SOLUTIONS

PROGRESS TEST-3

RB-1810 TO 1812 & RBK-1805

(JEE MAIN PATTERN)

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PHYSICS

1. **(B)**

2. **(A)**

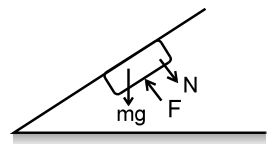
3. **(A)**

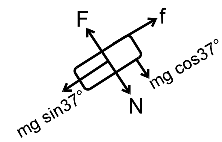
4. **(D)**

For block to be in equilibrium :

$F = N + mg\cos 37°$

$f = mg\sin 37°$ FBD :





For F to be minimum,

For no sliding $f \le \mu N$

$\Rightarrow mg\sin 37° \le \frac{1}{2}[F - mg\cos 37°]$

$\Rightarrow 2mg\sin 37° + mg\cos 37° \le F$

$\Rightarrow F \ge 2mg \times \frac{3}{5} + mg \times \frac{4}{5} \Rightarrow F \ge 2mg$ i.e. $F \ge 200$ N $\quad \Rightarrow F_{min} = 200N$

5. (D)

Such problems can be solved with or without using the concept of pseudo force.

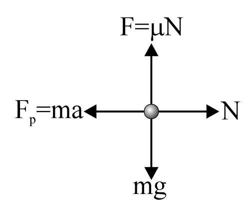
a = acceleration of (wedge + block) in horizontal direction

Non inertial frame of reference (Wedge)

F. B. D. of m with respect to wedge

(real + one pseudo force)

with respect to wedge block is stationary.



$\therefore \sum F_x = 0 = \sum F_y$

$\therefore mg = \mu N$ and $N = ma$

$\therefore a = \frac{g}{\mu}$ and $F = (M+m)a \quad = \mathbf{(M+m)\frac{g}{\mu}}$

6. (C)

Let m_A and m_B be the mass of blocks A and B respectively.

As the force F increases from 0 to $\mu_s m_A g$, the frictional force f on block A is such that f = F. When $F = \mu_s m_A g$, the frictional force f attains maximum value $f = \mu_s mg$.

As F is further increased to $\mu_s(m_A+m_B)g$, the block A does not move. In this duration frictional force on block A remains constant at $\mu_s m_A g$.

Hence C is correct choice.

7. (C)

The tension in the string initially is zero. If $\mu_1 > \tan\alpha$ and $\mu_2 > \tan\beta$, both the blocks will not move down the incline and the tension in the string shall continue to remain zero.

8. (B)

If we consider blocks 2 & 1 independently then their accelerations would be

for block (1)

$$a_1 = g\sin\theta - \mu_1 g\cos\theta = g\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\times\frac{1}{2}\right] = \frac{g\left[2\sqrt{3}-1\right]}{4}$$ for block (2)

$$a_2 = g\sin\theta - \mu_2 g\cos\theta = g\left[\frac{\sqrt{3}}{2} - \frac{2}{5}\times\frac{1}{2}\right] = \frac{g}{10}\left[5\sqrt{3}-2\right]$$

since $a_2 > a_1$ so both blocks will move separately.

9. (B) **10. (A)** **11. (C)** **12. (C)**

13. (B) **14. (C)** **15. (A)**

16. (B)

Here, D = 100 cm ; L = 30 cm

$$\therefore \quad f = \frac{D^2 - L^2}{4D} = \frac{(100)^2 - (30)^2}{4(100)} = \frac{91}{4} \text{ cm}$$

$I_1 = 16$ mm; $I_2 = 9$ mm

$\therefore$ object size = $\sqrt{I_1 I_2} = \sqrt{16\times 9}$ = **12 mm**

17. (D)

In case compound microscope

$$MP = m \times m_0$$

since the final image is formed at D,

$$m_0 = \left(1 + \frac{D}{f_e}\right)$$

$$\therefore \quad MP = m\times\left(1 + \frac{D}{f_e}\right)$$

Hence, MP = –30; D = 25 cm; f_e = 5cm

$$\therefore \quad -30 = m\times\left(1 + \frac{25}{5}\right)$$

or, m = –5

The negative sign implies that image formed by the objective is inverted.

18. (B) **19. (A)**

20. (D)

$$(P_m)_{eq} = 2P_L + P_m$$

$$P_L = \frac{1}{60} \times 100\ D = \frac{5}{3} D$$

$$P_m = 0$$

$$(P_m)_{eq} = 2P_L + 0 = 2 \times \frac{5}{3} D$$

$$(P_m)_{eq} = \frac{10}{3} D$$

$$f_{eq} = -\frac{3}{10} \times 100 = -30\,cm$$

The problem is reduced to a simple case where a point object is placed in front of a concave mirror.

Now, using the mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

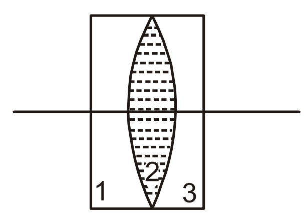
$$-\frac{1}{20} + \frac{1}{v} = -\frac{1}{30}$$

$$v = 60\,cm$$

21. (C)

As shown in figure, the system is equivalent to combination of three in contact,

i.e., $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$



By lens maker's formula $\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{\infty} - \frac{1}{20}\right] = \frac{1}{40}\ cm$

$$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left[\frac{1}{20} - \frac{1}{-30}\right] = \frac{5}{180}\ cm$$

$$\frac{1}{f_3} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{-30} - \frac{1}{\infty}\right] = \frac{1}{60}\ cm\ ]$$

$\therefore$ F = – 72 cm

Thus, the system will hehave as a concave lens of focal length 72 cm.

22. (C)

Here , $A = 60°$; m = $\sqrt{3}$

Now, $$m = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

or, $$\sqrt{3} = \frac{\sin\left(\frac{60°+\delta_m}{2}\right)}{\sin\frac{60°}{2}} = \frac{\sin\left(\frac{60+\delta_m}{2}\right)}{\sin 30°}$$

or, $\sin\left(\frac{60° + \delta_m}{2}\right) = \sin 30° =$

or, $=60^0$

δ_m = **60^0**

23. (D)

Angle of incidence = $i = 60°$

At point P, $\frac{\sin\ 60°}{\sin\ r_1} = \frac{\sqrt{3}}{1}$

$\Rightarrow$ $\sin r_1 = 30^0$

Using $r_1 + r_2 = A$, we get

$r_2 = A - r_1 = 60° - 30° = 30^0$

At point Q, $\frac{\sin r_2}{\sin\ e} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $\sin e = \frac{\sqrt{3}}{2}$

$\Rightarrow$ $e = 60^0$

24. (B)

Here

$i = 60°$; $\mu = \sqrt{3}$ and $t = 2\sqrt{3}$ m

Now, $\mu = \frac{\sin i}{\sin\ r}$ or, $\sin r = \frac{\sin i}{\mu} = \frac{\sin\ 60°}{\sqrt{3}} = \frac{1}{2}$

$\Rightarrow$ $r = 30^0$

Now, lateral shift

$$d = \frac{t}{\cos r}\sin(i-r) = \frac{2\sqrt{3}}{\sqrt{3}} = 2\text{mm}$$

25. (C)

26. (A)

Using $\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$

$\Rightarrow \frac{1}{v}-\frac{1}{15}=-\frac{1}{10}$ Þ v = –30 cm

Now, using $V_{im} = -\frac{v^2}{u^2}V_{om}$

$$\Rightarrow (V_i - V_m) = -\frac{v^2}{u^2}(V_o - V_m)$$

$$\Rightarrow V_i - (1) = -\frac{(-30)^2}{(-15)^2}[(-10)-(+1)]$$

$\Rightarrow V_i = 45$ cm/s

So the image will move with velocity 45 cm/s.

27. (A)

28. (A)

29. (B)

If r is the radius (in m) of the largest cicle from which light comes out and i_c is the critical angle for water–air interface, r = 0.5 × tan i_c and sin i_c = 1/1.33 = 0.75.

$$\text{Area} = \frac{\pi\times(0.8)^2\times(0.75)^2}{1-(0.75)^2}\,m^2 = 2.6\,m^2$$

30. (B)

CHEMISTRY

31. (B)

Let 1mole of mixture has x mole N_2O_4

2 × 27.6 = x(92) + (1 – x)46;

x = 0.2

32. (A)

Let mole % of ^{26}Mg be x

$\therefore \quad \dfrac{(21-x)25+x(26)+79(24)}{100}=24.31$

X = 10 %

33. (C)

$$2Fe(NO_3)_3 + 3Na_2CO_3 \longrightarrow Fe_2(CO_3)_3 + 6NaNO_3$$

mole	2.5	3.6
mole/stoichiometric coefficient	1.25	1.2

Limiting reagent is Na_2CO_3 so moles of $NaNO_3$ format = 3.6 × 2 = 7.2

$$\%\ \text{yield} = \frac{6.3}{7.2}\times 100 = 87.5$$

34. (B)

Total Cl^- = 0.2+0.4 mole (from $BaCl_2$) = 0.6 mole

$M_{Cl^-} - \dfrac{0.6}{500}\times 1000 = 1.2$

35. (B)

Mass of solution =10 ×1.2 = 12g

Mass of solute $=12\times\dfrac{37}{100}=4.44\,g$

36. (B)

Total moles of H_2SO_4 = 0.1 mole

$\text{Total volume} = \dfrac{150+400}{1.25} = \dfrac{550}{1.25} = 440$

$\therefore \quad M = \dfrac{0.1}{440}\times 1000 = \dfrac{1}{4.4} = 0.227M$

37. (A)

1000 mL solution contain 2 mole of ethanol or 1000×10.25 g solution contain 2 mole of ethanol

Mass of solvent = 1000×1.025 – 2 × 46

$m = \dfrac{2}{1000\times 1.025 - 2\times 46}\times 1000$

$m = \dfrac{2}{933}\times 1000 = 2.143$

38. (A)

$M_{0.98}O \rightarrow M^{2+}$ (x), M^{3+} (0.98 – x)

M^{2+} $\quad$ M^{3+}

x $\quad$ (0.98 – x)

$2x + (0.98 - x)3 - 2 = 0$

$x = 0.94$

$$\% \text{ of } M^{3+} = \frac{0.04}{0.98} \times 100 = 4.08$$

39. (B)

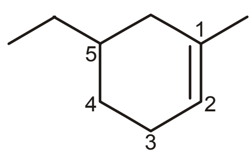
$$xMnO_4^- + yC_2O_4^{2-} + zH^+ \rightarrow xMn^{2+} + 2yCO_2 + \frac{z}{2}H_2O$$

V.F = 5 V.F = 2

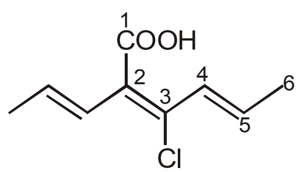
$x = 2,\ y = 5,\ z = 16$

40. (C)

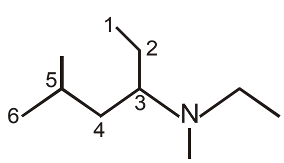
41. (B)



42. (A)



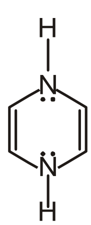
43. (C)



44. (C)

Order of –R effect $-NO_2 > -CN > -CHO > -Ph$

45. (D)



Do not follow Huckel's rule so is not aromatic.

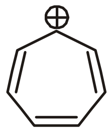
46. (A)

It becomes aromatic. Boron having vacant p-orbital and structure having 6π delocalised.

47. (B)

All π bond and lone pair are delocalised.

48. (B)

 Cyclic resonance &

(4n + 2) πe^-

$\pi e^- = 6$

49. (C)

50. (B)

51. (B)

$O^{2-} > Mg^{2+} > Al^{3+}$; (Z/e) ratio.

52. (B)

I.E. of Mg > I.E. of Al (Due to electronic configuration)

53. (C)

2 (inert gas configuration)

54. (A)

$E_n = \frac{I.E. + E_A}{2}$ (Here I.E. & E_A in ev/atom)

$E_n = \frac{I.E. + E_A}{540} KJ/mol$

55. (B)

(ΔHeg)II is always endothermic.

56. (C)

O^- ion will resist the addition of another electron due to inter-electronic repulsion.

57. (D)

All are iso-electronic species.

58. (A)

Ni = 3d

Pd = 4d

Pt = 5d

$1P_1 = 3d < 4d < 5d$

59. (B)

En = x

Ea = y

$$EN = \frac{IP + EA}{2} = x = \frac{IP + y}{2} = \therefore I.P. = 2x - y$$

60. (B)

$$\frac{2E_1}{N_O}, \frac{2E_2}{N_O}$$

$$x(g) \to x^+_{(g)} + 1e$$

$$\frac{N_o}{2} \longrightarrow E_1$$

$$\because \frac{N_o}{2} \to E_1$$

$$\therefore 1 \text{——} \frac{E_1}{\frac{N_o}{2}} = \frac{2E_1}{N_O}$$

$$X(g) + 1e - x^-_{(g)} - E_2$$

$$\frac{N_o}{2} \longrightarrow E_2$$

$$1 \longrightarrow \frac{E_2}{\frac{N_o}{2}} = \frac{2E_2}{N_0}$$

MATHEMATICS

61. (A)

$$n(A \times A) = n(A), n(A) = 3^2 = 9$$

So, the total number of subsets of A × A is 2^9 and a subset of A × A is a relation over the set A.

62. (B)

Since R is an equivalence relation on set A, therefore $(a, a) \in R$ for all $a \in A.$ Hence, R has at least n ordered pairs.

63. (B)

$$S = \sum_{r=1}^{n} \left(\frac{\sin\theta}{2\cos(r+1)\theta\cos r\theta\sin\theta} \right) = \frac{1}{2\sin\theta} \left(\sum_{r=1}^{n} \frac{\sin((r+1)\theta - r\theta)}{\cos(r+1)\theta\cos r\theta} \right)$$

$$=\frac{1}{2\sin\theta}\left(\sum_{r=1}^{n}\{\tan(r+1)\theta-\tan r\theta\}\right)=\frac{1}{2\sin\theta}\{\tan(n+1)\theta-\tan\theta\}=\left(\frac{\sin n\theta}{\sin 2\theta\cos(n+1)\theta}\right).$$

64. (C)

sec 40°, sec 80°, sec 160° are the roots of $\frac{8}{t^3}-\frac{6}{t}+1=0$

or $t^3-6t^2+8=0$

$\therefore$ Sum of roots = 6.

65. (C)

$\log_{\sin x}\frac{|x|}{x}\Rightarrow \sin x\in(0,1)$ and $x\in(0,\infty)$

$\therefore\ x\in\bigcup_{n\in W}\left(2n\pi,2n\pi+\frac{\pi}{2}\right)\cup\left(2n\pi+\frac{\pi}{2},(2n+1)\pi\right)$ and $y\in\{0\}$

66. (C) $\log_{\frac{1}{3}}(x^2+x+1)>-1\quad\Rightarrow\quad x^2+x+1<3$

$\Rightarrow\ x^2+x-2<0\quad\Rightarrow(x+2)(x-1)<0\quad\Rightarrow\quad x\in(-2,1)$

67. (D)

$f(x)=\log_{\sqrt{2}}\left(2-\log_2\left(16\sin^2x+1\right)\right)$

$1\le 16\sin^2x+1\le 17$

$\therefore\ 0\le\log_2(16\sin^2x+1)\le\log_2 17$

$\therefore\ 2-\log_2 17\le 2-\log_2(16\sin^2x+1)\le 2$

Now consider

$0<2-\log_2(16\sin^2x+1)\le 2$

$\therefore\ -\infty<\log_{\sqrt{2}}[2-\log_2(16\sin^2x+1)]\le\log_{\sqrt{2}}2=2$

$\therefore$ the range is $(-\infty,2]$

68. (B)

Put x = y = 1, $(f(1))^2=3f(1)-2\Rightarrow f(1)=1$ or 2

Let f(1) = 1, then put y = 1

$f(x).\,f(1)=f(x)+f(1)+f(x)-2$

$\Rightarrow f(x)=1$constant function

$\therefore\ f(1)\neq 1$, hence $f(1)=2$

69. (A)

Point of intersection is A (–2, 0). The required line will be one which passes through (–2,0) and is perpendicular to the line joining (–2, 0) and (2, 3)

70. (B)

We can assume that OP and OR are x-axis and y-axis respectively.

Let OP = a, then ar (sq. OPQR) = a^2

coordinates of M and N are $\left(a, \frac{a}{2}\right)$ and $\left(\frac{a}{2}, a\right)$ respectively

$\therefore$ ar(ΔOMN) = $\frac{1}{2}\begin{vmatrix} a & a/2 \\ a/2 & a \end{vmatrix} = \frac{3a^2}{8}$

$\therefore \frac{8}{3} = \frac{\lambda}{6}$ $\qquad \therefore \lambda = 16$

71. (C)

Let P be $(-2 + r\cos\theta, 0 + r\sin\theta) \Rightarrow r^2\sin^2\theta - 4(-2 + r\cos\theta) = 0$

$\Rightarrow r^2\sin^2\theta - 4r\cos\theta + 8 = 0 \Rightarrow r_1 + r_2 = \frac{4\cos\theta}{\sin^2\theta}$ and $r_1 r_2 = \frac{8}{\sin^2\theta}$

$\frac{1}{AP} + \frac{1}{AQ} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{\cos\theta}{2} = \frac{1}{4} \Rightarrow \tan\theta = \sqrt{3}$

Hence equation of incident ray is $y - 0 = -\sqrt{3}(x + 2) \Rightarrow y + x\sqrt{3} + 2\sqrt{3} = 0$

72. (A)

Let $y = 3\sec\theta - 2\tan\theta$

$\therefore 5\tan^2\theta - 4y\tan\theta + 9 - y^2 = 0$

The roots of this equation will be real if $D \geq 0$ i.e. $16y^2 - 20(9 - y^2) \geq 0$

$y^2 \geq 5$

$\therefore$ Minimum positive value of y is $\sqrt{5}$.

73. (A)

Image of A in x-axis is (1, –2)

$\therefore$ equation of reflected ray is

$y - 3 = \frac{5}{4}(x - 5)$

is $5x - 4y = 13$

74. (A)

Distance of (0,0) from the line $2x + 3y - 6 = 0$

$$\frac{6}{\sqrt{4+9}} = \frac{6}{\sqrt{13}}$$

$\therefore$ area of the Δ is $\left(\frac{6}{\sqrt{13}}\right)^2 = \frac{36}{13}$

75. (A)

$$\cos A.\cos(45^\circ - A) = \cos A\left(\frac{\cos A + \sin A}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}(\cos^2 A + \sin A.\cos A)$$

$$= \frac{1}{2\sqrt{2}}((1 + \cos 2A) + \sin 2A)$$

as $\cos 2A + \sin 2A + 1 \le \sqrt{2} + 1$

$\therefore$ max. value of $\cos A.\cos B = \frac{1}{2\sqrt{2}}(1 + \sqrt{2})$

76. (D)

The point Q is (–b, –a) and the point R is (–a, –b)

$\therefore$ mid point of PR is (0, 0)

77. (C)

It is obvious that a, b and c are the roots of the equation $mt^3 + (l - p)\,t - kq = 0$, where (p, q) is the point of concurrency.

Obviously sum of roots = a + b + c = 0

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

78. **(A)** $5\{x\} = x + [x]$ .....(i)

$[x] - \{x\} = \frac{1}{2}$ .....(ii)

$\therefore\ 0 \le \{x\} < 1$

$\Rightarrow\ 0 \le [x] - \frac{1}{2} < 1$(by(ii)

$\Rightarrow\ [x] = 1 \qquad \therefore\ \{x\} = \frac{1}{2}$

$\therefore$ from (i) we get $\frac{5}{2} = x + 1$

$\therefore\ x = \frac{3}{2}$, (one value)

79. **(C)** $\cos x + \sin x = \sqrt{2}\cos x$

$\sin x = (\sqrt{2} - 1)\cos x$

$\cos x = \frac{1}{(\sqrt{2}-1)}\sin x$

$\cos x = (\sqrt{2}+1)\sin x$

$\cos x - \sin x = \sqrt{2}\sin x$

80. **(B)**

$$\frac{\cos x \tan x}{k^2} + \frac{1}{\tan x} + \frac{\sin}{1+\cos x} = \frac{\sin x}{k^2} + \frac{\cos x(1+\cos x)+\sin^2}{\sin x(1+\cos x)} = \frac{a}{k} + \frac{1}{\sin x} = \frac{1}{k}\left(a + \frac{1}{a}\right)$$

81. **(D)**

Since $\sin\theta - \cos\theta \ne 0$

$\tan\theta \ne 1$

$\therefore\ \theta \ne \frac{\pi}{4}, \frac{5\pi}{4}$

Now $\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta - |\sin\theta|\cos\theta - 2\tan\theta\cot\theta = -1$

$\Rightarrow\ 1 + \cos\theta(\sin\theta - |\sin\theta|) - 2 = -1 \qquad \Rightarrow\ \cos\theta(\sin\theta - |\sin\theta|) = 0$

$\therefore\ \theta \in (0, \pi) - \left\{\frac{\pi}{4}, \frac{\pi}{2}\right\}$

82. **(B)**

$\therefore\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \qquad \Rightarrow\ -1 \le \sin\theta \le 1$

Here $0 < \sin\theta < 1 \qquad \Rightarrow\ \log_{\sin\theta}\cos 2\theta = 2$

$\cos 2\theta = \sin^2\theta \qquad \Rightarrow\ 1 - 2\sin^2\theta = \sin^2\theta$

$3\sin^2 \Rightarrow = 1 \qquad \Rightarrow \sin^2\theta = \frac{1}{3}$

$\therefore \quad \sin\theta = \frac{1}{\sqrt{3}}$ {$\because$ 0 < sinθ < 1} a unique solution

83. (D)

$0 \le \log_e [2x] \le 1$

$1 \le [2x] \le e \quad \Rightarrow [2x] = 1, 2 \qquad \Rightarrow 1 \le 2x < 3$

$\frac{1}{2} \le x < \frac{3}{2}$

84. (A) $|a| + |b| = |a - b|$

$\Rightarrow \quad ab \le 0$

$(x^2 - 5x + 7)(x^2 - 5x - 14) \le 0$

$(x - 7)(x + 2) \le 0$

$\Rightarrow \quad x \in [-2, 7]$

85. (A)

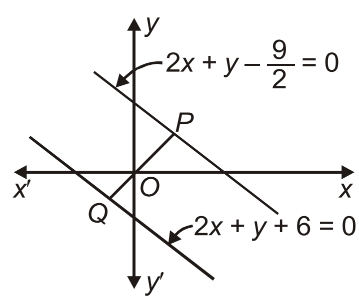
Taking *PQ* in any direction, ratio of *OP* and *OQ* will be same, so taking *PQ* ⊥ to the given parallel lines

OP = ⊥ from (0, 0) on $2x + y - \frac{9}{2} = 0$.

$= \left|\frac{-9/2}{\sqrt{4+1}}\right| = \frac{9}{2\sqrt{5}}$

OQ = ⊥ from (0, 0) on $2x + y + 6 = 0$

$= \left|\frac{6}{\sqrt{4+1}}\right| = \frac{6}{\sqrt{5}}$



$\therefore \quad \frac{OP}{OQ} = \frac{9/2\sqrt{5}}{6/\sqrt{5}} = \frac{9}{12} = \frac{3}{4}$

86. (C)

Let $P = \cos\theta \cos 2\theta \cos 3\theta \ldots \cos 1004\theta$

and $Q = \sin\theta \sin 2\theta \sin 3\theta \ldots \sin 1004\theta$

Then $2^{1004} PQ = \sin 2\theta \sin 4\theta \ldots \sin 2008\theta$

$$= (\sin 2\theta \ \sin 4\theta ... \ \sin 1004\theta) \ [\sin(2\pi - 1003\theta)\sin(2\theta - 1001\theta)...\sin(2\pi - \theta)]$$

$$= (\sin 2\theta \ \sin 4\theta \ ... \ \sin 1004\theta) \ [-\sin 1003\theta][-\sin 1001\theta] \ ... \ [-\sin\theta] = Q$$

$$\Rightarrow \quad P = \frac{1}{2^{1004}}.$$

87. (A)

$$\frac{(x-1)^3\left(x^2+3x+2\right)^5|x+4|}{\left(x^2+4x+4\right)^7}<0$$

$$\Rightarrow \frac{(x-1)^3(x+2)^5(x+1)^5|x+4|}{\left((x+2)^2\right)^7}<0 \ \Rightarrow \frac{(x-1)^3(x+2)^5(x+1)^5|x+4|}{(x+2)^{14}}<0$$

$(x-1)(x+2)(x+1)<0$

$x\in(-\infty,-2)\cup(-1,1)$ and $1<|x-3|<5$

$\Rightarrow 1<x-3<5$ or $-5<x-3<-1 \ \Rightarrow \ 4<x<8$ or $-2<x<2$

Hence common solution is (–1, 1)

88. (A) $\sin^2 x + a\cos x + a^2 \ge 1+\cos x \ \Rightarrow \ \cos^2 x - (a-1)\cos x - a^2 \le 0$

Let cos x = t then, $f(t) = t^2 - (a-1)t - a^2$, then $f(t) \le 0, \ \forall t \in [-1,1]$, which is possible iff $f(-1) \le 0$ and $f(1) \le 0$.

$f(-1) \le 0 \Rightarrow 1 + a - 1 - a^2 \le 0 \ \Rightarrow \ a \le 0$ or $a \ge 1$ ...(i)

$f(1) \le 0 \ \Rightarrow \ 1 - a + 1 - a^2 \le 0 \ \Rightarrow \ a \le -2$ or $a \ge 1$ ...(ii)

From (i) & (ii), $a \in (-\infty, -2] \cup [1, \infty)$

89. (A)

$\sin x(1+\sin^2 x) = \cos^2 x \ \Rightarrow \ \sin x(2-\cos^2 x) = \cos^2 x$

$\Rightarrow \ \sin^2 x(2-\cos^2 x)^2 = \cos^4 x \ \Rightarrow \ (1-\cos^2 x)(4 - 4\cos^2 x + \cos^4 x) = \cos^4 x$

$\Rightarrow \ \cos^6 x - 4\cos^4 x + 8\cos^2 x = 4 \ \Rightarrow \ \frac{1}{4}(\cos^6 x - 4\cos^4 x + 8\cos^2 x) = 1$

90. (B)

Case I

$[x] - 2x = 4$ ......(i)

$\Rightarrow [x] - 2([x] + \{x\}) = 4$

$\Rightarrow [x] + 2\{x\} + 4 = 0$ ......(ii)

$\therefore 0 \le 2\{x\} < 2$

$\therefore 0 \le -[x] - 4 < 2$ (from (ii))

$\Rightarrow [x] = -4, -5$

$\therefore$ from (i) we get $x = -4, \frac{-9}{2}$

Case II

$[x] - 2x = -4$ ......(iii)

$\Rightarrow [x] = 2x - 4$

$\Rightarrow [x] = 2([x] + \{x\}) - 4$

$\Rightarrow 2\{x\} = 4 - [x]$ .....(iv)

$\therefore 0 \le 2\{x\} < 2$

$\Rightarrow 0 \le 4 - [x] < 2$

$\Rightarrow 2 < [x] \le 4$ $\therefore [x] = 3, 4$

$\therefore$ from (iii) we get $x = 4, \frac{7}{2}$

$\therefore$ Number of solutions $|[x] - 2x| = 4$ are 4.