SOLUTIONS

PHASE TEST-1

CD-1801(α) & CD-1801(β)

(JEE ADVANCED PATTERN)

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PHYSICS

1. (A)

$$\frac{1}{2}(100)\left(\frac{5}{100}\right)^2 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{5}{2}}; \qquad x = v\sqrt{\frac{2h}{g}} = 1\,\text{m}$$

2. (A)

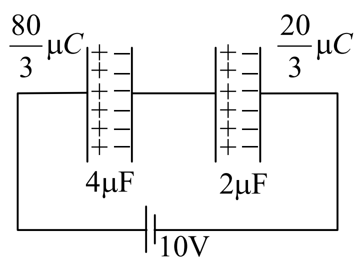
Work done = $-mgH_{\max}$ = $-mg\frac{u^2\sin^2\theta}{2g}$ = $\frac{-mu^2\sin^2\theta}{2}$

3. (D)

$w = \int_0^5 F.dx = \int_0^5 (7-2x+3x^2)dx$ = 135 J

4. (B)

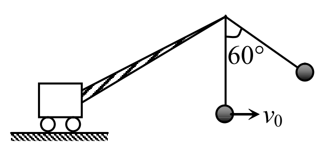
Using Kirchhoff's loop law and conservation of charge, final distribution of charge on the capacitors will be as shown in the figure.



Charge q flown through the battery = charge on 2μF capacitor and work done by the battery $=$ qV

5. (B)

By conservation of energy



$$mgl(1-\cos 60°) = \frac{1}{2}mv_0^2$$

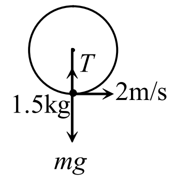
$v_0 = 7$ m/s

6. (B)

$V_0 + 10 - 5 + 10 = V_x \Rightarrow V_x = 15$ V

7. (A)

The maximum tension will be at the lowest point.



$$T - mg = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r} + mg = \frac{1.5 \times 4}{0.5} + 1.5 \times 10 = \mathbf{27\ N}$$

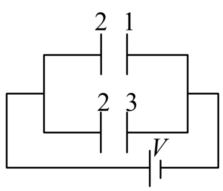
8. (A)

Its velocity becomes $\frac{v_0}{2}$ under a retardation of μg in time t_0.

$\therefore \quad \frac{v_0}{2} = v_0 - \mu g t_0$ or $\mu g t_0 = \frac{v_0}{2}$ or $\mu = \frac{v_0}{2gt_0}$

9. (B)

Both the capacitor are in parallel



Charge on plate (2) initially

$= \frac{2\varepsilon_0 A}{d} V$

10. (C)

Charge on plate (2) finally = $\left(\frac{\varepsilon_0 A}{3d/2} + \frac{\varepsilon_0 A}{d/2}\right)V = \frac{8\varepsilon_0 A}{3d} V$

11. (A)

$x = A\sin\omega t$, $y = A\cos\omega t$

$x = A\sqrt{1-\cos^2\omega t}$

$x = A\sqrt{1-\frac{y^2}{A^2}}$

$x = \sqrt{1-y^2}$

$x^2 = 1 - y^2$, $x^2 + y^2 = 1$ this equation of circle.

12. (A)

$$H_1 = \frac{v^2 \sin^2 \theta_1}{g}, \quad H_2 = \frac{v^2 \sin^2 \theta_2}{g}$$

$$\frac{H_1}{H_2} = \frac{\sin^2 \theta_1}{\sin^2 \theta_2}, \quad \theta_1 + \theta_2 = \pi_2 = \frac{\sin^2 \theta_1}{\cos^2 \theta_1} = \tan^2 \theta_1$$

13. (D)

When switch in position 1 and steady state is reached

$q_0 = CE_1 =$ 10 C

W_1 = Energy supplied by battery = q_0E_1 = 1000 J

Energy stored on the capacitor $U_1 = \frac{q_0^2}{2C}$ = 500 J

$H_1 = W_1 - U_1$ = 500 J

$H_{99\Omega} = \frac{99}{100} \times 500$ = 495 J

14. Position (2), in steady state

q =5C

$W_2 = E_2(q_0 - q)$ = 250 J

∴ **(A)**

15. (B) and (D)

Velocity of *A* with respect to river $= \frac{10}{120}$ m/s = $\frac{1}{12}$ m/s

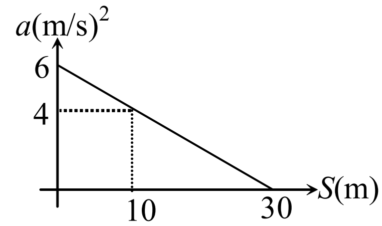
Flow velocity $= \frac{30}{120} = \frac{1}{4}$ m/s

Velocity of *B* along *y*-axis with respect to earth $= \frac{10}{120}$ m/s $= \frac{1}{12}$ m/s

Velocity of *B* along *x*-axis with respect to earth $= \frac{30-5}{120}$ m/s $= \frac{5}{24}$ m/s

16. **(B) and (C)**

Area $= \frac{1}{2} \times 10 \times (6+4) = \frac{v^2}{2}$



v = 10 m/s

Area upto 30 m $= \frac{1}{2} \times 30 \times 6 = \frac{v^2}{2}$

$v^2 = 180$

$v_{max} = \sqrt{180} < 14$

17. Let V_D be the potential of D, then

$$\frac{V_A - V_D}{10} + \frac{V_B - V_D}{20} + \frac{V_C - V_D}{30} = 0 \Rightarrow V_D = 40 \text{ V}$$

Also, ratio of current in *AD*, *DB* and *DC* are $\frac{70-40}{10} : \frac{40}{20} : \frac{40-10}{30}$

i.e. 3 : 2 : 1

Also total power network draws, $P = \sum I^2 R = 200$ W

$\therefore$ **(A) (B) (D)**

18. Current across *NP*, $I_{NP} \times 10 = 20 \times 1$ or $I_{NP} = 2A$

Across *MP*, $0.5R_1 = 20$ or $R_1 = 40\Omega$

Total current $= 2 + 0.5 + 1.0 = 3.5\Omega$

$3.5 = \frac{69}{R + 40/4}$ yields $R = 4\Omega$

$\therefore$ **(B) (C) and (D)**

19. Electric potential increased continuously but electric field strength decreases in dielectric but direction is same from $x = 0$ to $x = 3d$.

$\therefore$ **(B) (C)**

20. **(B) and (D)**

As work done by the force is zero.

CHEMISTRY

21. (A)

22. (D)

k > Mg > Al >B

Metallic character $\alpha \frac{1}{IP}$

23. (B)

Within a period, the oxidising charactor increases from left to right, therefore among F,O and nitrogen oxidising power decreases in the order F > O > N. However within a group oxidising power decreases from top to bottom. Thus, fluorine is more oxidising aent than CL. Further because 'O' is more eletronegative than Cl, therefore oxygen is more oxidising agent than CL.

Order of oxidising property =F > O > Cl > N

24. (C)

$$k = \frac{2.303}{2\times10^4}\log_{10}\frac{800}{50}$$

or, $k = 1.386 \times 10^{-4}$

25. (C)

He is inert gas.

26. (A)

Molecular weight of naphthoic acid

$C_{11}H_8O_2$ = 172 g/mol

The theoretical value of depression in freezing point = $K_f \frac{W_B}{M_B}\times\frac{1000}{W_A} = 1.72\times\frac{20\times1000}{172\times50} = 4K$

Van't Hoff factor :

$$i = \frac{\text{Observed value of colligative property}}{\text{Theoretical value of colligative peoperty}} = \frac{2}{4} = 0.5$$

27. (A)

28. (C)

Put $x_A = 0 \quad P_B^0 = 254$

Put $x_A = 1 \quad P_A^0 = 135$

29. (A)

30. (B)

31. (D)

32. (B)

33. (D)

34. (D)

35. (C), (D)

No of particle = i × c

no of particle are same solution are isotonic

(A) $.1 \times 1 \neq .1 \times 2$

(B) $.1 \times 1 \neq .1 \times 3$

(C) $.1 \times 3 = .1 \times 3$

(D) $.1 \times 3 = .1 \times 3$

(C), (D) have example of isotoric solution

36. (A), (B), (D)

37. (A,B,C)

38. (A,B,C)

39. (A), (B), (D)

(A) $MgC_2O_4 < CaC_2O_4 < SrC_2O_4 < BaC_2O_4 < BeC_2O_4$

(B) LiI > KI > RbI > CsI

(C) $LiClO_4 > NaClO_4 > KClO_4 > RbClO_4 > CsClO_4$

(D) $BeCO_3 > MgCO_3 > CaCO_3 > SrCO_3 > BaCO_3$

40. (A,B,C)

MATHEMATICS

41. (C)

For continuity at x = 1; a + b = 3 –4 + 1 $\Rightarrow$ a + b = 0 ... (i)

For differentiability at x = 1; a = $\left(6x - \frac{4}{2\sqrt{x}}\right)_{x=1}$ $\Rightarrow$ a = 4 ... (ii)

From (i) and (ii); a = 4, b = – 4

42. (B)

$\{x\} \in [0, 1)$

$$\Rightarrow \sin\{x\} \in [0, \sin 1) \Rightarrow \frac{1}{\sin\{x\}} \in \left(\frac{1}{\sin 1}, \infty\right) \quad \left\{\because \sin 1 < 1; \Rightarrow \frac{1}{\sin 1} > 1\right\}$$

$$\therefore \left[\frac{1}{\sin\{x\}}\right] \in \{1, 2, 3, ...\}$$

43. (B)

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{f(h)}{h}$$

and $\because$ $f(0)=0 \quad \Rightarrow \quad f'(x) = \lim_{h\to 0}\left(\frac{f(h)-f(0)}{h}\right)$

$\Rightarrow \quad f'(x) = f'(0) \quad \Rightarrow \quad f(x) = 2x$

44. (A)

$$\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}} = e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left(\frac{1+5x^2}{1+3x^2}-1\right)} = e^{\lim\limits_{x\to 0}\frac{1}{x^2}\cdot\frac{2x^2}{1+3x^2}} = e^2$$

45. (D)

$$A = \lim_{x\to 0}\frac{\int_0^{x^2}\left(\tan^{-1}t\right)^2 dt}{\int_0^{x^4}\left(\sin\sqrt{t}\right)dt}$$

Applying, *L*' Hospital's rule

$$A = \lim_{x\to 0}\frac{\left(\frac{d(x^2)}{dx}.(\tan^{-1}x^2)^2\right)}{\frac{d}{dx}(x^4).\sin(\sqrt{x^4})} = \lim_{x\to 0}\frac{2x\{\tan^{-1}(x^2)\}^2}{4x^3\sin x^2}$$

$$= \lim_{x\to 0}\frac{1}{2}.\left\{\frac{\tan^{-1}(x^2)}{x^2}\right\}^2.\frac{x^2}{\sin(x^2)} \Rightarrow A = \frac{1}{2}$$

46. (B)

$$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2}-1\right) + \log(\cos x)$$

f(x) is defined if $-1 \le \left(\frac{x}{2}-1\right) \le 1$ and cos x > 0

or $\quad 0 \le \frac{x}{2} \le 2$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

or $\quad 0 \le x \le 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

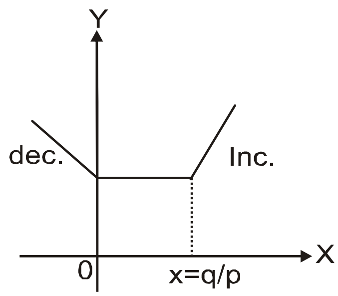
$\therefore \quad x \in \left[0, \frac{\pi}{2}\right).$

47. (C)

f(x) = |px – q| +r |x|

$$= \begin{cases} -px + q - rx, & x \le 0 \\ -px + q + rx, & 0 < x \le q/p \\ px - q + rx, & q/p < x \end{cases}$$

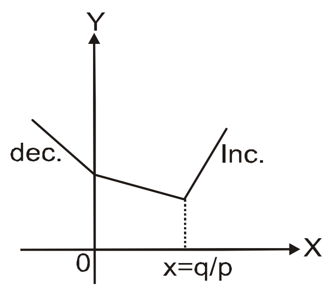
Case I : r = p



(i)

From graph (i) infinite many points for min value of f(x)

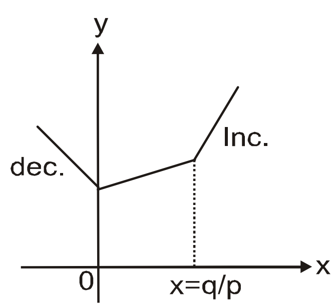
Case II : r < p



(ii)

From graph (ii) only pt. of min of f(x) at x = q/p

Case III : r >p,



(iii)

From graph (iii) only one pt. of min of f (x) at x = 0

48. **(A)**

$0 \le \sqrt{x^2+x+1} \le 1$ and $\sqrt{x^2+x} \ge 1$

$\Rightarrow x^2+x+1 \le 1$ and $x^2+x \ge 1$

$\Rightarrow x^2+x \le 0$ and $x^2+x \ge 1$

$\Rightarrow x \in \phi$

49. **(A)**

$f(x) = (x-1)^2 - 2, a = 1, b = -2$

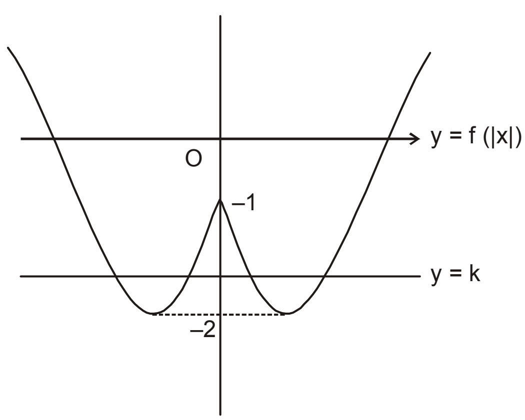
$f : [1, \infty) \to [-2, \infty)$

then $f^{-1} : [-2, \infty) \to [1, \infty)$, $f(x) = y \Rightarrow x^2 - 2x - (1+y) = 0$

$\therefore x = \dfrac{2 \pm \sqrt{4+4(1+y)}}{2}$, $\quad x = 1 \pm \sqrt{2+y}$

$\because f^{-1}(y) = 1 + \sqrt{2+y} \Rightarrow f^{-1}(x) = 1 + \sqrt{2+x}$

50. **(A)**



For f (|x|) = k to be four distinct solutions, $k \in (-2, -1)$

51. **(C)**

52. **(A)**

From the given relations, we see that θ and ϕ are the roots of the equation

$x\cos\alpha + y\sin\alpha = 2a$

$\Rightarrow (x\cos\alpha - 2a) = -y\sin\alpha$

$\Rightarrow (x\cos\alpha - 2a)^2 = y^2\sin^2\alpha = y^2(1-\cos^2\alpha)$

$\Rightarrow (x^2+y^2)\cdot\cos^2\alpha - 4ax\cos\alpha + 4a^2 - y^2 = 0$

Which is quadratic in $\cos\alpha$ with roots $\cos\theta$ and $\cos\phi$

$\because \cos\theta + \cos\phi = \dfrac{4ax}{x^2+y^2}$

But $2\sin\dfrac{\theta}{2}.\sin\dfrac{\phi}{2} = 1$ (given)

$\Rightarrow 4\sin^2\dfrac{\theta}{2}\cdot\sin^2\dfrac{\phi}{2} = 1$ $\Rightarrow (1-\cos\theta)(1-\cos\phi) = 1$

$\Rightarrow \cos\theta + \cos\phi = \cos\theta\cdot\cos\phi$

$\Rightarrow \dfrac{4ax}{x^2+y^2} = \dfrac{4a^2-y^2}{x^2+y^2}$ $\Rightarrow 4ax = 4a^2 - y^2 \Rightarrow y^2 = 4a(a-x)$

53. (D)

f(0) = –3, f(1) = 1, f(2) = –1 and f(3) = 3.

So f(x) = 0 has one root in each of the intervals (0, 1), (1, 2) and (2, 3) and hence no root in (3, 4).

54. (C)

Let $\alpha \le \beta \le \gamma$, then

$\alpha \in (0, 1)$, $\beta \in (1, 2)$, $\gamma \in (2, 3)$

$\therefore$ $[\alpha] = 0$, $[\beta] = 1$ and $[\gamma] = 2$

$\therefore$ $\{\alpha\} + \{\beta\} + \{\gamma\} = (\alpha+\beta+\gamma) - ([\alpha]+[\beta]+[\gamma]) = \dfrac{9}{2} - (0+1+2) = \dfrac{3}{2}$.

55. (B, D)

Let $I = (\sec^{-1}x)^2 + (\operatorname{cosec}^{-1}x)^2 = (\sec^{-1}x + \operatorname{cosec}^{-1}x)^2 - 2\sec^{-1}x.\operatorname{cosec}^{-1}x$

$= \dfrac{\pi^2}{4} - 2\sec^{-1}x\left(\dfrac{\pi}{2} - \sec^{-1}x\right) = \dfrac{\pi^2}{4} + 2\left(\sec^{-1}x\right)^2 - \pi\sec^{-1}x$

$= \dfrac{\pi^2}{4} + 2\left[\left(\sec^{-1}x\right)^2 - 2\dfrac{\pi}{4}\sec^{-1}x + \left(\dfrac{\pi}{4}\right)^2\right] - \dfrac{\pi^2}{8} = 2\left(\sec^{-1}x - \dfrac{\pi}{4}\right)^2 + \dfrac{\pi^2}{8} \Rightarrow I \ge \dfrac{\pi^2}{8}$

and maximum value is $\dfrac{5\pi^2}{4}$

56. **(A, D)**

$[x]^2 + [x+1] - 3 = 0 \Rightarrow [x]^2 + [x] - 2 = 0$

$\Rightarrow ([x]+2)([x]-1) = 0 \Rightarrow [x] = -2, 1 \Rightarrow x \in [-2,-1) \cup [1,2)$

Also, domain of $f(x) = \log_{\left[x+\frac{1}{2}\right]}(2x^2 + x - 1)$ is $\left[\frac{3}{2}, \infty\right)$

Hence, Answer is $[-2,-1) \cup \left[1, \frac{3}{2}\right)$.

57. **(B, D)**

f(x) is continuous at the points where $x^2 = x^3$. i.e., x = 0 or 1.

Now $f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h)}{h} = 0$ also f is not differentiable at x = 1.

58. **(A,B,C,D)**

Let α be a solution of $\sin^{-1} x = 2 \sin^{-1} a$. Then, $\sin^{-1}\alpha = 2\sin^{-1} a$

$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1} a \le \frac{\pi}{2}$ $\left[\because -\frac{\pi}{2} \le \sin^{-1}\alpha \le \frac{\pi}{2}\right]$

$\Rightarrow -\frac{\pi}{4} \le \sin^{-1} a \le \frac{\pi}{4}$

$\Rightarrow -\frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}} \Rightarrow |a| \le \frac{1}{\sqrt{2}}$

59. **(D)**

Given limit $= 0 + (2^2 - 1) + (3^2 - 1) + ... + (10^2 - 1)$

$$= \sum_{n=2}^{10} (n^2 - 1) = \frac{10 \times 11 \times 21}{6} - 1 - 9 = 385 - 10 = 375$$

$$\frac{\cos\alpha}{\sin\alpha + \sin 3\alpha} = \frac{1}{4\sin\alpha\cos\alpha} = \frac{1 + \tan^2\alpha}{4\tan\alpha} = \frac{1+\alpha^2}{4\alpha}$$

60. **(A, B,C)**

(A) $\cos(\tan^{-1}(\tan(4 - \pi))) = \cos(4 - \pi) = \cos(\pi - 4) = -\cos 4 > 0$

(B) $\sin(\cot^{-1}(\cot(4-\pi))) = \sin(4 - \pi) = -\sin 4 > 0$ (as $\sin 4 < 0$)

(C) $\tan(\cos^{-1}(\cos(2\pi - 5))) = \tan(2\pi - 5) = -\tan 5 > 0$ (as $\tan 5 < 0$)

(D) $\cot(\sin^{-1}(\sin(\pi - 4))) = \cot(\pi - 4) = -\cot 4 < 0$