SOLUTIONS

WEEKLY TEST-10

GRA

(JEE ADVANCED PATTERN)

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PHYSICS

1. (A, D)

Increasing the accelerating voltage means increasing speed of the electron, thereby decreasing time spent between the plates. It will reduce X.

Increasing deflecting voltage means increasing electric field between the plates, making acceleration of electron greater.

2. (A,D)

3. (A, B)

4. (C)

$$C = \frac{\varepsilon_0 A}{d + \frac{1}{2}at^2}$$

charge on capacitor

$$q = \frac{\varepsilon_0 VA}{d + \frac{1}{2}at^2}.$$

5. (B)

$$H = \frac{C}{2}V_2^2.$$

6. (A, B, D)

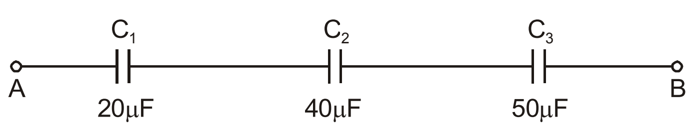
When k_2 is closed.

$$C_{eq} = \frac{5C}{3}$$

$$\therefore \quad U_2 = \frac{1}{2}\left(\frac{5C}{3}\right)V^2 = \frac{5CV^2}{6}.$$

7. (B, C, D)

All capacitors are in series, therefore charge on each capacitor will be same, when connected across a potential source. The capacitor with minimum capacitance will have maximum potential difference across it (evident from $V = \frac{Q}{C}$; Q being same for all). So 100 V potential difference can exist across C_1, then charge on it would be 100 V × 20 μF = 2000μC.



2000μC	2000μC	2000μC
V_1 = 100 V	V_2 = 50 V	V_3 = 40 V
$E_1 = \frac{Q^2}{C_1}$	$E_2 = \frac{Q^2}{C_2}$	$E_3 = \frac{Q^2}{C_3}$

$E = E_1 + E_2 + E_3 = 190$ mJ.

8. **(C,D)**

9. **(C)** $C = \frac{\in_0 (L-y)L}{L} + \frac{k \in_0 yL}{L} = \in_0 (L - y + ky) = \frac{\in_0}{9}(t^2 - 54t + 810)$.

10. **(B)** $q = C\varepsilon$, $i = -\frac{dq}{dt}$, $i = 9 \in_0 (54 - 2t)$.

11. **(A)**

12. **(A)**

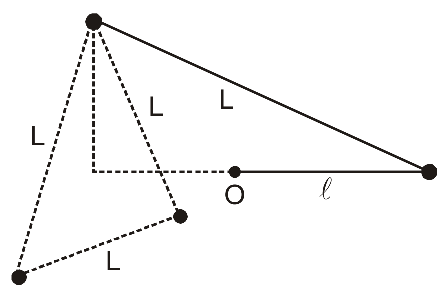
13. **(B)**

In the final state the four charges will lie at the vertices of a tetrahedron with O at geometric centre.

At this state distance between any two charge, will be

$$L = \frac{2\sqrt{2}}{\sqrt{3}}\ell$$

Now final potential energy of system will be



$$U_f = \frac{kq^2}{L} \times 3 \times 4 \times \frac{1}{2} = \frac{6kq^2}{L} = \frac{6kq^2}{\frac{2\sqrt{2}}{\sqrt{3}}\ell} = \frac{3\sqrt{3}kq^2}{\sqrt{2}\ell}$$

$$U_i = \frac{kq^2}{\ell} \times 2 \times 3 \times \frac{1}{2} = \frac{3kq^2}{\ell}$$

∴ work done

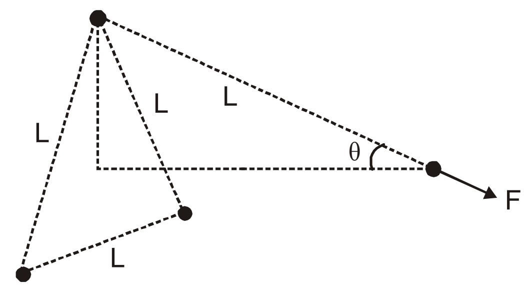
$$W_{ext} = U_f - U_i = \frac{3kq^2}{\ell}\left[\frac{\sqrt{3}}{\sqrt{2}} - 1\right]$$

14. (C)

$$T = 3F\cos\theta$$

$$= 3\frac{kq^2}{L^2}\frac{\sqrt{2}}{\sqrt{3}}$$

$$= \frac{3kq^2}{\left(\frac{2\sqrt{2}}{\sqrt{3}}\ell\right)^2}\frac{\sqrt{2}}{\sqrt{3}} = \frac{3\sqrt{3}}{4\sqrt{2}}\frac{kq^2}{\ell^2}$$



15. (B)

16. (D)

17. (D)

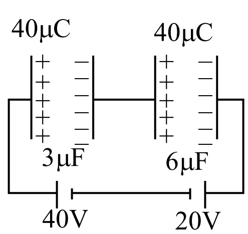
If S_1 is closed, then $\frac{kQ_A}{a} + \frac{kQ}{2a} = 0$ $\quad Q_A = -\frac{Q}{2}$

If S_2 is closed, then $\frac{kQ_B}{2a} = 0$ $\quad Q_B = 0$

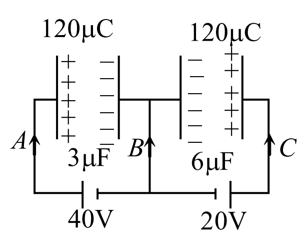
If S_3 is closed, then $\frac{kQ}{3a} + \frac{kQ_C}{3a} = 0$ $\quad Q_C = -Q$

If S_4 is closed, charge on shell B is $\underline{Q}$

18. (A)



Before closing



After closing

q_A = 80 mC, $\quad q_B$ = –240 mC $\quad q_C$ = 160 mC

W_1 = 40 ´ 80 = 3200 mJ

W_2 = 20 ´ 160 = 3200 mJ

$\therefore$ A-Q, B-P, C-S, D-R

19. (A)

20. (B)

CHEMISTRY

21. (A), (B), (C)

Higher the solvation, lower will be the ionic mobility

solvation increases in the order $Li^+ > Na^+ > Cs^+$ and $F^- > Cl^- > Br^- > I^-$

The ionic moblity of H^+ is very high due to proton jump.

Hence, correct order is :

$H^+ > Ci^+ > Na^+ > Li^+$

$I^- > Br^- > Cl^- > F^-$

On increasing temperature, electrolytic resistance decreases, hence, electrolytic conductance increases.

Kohlrausch's law is applicable for both strong as well as weak electrolyte.

22. (A,D)

23. (C), (D)

E_{cell} should be +ve.

E_{cell} will be negative with F^-/F_2 .

24. (A,B,C)

25. (A, B, C)

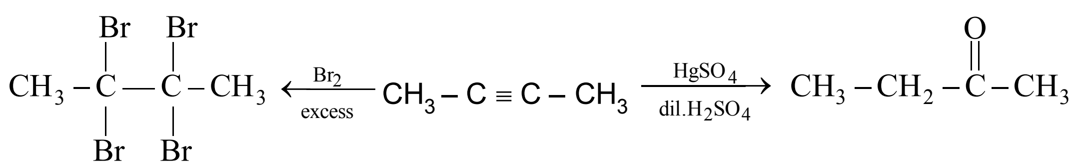
More the hyper-conjugation more stable is the compound and less is the $\Delta H_{C.}$

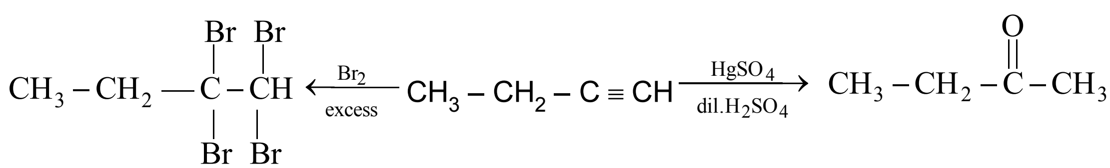
26. (A,C,D)

These molecules are unsymmetrical and having odd number of C-atoms.

27. (D)

$C_4H_6 \longrightarrow C_4H_8O$





28. (A), (C), (D)

$$\lambda_m = \frac{1000 \times K}{M}, \ T \uparrow M \downarrow \lambda_m \uparrow$$

Polarity of solvent $\uparrow$ no. of ions $\uparrow \lambda_m \uparrow$

29. (A)

Levigation is process by which lighter earthy particle are free from heavier ore particle by washing with H_2O.

30. (B)

Baeyer process : NaOH

Hall process : Na_2CO_3

31. (A)

When N is connected to SHE electron flow from N to SHE i.e. E^o_{oxi} of N is positive

$N^{+2} + 2e^- \rightarrow N \qquad E_{N^{+2}/N} = -0.25V$

Also reduction potential of M is greater than reduction potential of N (as e^- flow from N to M)

$E^o_{M^{+2}/M} = +0.34V, E^o_{N^{+2}/N} = -0.25V$

Now, $N_{(s)} | N^{+2}(0.1M) || M^{+2}(1M) | M_{(s)}$

$$Ecell = E^o cell - \frac{0.0591}{2}\log_{10}\frac{[N^{+2}]}{[M^{+2}]}$$

$$= (0.34 + 0.25) - \frac{0.0591}{2}\log_{10}\frac{0.1}{1}$$

$Ecell = 0.62V$

32. (D)

$M^+ + e^- \rightarrow M \quad -(1) \qquad E_1^0 = ? \qquad \Rightarrow \Delta G_1^o = -1 \times F \times E_1^o$

$M^{+2} + e^- \rightarrow M^+ - (2) \qquad E_2^0 = 0.15V \qquad \Rightarrow \Delta G_2^o = -1 \times F \times 0.15$

$M^{+2} + 2e^- \rightarrow M - (3) \qquad E_3^0 = 0.34V \qquad \Rightarrow G_3^o = -2 \times F \times 0.34$

add eq. (3) - eqn. (2)

$M^+ + e^- \rightarrow M \qquad \Delta G_1^0 = \Delta G_3^0 - \Delta G_2^0$

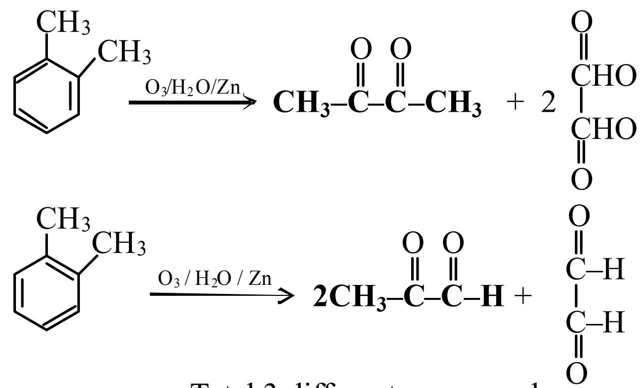
$-1 \times F \times E_1^0 = -2F(0.34) - (-1F \times 0.15)$

$E_1^0 = +0.68 - 0.15$

$E_1^0 = +0.53V$

33. (A)

34. (C)



Total 3 different compounds

35. (C)

36. (C)

37. (B)

(A) — (Q) ; (B) — (R) ; (C) — (S) ; (D) — (P)

(A) For concentration cell,

$$E^o_{cell} = 0,\ E_{cell} = -\frac{0.0591}{n}\log\frac{[\text{Cation}]_{anode}}{[\text{Cation}]_{cathode}}$$

(B) E^o_{red} depend upon nature of substance

(C) Daniel cell : Galvanic cell

(D) H_2 & O_2 is used in fuel cell

38. (C)

(A) — (S); (B) — (Q); (C) — (R); (D) — (P)

Temp. co-efficient = $\dfrac{\Delta Ecell}{\Delta T} = \dfrac{(.21-.23)}{308-288K}$ = – 1 × $10^{-3}VK^{-1}$ = –1 mVK^{-1}

$$\Delta G = \Delta H + T\left[\frac{d\Delta G^0}{dT}\right]$$

$$\Delta S = \frac{nFdE^0_{cell}}{dt} = -1 \times 96500 \times 1 \times 10^{-3} = -96.5\ JK^{-1}$$

$$\Delta G^o = \Delta H^o - T\Delta S^o$$

$$-n\underset{cell}{\overset{0}{E}}F = \Delta H^o - T\Delta S^o$$

$-1 \times 96500 \times 0.23 = \Delta H^o - 288(-96.5)$

$\Delta H = -50000J = -50KJ$

$E^o_{cell} = .22V$ $\quad AgCl + e \rightarrow Ag^+ + Cl^- \quad E^o = .22V$

$E^o_{cell} = \frac{.0591}{1} \log KSp$ $\quad Ag \rightarrow Ag^+ + e \quad E^o = -.80V$

$AgCl \rightarrow Ag^+ + Cl^{-1} \quad E^o = -.58V$

$\log Ksp = \frac{-.58}{.0591} = -9.81$

39. (A)

(A) — (Q); (B) — (R); (C) — (P); (D) — (S)

40. (C)

(A) – P ; (B) – S ; (C) – Q ; (D) – R

MATHEMATICS

41. (A, B, D)

$$I = \int \frac{(x^2+1)^2 - 2x^2}{(x^2+1)(x^4-x^2+1)} dx = \int \frac{(x^2+1)dx}{x^4-x^2+1} - 2\int \frac{x^2 dx}{x^6+1}$$

$$= \int \frac{(1+\frac{1}{x^2})dx}{x^2 - 1 + \frac{1}{x^2}} - 2\int \frac{x^2 dx}{(x^3)^2+1}$$

$\downarrow$ $\quad\quad$ $\downarrow$

Put $x - \frac{1}{x} = t$ $\quad\quad$ Put $x^3 = u$

$$= \tan^{-1}\left(x - \frac{1}{x}\right) - \frac{2}{3}\tan^{-1}(x)^3 + C$$

$\Rightarrow f(x) = x - \frac{1}{x}$ $\quad\quad$ $g(x) = x^3$

Hence option (C) is incorrect, rest of the option are correct.

42. (D)

$$\int \frac{1}{1-\sin x}\sqrt{\frac{\cos x}{1+\cos x+\sin x}}\, dx$$

$$= \int \frac{1}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2} \sqrt{\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{2\cos^2\frac{x}{2} + 2\cos\frac{x}{2}\sin\frac{x}{2}}}\, dx$$

$$= \int \frac{dx}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^{3/2} \sqrt{2\cos\frac{x}{2}}} = \frac{-2}{\sqrt{2}} \int \frac{-\frac{1}{2}\sec^2\frac{x}{2}\, dx}{\left(1 - \tan\frac{x}{2}\right)^{3/2}}$$

$$= -\sqrt{2}\, \frac{\left(1 - \tan\frac{x}{2}\right)^{-1/2}}{-1/2} + c = \frac{2\sqrt{2}}{\sqrt{1 - \tan\frac{x}{2}}} + c$$

$$\therefore \quad f(x) = 1 - \tan\frac{x}{2}$$

43. (A, C)

$$I = \int \frac{x^3 - x^2 + x - 1}{x^5 + 1}\, dx = \int \frac{(x^4 - 1)}{(x^5 + 1)(x + 1)}\, dx$$

$$= \int \frac{x^4}{x^5 + 1}\, dx - \int \frac{1}{x + 1}\, dx$$

$$= \frac{1}{5}\ln\left|x^5 + 1\right| - \ln\left|x + 1\right| + C$$

44. (B, D)

$$\int \frac{\cos x + x\sin x}{x(x + \cos x)}\, dx = \int \frac{(x + \cos x) - x + x\sin x}{x(x + \cos x)}\, dx$$

$= \int \frac{1}{x}\, dx - \int \frac{1 - \sin x}{x + \cos x}\, dx$ = log |x| – log |x+cosx| + c. Hence $f'\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$.

45. (A,C,D)

$$f(x + y) = 2^x f(y) + 4^y f(x)$$

Interchaning x and y, we get $f(x + y) = 2^y f(x) + 4^x f(y)$

$$\frac{f(x)}{4^x - 2^x} = \frac{f(y)}{4^y - 2^y} = k$$

$$f(x) = k\left(4^x - 2^x\right)$$

since $f'(0) = \ell n2$ we get k = 1

$f(x) = 4^x - 2^x$

46. (B, C)

$$f'(x) = e^{\tan x}(1 - \sin 2x) \geq 0 \ \forall \ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$\therefore$ f is increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$f'(x) = 0 \quad \Rightarrow \quad x = \frac{\pi}{4}$$

47. (A, C,D)

$$I = \int \frac{dx}{\sqrt{(x^2+1)^3}} = \int \frac{dx}{(x^2+1)\sqrt{x^2+1}}$$

put $x = \frac{1}{y}$ to get

$$I = -\int \frac{y\,dy}{(y^2+1)\sqrt{y^2+1}}$$

$$= -\int \frac{dt}{t^2} \qquad \text{(put } y^2 + 1 = t^2 \text{)}$$

$$= \frac{1}{t} + c = \frac{1}{\sqrt{y^2+1}} + c = \frac{x}{\sqrt{x^2+1}} + c$$

$$= \frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1}} + c_1 = \frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}} + c_2$$

48. (C)

$$I = \int (x^6 + x^3)\sqrt[3]{x^3 + 2}\,dx = \int (x^5 + x^2)\sqrt[3]{x^6 + 2x^3}\,dx$$

Let $x^6 + 2x^3 = t$

$6(x^5 + x^2)dx = dt$

$I = \frac{1}{6}\int t^{\frac{1}{3}}dt = \frac{t^{\frac{4}{3}}}{6 \times \frac{4}{3}} + c = \frac{1}{8}t^{4/3} + c$

$I = \frac{1}{8}(x^6 + 2x^3)^{4/3} + c$

49. (A)

$f(x) = \left(1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!} + ...\text{to } \infty\right) + 1$

and $g(x) = \left(1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!} + ...\text{ to} \infty\right) - 1$

Hence $f(x) = e^{2x} + 1$ and $g(x) = e^{2x} - 1$

$\int \frac{e^{2x} - 1}{e^{2x} + 1}dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx = \log|e^x + e^{-x}| + C.$

50. (C)

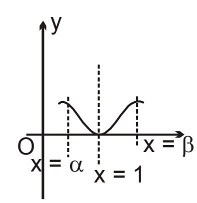
$\int \frac{e^{2x} + 1}{\sqrt{e^{2x} - 1}}\,dx = \int \frac{e^{2x}}{\sqrt{e^{2x} - 1}}dx + \int \frac{1}{\sqrt{e^{2x} - 1}}dx$

$= \int \frac{e^{2x}}{\sqrt{e^{2x} - 1}}dx + \int \frac{e^x}{e^x\sqrt{e^{2x} - 1}}dx$

$= \sqrt{e^{2x} - 1} + \sec^{-1}(e^x) + C$

51. (D)

Since f(x) is symmetric about x = 1 and it is twice differentiable. so $f'(x)$ must have one root at x = 1.



$f'(x) = a(x-1)(x-\alpha)(x-\beta) = a(x-1)\left(x^2 - (\alpha+\beta)x + \alpha\beta\right)$

Here $\frac{\alpha + \beta}{2} = 1$ so $\alpha + \beta = 2$

$f'(x) = a\left(x^3 - 3x^2 + (\alpha\beta + 2)x - \alpha\beta\right)$

$f''(2) = 0 \Rightarrow \alpha\beta = -2$

$\therefore$ Sum of roots of $f'(x) = 0$ is $1 + \alpha + \beta$ i.e., 3.

52. (C)

$$f'(x) = a(x-1)\left(x^2-2x-2\right) = a(x-1)\left((x-1)^2-3\right)$$

$$f(x) = a\left(\frac{(x-1)^4}{4} - \frac{3}{2}(x-1)^2\right) + C$$

$$\because f(1) = 0 \text{ so } c = 0;\ f(2) = 1 \text{ so } a = -\frac{4}{5}$$

$$f(x) = -\frac{4}{5}\left[\frac{(x-1)^4}{4} - \frac{3}{2}(x-1)^2\right]$$

$$f(3) = \frac{8}{5}$$

57. (B)

Let Asinx + Bcosx = a (3sinx – 4cosx) + b(3cosx + 4sinx)

$\Rightarrow$ 3a + 4b = A and – 4a + 3b = B

I = ax + b ln |3sinx – 4cosx| + C

If a = 1 and b = 1 then A = 7, B = –1

If a = –1 and b = –1, then A = –7, B = 1

If a = 3 and b = –1, then A = 5, B = –15

If a = 2 and b = –2, then A = –2, B = –14

58. (A)

(P) Let $\lim\limits_{x\to\infty} f(x) = l$, then $l = 2l - \frac{4}{l} \Rightarrow l = 2$

(Q) $\lim\limits_{h\to 0} \frac{f(h^2+h+2)-f(2)}{f(1-2h)-f(1)} = \lim\limits_{h\to 0} \frac{f'(h^2+h+2)(2h+1)}{f'(1-2h)(-2)} = \frac{f'(2)}{-2f'(1)} = \frac{-1}{4}$

(R) $\lim\limits_{x\to 0}\left[\frac{x^3}{x-\sin x}\right] = \lim\limits_{x\to 0}\left[\frac{x^3}{x-\left(x-\frac{x^3}{\underline{|3}}+\frac{x^5}{\underline{|5}}-....\right)}\right] = \lim\limits_{x\to 0}\left[\frac{1}{\frac{1}{\underline{|3}}-\frac{x^2}{\underline{|5}}+...}\right] = 6$

(S) $\lim\limits_{x\to 0^+} \frac{\sqrt{\tan x - x} - ax^{3/2}}{x^b} = \lim\limits_{x\to 0^+} \frac{\sqrt{\frac{1}{3}x^3 + \frac{2}{15}x^5 + ...} - ax^{3/2}}{x^b}$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\sqrt{3}}\left(1+\frac{2}{5}x^2+...\right)^{1/2}-a}{x^{b-\frac{3}{2}}} = \lim_{x\to 0^+} \frac{\frac{1}{\sqrt{3}}\left(1+\frac{1}{5}x^2+...\right)-a}{x^{b-\frac{3}{2}}}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{5\sqrt{3}}x^2+...}{x^{b-\frac{3}{2}}} \qquad \left(\frac{1}{\sqrt{3}}-a=0 \Rightarrow a=\frac{1}{\sqrt{3}}\right)$$

$$\therefore\ b-\frac{3}{2}=2 \Rightarrow b=\frac{7}{2} \qquad \therefore\ a^2+b=\frac{1}{3}+\frac{7}{2}=\frac{23}{6}$$

59. (B)

(P) If $\frac{\pi}{4} < x < \frac{3\pi}{8}$, then $\sin x > \cos x$

$$\therefore \int \frac{\sin x - \cos x}{|\sin x - \cos x|}\,dx = \int 1.\,dx = x + c$$

(Q) $$\int \frac{x^2\,dx}{(x^3+1)(x^3+2)} = \frac{1}{3}\int 3x^2\left(\frac{1}{x^3+1}-\frac{1}{x^3+2}\right)dx = \frac{1}{3}\ \ell n\left|\frac{x^3+1}{x^3+2}\right| + c$$

$\therefore$ f(x) = $\ell n\ |x|$

(R) $$f(x) = \int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx = \int (\tan x)^{\frac{-1}{2}}\sec^2 x\,dx = 2\sqrt{\tan x}+c$$

(S) $$\int \frac{dx}{x\ell n|x|} = \ell n\ |\ell n|x|| + c$$

$\therefore$ f(x) = $\ell n|x|$

60. (C)

(P) $$I = \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2}dx$$

Put $x-\frac{1}{x}=t$

$$\Rightarrow\ I = \int \frac{dt}{t^2} = -\frac{1}{t}+C = -\frac{1}{x-\frac{1}{x}}+C = \frac{x}{1-x^2}+C = -\frac{1}{P}+C$$

(Q) $I = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1}dx$

Put $x - \frac{1}{x} = t$

$$I = \int \frac{dt}{t^2+3} = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + C$$

$$\Rightarrow I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right) + C = \frac{1}{\sqrt{3}}\tan^{-1}\frac{P}{\sqrt{3}} + C$$

(R) $I = \int \frac{x^2+1}{x^4-x^2+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-1}dx$

Put $x - \frac{1}{x} = t$

$$\therefore I = \int \frac{dt}{t^2+1} = \tan^{-1}\frac{x^2-1}{x} + C = \tan^{-1}P + C$$

(S) $I = \int \frac{x^2+1}{x^4+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$

Put $x - \frac{1}{x} = t$

$$\Rightarrow I = \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right) + C = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{P}{\sqrt{2}}\right) + C$$