \lambda(x+1)$

Hence differential equation of the family is $y - 1 = y'(x+1)$

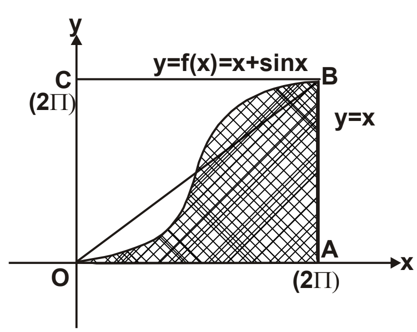
$\Rightarrow$ orthogonal trajectory is $(x+1)^2 + (y-1)^2 = \alpha$

Also for 2a – b + 2c = 0 orthogonal trajectory $(x-1)^2 + (y+1)^2 = \beta$, where α and β are parameters.

49. (B, C)

50. (B, C)

The required area is equivalent to the area bounded by f(x) with x-axis from x = 0 to $x = 2\pi$.



Thus Required Area = $\int_0^{2\pi} f(x)dx \;= \int_0^{2\pi}(\sin x + x)dx = \left[-\cos x + \frac{x^2}{2}\right]_0^{2\pi}$ = $2\pi^2$sq.units

51. **(B)**

52. **(C)**

$$I_n = \int_0^{\pi/4} \tan^{n-2} x(\sec^2 x - 1)\,dx = \int_0^1 t^{n-2} dx - I_{n-2}$$

$$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1} \Rightarrow I_{n+1} + I_{n-1} = \frac{1}{n}$$

$$\because I_n < I_{n-2} \Rightarrow 2I_n < I_n + I_{n-2} = \frac{1}{n-1}$$

Also, $I_n > I_{n+2} \Rightarrow 2I_n > I_n + I_{n+2} = \dfrac{1}{n+1}$

Hence $\dfrac{1}{n+1} < 2I_n < \dfrac{1}{n-1}$

53. **(C)**

54. **(B)**

$$y = vx \Rightarrow v + x\frac{dv}{dx} = v + \tan v$$

$$\Rightarrow \cot v\,dv = \frac{dx}{x} \Rightarrow \ell n(\sin v) = \ell n(x) + \ell n(k)$$

$$\Rightarrow \sin v = kx \Rightarrow y = x\sin^{-1}(kx)$$

putting x = 1, y = $\pi/2$ we have k = 1

$\Rightarrow$ Solution is y = x $\sin^{-1} x$