SOLUTIONS

WEEKLY TEST-14

GRA, GRS-1801 & GRKS-1801

[TOP 170 STUDENTS]

(JEE ADVANCED PATTERN)

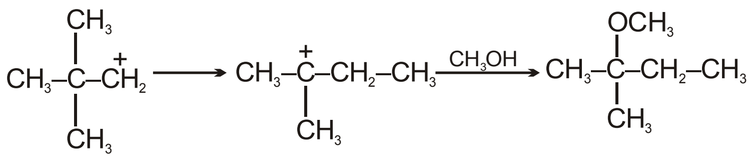
Test Date: 16-09-2017

Mentors® Eduserv™

Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

CHEMISTRY

1. (B)



2. (B)

$$\frac{P^0 - P_s}{P_s} = \frac{n}{N} \Rightarrow \frac{5}{95} = \frac{m_{solute} / M_{solute}}{m_{solvent} / (0.3\ Msolute)} \Rightarrow \frac{m_{solvent}}{m_{solute}} = \frac{0.3 \times 95}{5} = \frac{57}{10} = 57:10$$

3. (B)

$T_b - T_f = 105$

$\Rightarrow \quad (100 + \Delta T_b) + (\Delta T_f) = 105 \qquad \Rightarrow \quad \Delta T_b + \Delta T_f = 5$

$\Rightarrow \quad (K_b + K_f) \cdot m = 5$

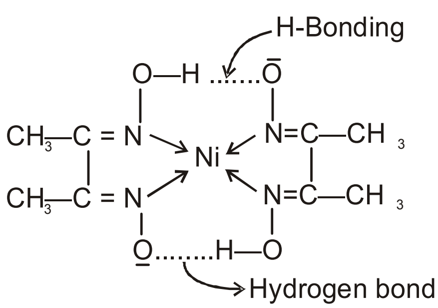
$\Rightarrow \quad 2.5 \times m = 5 \qquad \therefore \quad m = 2$

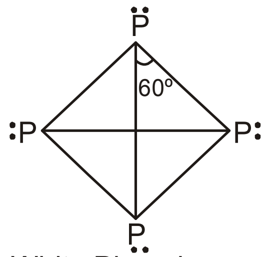
$\therefore \quad 2 = \frac{(W / 342)}{0.1} \qquad \therefore W = 2 \times 342 \times 0.1$ g

i.e., w = 68.4 g

4. (B)

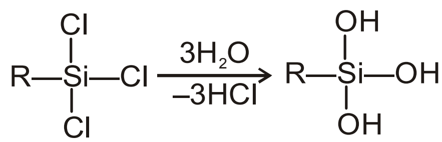
(i) $[Ni(dmg)_2]$ has two hydrogen bond

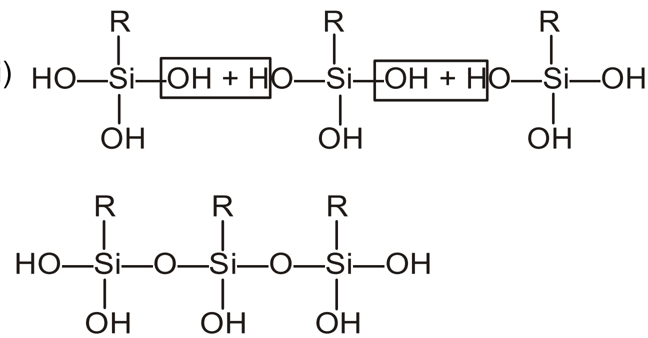


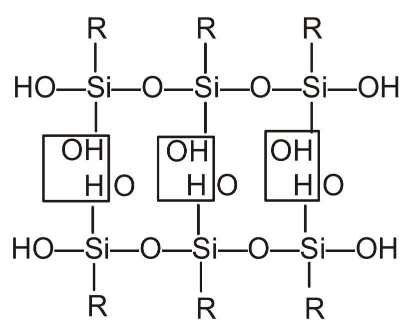
(iii) 

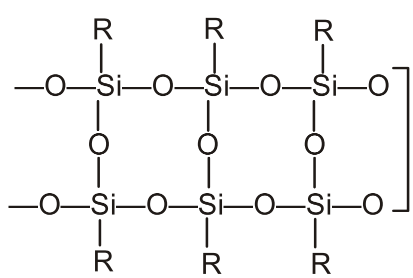
White Phosphorous

(iv) $RSiCl_3$ on Hydrolysis gives a cross linked silicon

(i) 

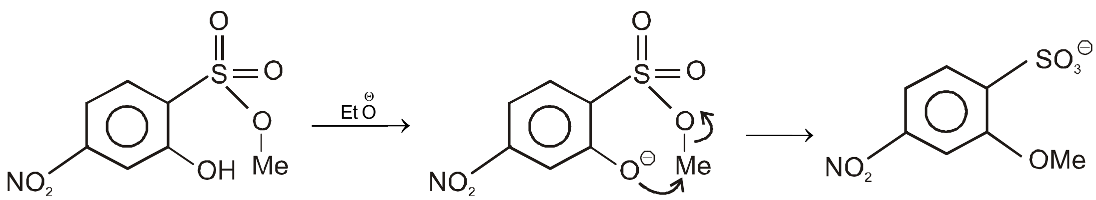
(ii) 

(iii) 



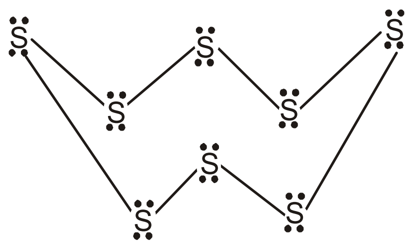
Cross linked silicon

5. **(A)**



6. **(A)**

S_8



Total = 16 lone pair.

7. **(5)**

$$\frac{500}{105.3}=\frac{\frac{n}{v_i}\times R\times 283}{\frac{n}{v_f}\times R\times 298}\Rightarrow\frac{v_f}{v_i}=\frac{500\times 298}{105.3\times 283}=5:1=x:1$$

$\therefore\ x=5$

8. **(8)**

$$K_b=\frac{2\times 320\times 320}{1000\times 80}=\frac{64}{25}$$

$\therefore\quad \Delta T_b=K_b\cdot m\ \Rightarrow\ 0.32=\left(\frac{64}{25}\right)\times m$

$\therefore\quad m=\frac{1}{8}=\frac{6.4/(32x)}{0.2}$

$\Rightarrow\quad x=8$

$\therefore$ M.F. of sulphur = S_8

9. (1)

	$A(g) \longrightarrow$	$2B(g) +$	$3C(\ell) +$	$4D(s)$	Inert gas
t = 0	P	0	0	0	i mm Hg
t = 10 mins	P – x	2x	20	0	i mm Hg
t = ∞	0	2P	20	0	i mm Hg

P + i = 210

P – x + 2x + 20 + i = 330

2P + 20 + i = 430

On solving; P = 200, x = 100

Hence 10 mins must be $t_{1/2}$ and y = 1.

10. (3)

Leaching (x) = 1

Smelting (y) = 3

Amalgamation (z) = 1

x. $Al_2O_3.2H_2O + NaOH \rightarrow NaAl(OH)_4 + H_2O$

y. $ZnO + C \xrightarrow{>710^\circ} Zn + CO$

$SnO + C \xrightarrow{>710} Sn + CO$

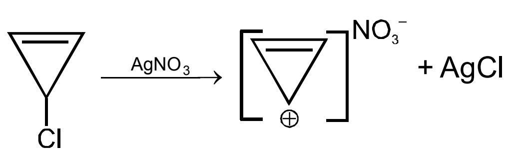
$Fe_2O_3 + CO \xrightarrow{<710} 2Fe + 3CO_2$

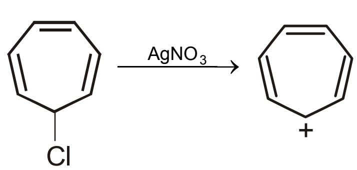
z. $Ag_2S + CuCl \longrightarrow AgCl + Cu_2S$

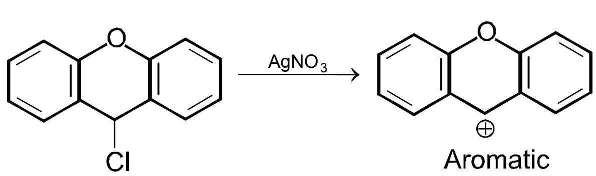
$AgCl + Hg \longrightarrow Ag.xHg + HgCl_2$

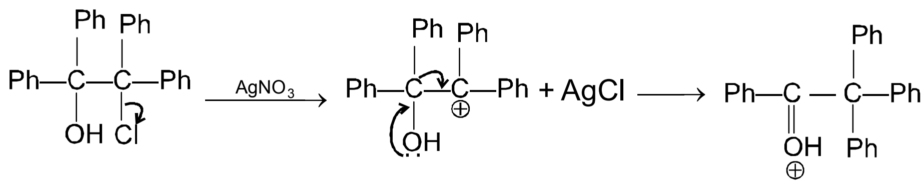
$Ag.xHg \xrightarrow{\Delta} Hg\uparrow + Ag(s)$

11. (4)









(To gain stability of carbocation)

12. (D)

13. (C)

1° Alkyl halide

14. (C)

In SN^2 reaction 1° Alkyl halide more reactive than 2° Alkyl halide

15. (A)

$Pb(s) + PbO_2(s) + 2H_2SO_4(aq) \rightarrow 2\ PbSO_4(s) + 2H_2O\ (l)$

Mass of H_2SO_4 before discharging = $3.5 \times 1000 \times x \times \frac{39}{100} = 1365\ x$

Mass of H_2SO_4 after discharging = $3.5 \times 1000 \times y \times \frac{20}{100} = 700\ y$

Mass of H_2SO_4 consumed = 1365 x – 700 y

$1365\ x - 700\ y = \frac{98}{96500} \times Q$

$Q = (1365\ x - 700\ y)\ \frac{96500}{98} C$

16. (A)

For charging, the reaction followed at cathode is

$PbSO_4(s) + H^+ (aq) + 2e^- \longrightarrow HSO_4^-(aq) + Pb(s)$

$Q = 10 \times 1.5 \times 60 \times 60$

$m = ZQ = \frac{M \times 10 \times 1.5 \times 60 \times 60}{2 \times 96500}$

$m = \frac{169.55}{2} g = 84.77g$

17. (B)

m = ZQ

$m_{Pb} = \frac{207 \times 2 \times 100 \times 60 \times 60}{2 \times 96500} = 772.23g$

Hence, the actual lead required is $772.23 \times \frac{100}{50}$ = **1544.46 g**

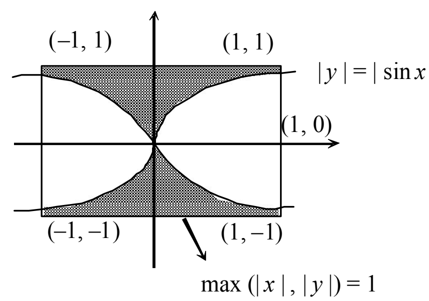
18. **(A) - p,q ; (B) - p,q ; (C) - p,q,r ; (D) - r,s**

19. **(A) – p,s ; (B) – p,q,s ; (C) – q,r,t ; (D) – p,s,t**

MATHEMATICS

20. **(A)**

$$\text{Required area} = 4\left[1-\int_0^1 \sin x \, dx\right] = 4\cos 1$$



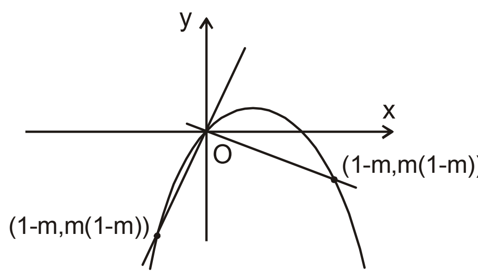
21. **(A)**

Either $\int_{1-m}^{0} (x - x^2 - mx)dx = \frac{4}{3}$

or $\int_{0}^{1-m} (x - x^2 - mx)dx = \frac{4}{3}$

Hence m = 3 or – 1

Hence $| m_1 - m_2 | = 4$



22. **(A)**

$$\int_0^{\pi/4} (\tan^n(x-[x]) + \tan^{n-2}(x-[x]))\,dx = \int_0^{\pi/4} (\tan^n x + \tan^{n-2} x)\,dx$$

$$= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x \, dx = \left.\frac{\tan^{n-1} x}{n-1}\right|_0^{\pi/4} = \frac{1}{n-1}$$

23. **(D)**

f(1) = –6

for maximum at x = 1

$$\lim_{x\to 1^-} f(x) = \tan^{-1}\alpha - 5 < -6$$

$$\tan^{-1}\alpha < -1 \Rightarrow \alpha < -\tan 1$$

24. (A)

$$I = \int_{\sin^{-1}a}^{\cos^{-1}a} \left(\frac{[2x]}{[2x] + [\pi - 2x]} \right) dx$$

$$I = \int_{\sin^{-1}a}^{\cos^{-1}a} \left(\frac{[\pi - 2x]}{[\pi - 2x] + [2x]} \right) dx \quad \left(\int_a^b f(x)dx = \int_a^b f(a+b-x)dx \right)$$

$$2I = \int_{\sin^{-1}a}^{\cos^{-1}a} 1.dx$$

$$I = \frac{1}{2}(\cos^{-1}a - \sin^{-1}a) = \frac{\pi}{4} - \sin^{-1}a$$

25. (C)

$$I = \int \frac{(2x^3 \sec^2 x - 6x)\tan x}{(x^3 \sec^2 x - 3x^2 \tan x)^2} \cdot \frac{x^3}{\tan x} dx$$

since $\frac{d}{dx}(x^3 \sec^2 x - 3x^2 \tan x) = (2x^3 \sec^2 x \tan x - 6x \tan x)$

$$I = \frac{x^3}{\tan x} \times \frac{-1}{(x^3 \sec^2 x - 3x^2 \tan x)} - \int \cot^2 x dx$$

$$I = \frac{x}{3\tan^2 x - x \sec^2 x \tan x} + \cot x + x + c$$

26. (0)

$$I = \int_a^b \frac{e^{x/a} - e^{b/x}}{x} dx$$

Put $\frac{x}{a} = \frac{b}{y} \Rightarrow I = \int_b^a \frac{e^{\frac{b}{y}} - e^{\frac{y}{a}}}{\frac{ab}{y}} \left(-\frac{ab}{y^2} \right) dy = \int_a^b \frac{e^{b/x} - e^{x/a}}{x} dx = -I$

$\Rightarrow 2I = 0 \Rightarrow I = 0.$

27. (4)

$$I = \int_x^{xy} f(t)dt \Rightarrow \frac{dI}{dx} = f(xy).y - f(x)$$

$\because \dfrac{dI}{dx} = 0$

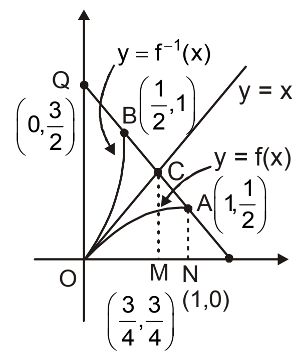
$f(xy).y = f(x)$ putting $y = \dfrac{2}{x}$

$f\left(x \cdot \dfrac{2}{x}\right) \cdot \dfrac{2}{x} = f(x) \Rightarrow f(x) = \dfrac{4}{x}$

$\int_1^e f(t)dt = 4\int_1^e \dfrac{1}{x}dx = 4[\ell nx]_1^e = 4\ \ell ne = 4$

28. (5)

Clearly $f(x) = \dfrac{2x - x^2}{2}$.



A = Area of OABO

= 2 [Area of OCM + Area of CMNA – Area of ONAO]

$= 2\left[\dfrac{1}{2} \cdot \dfrac{3}{4} \cdot \dfrac{3}{4} + \dfrac{1}{2}\left(\dfrac{3}{4} + \dfrac{1}{2}\right) \times \dfrac{1}{4} - \dfrac{1}{2}\int_0^1 (2x - x^2)dx\right] = \dfrac{5}{24}$

29. (1)

$y' = f'(x) - 4f'(4x)$

$y'(1) = f'(1) - 4f'(4) = 5$ ... (i)

$y'(4) = f'(4) - 4f'(16) = 7$ ... (ii)

Now, let y = f(x) – f(16x)

$y' = f'(x) - 16f'(16x)$, then ...(iii)

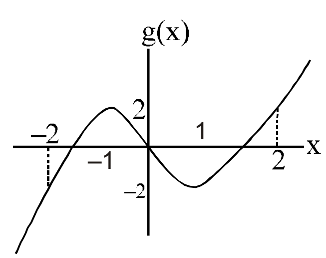
Using (i) and (ii)

$\Rightarrow\ f'(1) - 16f'(16) = 33$

$\therefore$ Derivative of f(x) – f(16x) – 32x at x = 1 is 1.

30. (5)

$f(x) = \dfrac{(x^3 - 3x)(x - \lambda)}{x - \lambda}$; consider $g(x) = x^3 - 3x$;



It is clear that $\lambda \in [-2, 2]$ number of integral values of λ is 5

31. (C)

$$\int_0^1 x^{3/2} f(x)(2-\sqrt{x}f(x))dx = \frac{1}{2} = \int_0^1 x\,dx$$

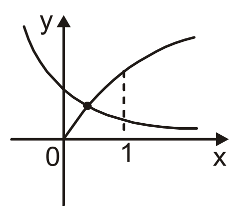
$$\Rightarrow \int_0^1 ((xf(x))^2 - 2\sqrt{x}\cdot xf(x) + (\sqrt{x})^2)dx = 0$$

$$\Rightarrow \int_0^1 (xf(x)-\sqrt{x})^2 dx = 0 \quad \Rightarrow \quad xf(x)-\sqrt{x}=0 \quad \Rightarrow \quad f(x)=\frac{1}{\sqrt{x}}$$

$$\therefore \int_0^1 x f(x)dx = \int_0^1 \sqrt{x}dx = \frac{2}{3}$$

32. (B)

$f(x) = e^x$

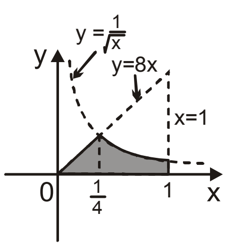


$$\Rightarrow \frac{1}{\sqrt{x}} = e^x$$

$$\Rightarrow \sqrt{x} = e^{-x}$$

which has exactly one solution in (0, 1)

33. (A)



Required area = $\int_0^{1/4} 8x\,dx + \int_{1/4}^{1} \frac{1}{\sqrt{x}}dx$ = $\frac{1}{4}+1$ = $\frac{5}{4}$

34. (D)

$$\int_\alpha^\beta f(x)dx = \frac{f(\alpha)+f(\beta)}{2}(\beta-\alpha),\ \forall \alpha,\beta \in R$$

Differentiating w.r.t. β,

$$f(\beta) = \frac{f'(\beta)}{2}(\beta-\alpha) + \frac{f(\alpha)+f(\beta)}{2}$$

$\Rightarrow f'(\beta) = \dfrac{f(\beta)-f(\alpha)}{\beta-\alpha} \Rightarrow \dfrac{f'(\beta)}{f(\beta)-f(\alpha)} = \dfrac{1}{\beta-\alpha}$

Integrating both sides w.r.t. β,

$\ln|f(\beta)-f(\alpha)| = \ln|\beta-\alpha| + \ln|c|$

$\Rightarrow f(\beta)-f(\alpha) = c(\beta-\alpha)$

Putting $\beta = x$ and $\alpha = 0$,

$f(x) = cx+\lambda$, where $\lambda = f(0)$

$f'(0) = 1 \Rightarrow c = 1$

$\int_{-1}^{1} xf(x)dx = \int_{-1}^{1}(x^2+\lambda x)dx = \dfrac{2}{3}$

35. (C)

$y = f(x) = cx+\lambda$ touches the ellipse $\dfrac{x^2}{4}+y^2 = 1$ in the first quadrant.

$\therefore$ $c<0$ and $\lambda = \sqrt{4c^2+1}$

$\therefore$ $\int_0^2 f(x)dx = \int_0^2\left(cx+\sqrt{4c^2+1}\right)dx$

$= 2\left(c+\sqrt{4c^2+1}\right) = g(c)$ (say)

$g'(c) = 0 \Rightarrow 1+\dfrac{8c}{2\sqrt{4c^2+1}} = 0 \Rightarrow c^2 = \dfrac{1}{12} \Rightarrow c = \dfrac{-1}{2\sqrt{3}}$

$\therefore$ minimum value of $g(c) = g\left(\dfrac{-1}{2\sqrt{3}}\right) = \sqrt{3}$

36. (D)

$f(x) = cx+2$ $(\because f(0) = 2)$

$f(x) = x^2 \Rightarrow x^2-cx-2 = 0$

Let α,β be the roots, where $\alpha<\beta$, then area of the region bounded by y = x^2 and y = f(x) is

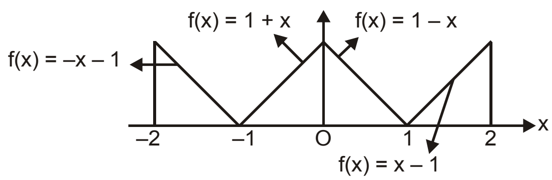
$\int_\alpha^\beta (cx+2-x^2)dx = \dfrac{c}{2}(\beta^2-\alpha^2)+2(\beta-\alpha)-\dfrac{\beta^3-\alpha^3}{3}$

$$= (\beta - \alpha)\left[\frac{c}{2}(\beta + \alpha) + 2 - \frac{\alpha^2 + \alpha\beta + \beta^2}{3}\right]$$

$$= \sqrt{c^2 + 8}\left[\frac{c^2}{2} + 2 - \frac{c^2 + 2}{3}\right] = \sqrt{c^2 + 8}\left(\frac{c^2}{6} + \frac{4}{3}\right) \geq \frac{8\sqrt{2}}{3}$$

37. A $\rightarrow$(q); B $\rightarrow$(p); C $\rightarrow$(t) ; D $\rightarrow$(s)

(A) $f(x) = \begin{cases} -x - 1 \text{ if } x \leq -1 \\ x + 1 \text{ if } -1 \leq x < 0 \\ 1 - x \text{ if } 0 \leq x < 1 \\ x - 1 \text{ if } x \geq 1 \end{cases}$



Area $= \int_{-2}^{2} f(x)dx = 4 \times \frac{1}{2} = 2$

(B) Differentiating, $2x \cdot x^2 f(x^2) = 5x^4 - 3x^2$

$x = 1 \Rightarrow f(1) = 1$

(C) We have $f(x) = x^3 + 6x^2 + px + 7$

$\therefore$ $f'(x) = 3x^2 + 12x + p < 0$

$\Rightarrow$ $3x^2 + 12x + p = k(x + 3)(x + 1)$

$\therefore$ On comparing coefficients, we get

$k = 3$ and $p = 3k$

Hence $p = 9$

(D) $I_2 = \int_0^1 \frac{\tan^{-1} x}{x} dx$, put $x = \tan\theta$

$= \int_0^{\pi/4} \frac{2\theta d\theta}{\sin 2\theta}$, put $2\theta = x$

$= \frac{1}{2}\int_0^{\pi/2} \frac{x}{\sin x} dx = \frac{1}{2} I_1 \quad \therefore \frac{I_1}{I_2} = 2$

38. $A \to (r)$; $B \to (r,s,t)$; $C \to (q)$; $D \to (q)$

(A) $$\int_{\sqrt{2}-1}^{\sqrt{2}+1} \frac{(x^2+1)^2-(x^2-1)}{(x^2+1)^2}dx = \int_{\sqrt{2}-1}^{\sqrt{2}+1}\left(1-\frac{x^2-1}{(x^2+1)^2}\right)dx$$

$$2-\int_{\sqrt{2}-1}^{\sqrt{2}+1}\frac{x^2-1}{(x^2+1)^2}dx$$

Now $I_1 = \int_{1/a}^{a}\frac{x^2-1}{(x^2+1)^2}dx$ where $a=\sqrt{2}+1$ Putting $x=\frac{1}{t}; dx = -\frac{1}{t^2}dt$

$$\int_a^{1/a}\frac{\frac{1}{t^2}-1}{\left(\frac{1}{t^2}+1\right)^2}\left(-\frac{1}{t^2}\right)dt = -\int_a^{1/a}\frac{1-t^2}{(t^2+1)^2}dt = -\int_{1/a}^{a}\frac{t^2-1}{(t^2+1)^2}dt = -I_1 \qquad \therefore I_1 = 0$$

(B) $\ell n f(x) = 1 \Rightarrow f(x) = e$ .... constant function and $D_f = (0,1)\cup(1,\infty)$

(C) $f'(x) = \frac{2^x}{x} + 2^x(\ell n 2)(\ell n x)$; $g'(x) = x^{2x}\left(2x\times\frac{1}{x}+2\ell n x\right)$

Common point is (1, 0), $f'(1) = g'(1) = 2$

(D) W $3y^2\frac{dy}{dx} - 3y - 3x\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{y}{y^2-x}$

$\frac{dy}{dx} = 0 \Rightarrow y = 0 \Rightarrow$ no real x.

$\frac{dx}{dy} = 0 \Rightarrow y^2 = x \Rightarrow y^3 = 1 \Rightarrow y = 1$

PHYSICS

39. (A)

(Hint: This can be answered using only mental calculation)

40. (D)

$i \propto \frac{1}{AE} \Rightarrow j_A : j_B : j_C : j_D = 2:2:1:1.$

41. (B)

Q = quantity of energy required

$P_1 t_1 = Q,\ P_2 t_2 = Q$

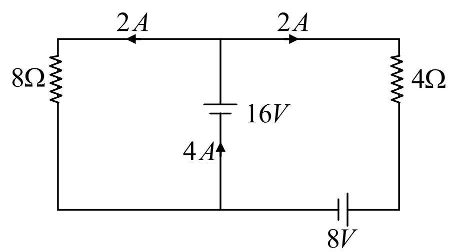
$$P_{\text{series}} = \frac{P_1 P_2}{P_1 + P_2}$$

$P_{series}\, t_0 = Q$, $\left(\frac{P_1 P_2}{P_1 + P_2}\right) t_0 = Q$

Solving $t_0 = t_1 + t_2$.

42. (B)

The simplified circuit can be drawn as



$I_1 = 4A$

$I_2 = \frac{1}{2}A$.

43. (B)

44. (A)

$\varepsilon_1 + \varepsilon_2 = I_1 R$ ...(i)

$\varepsilon_1 - \varepsilon_2 = I_2 R$ ...(ii)

From (i) and (ii)

$$\varepsilon_1 = \frac{I_1 + I_2}{I_1 - I_2} . \varepsilon_2$$

45. (2)

Given $\frac{kq}{r} = 2$

let. $\frac{kQ}{R} = V$

According to question

$\frac{4}{3}\pi r^3 \times N = \frac{4}{3}\pi R^3 \Rightarrow R = r\,N^{1/3} \Rightarrow 1 = 10^{-3} N^{\frac{1}{3}} \Rightarrow \boxed{N = 10^9}$

Hence $\boxed{Q = Nq}$

$$\Rightarrow V = \frac{KQ}{R} = \frac{KNq}{rN^{1/3}} = \left(\frac{kq}{r}\right) N^{2/3} = 2 \times 10^6 \text{ volts}$$

$\therefore \boxed{V = 2 \times 10^6 \text{ volts}}$.

46. (5)

If loop FGHDABF

iR + iR + iR + iR = 16 – 4

iR = 3

Now , $V_{GC} = V_{GF} + V_{FB} + V_{BC}$ = 16 – iR – 8 = 16 – 3 – 8 = 5 volt.

47. (2)

$$I_0 = \frac{\varepsilon}{R_2}$$

$$I_\infty = \frac{I_0}{5/2} = \frac{\varepsilon}{R_2 + 3R_1}.$$

48. (3)

$$P_1 = \frac{v^2}{R_1 + R_2}$$

$$P_2 = \frac{v^2(R_2 + R_3)}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$P_3 = \frac{v^2}{R_2 + R_3}$$

and $P_4 = \frac{v^2(R_1 + R_2)}{R_1R_2 + R_2R_3 + R_3R_1}$

This gives, $P_2P_3 = P_1P_4$

$$\therefore \quad P_4 = \frac{P_2}{P_1}P_3$$

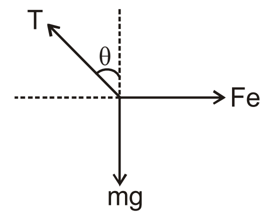
$$P_4 = \frac{120}{40} \times 20 \Rightarrow \quad P_4 = 60\,w$$

$$P_4 = \frac{3}{2}P_1$$

$\therefore \quad k = 3$.

49. (1)

F.B.D of wire



$$\tan\theta = \frac{Fe}{mg} = \frac{2K\lambda^2\ell}{mg\,d}$$

$$\Rightarrow \quad \theta = \sqrt{\frac{K\lambda^2}{mg}} = 10^{-4}\,\text{rad.}$$

50. (C)

$$2\times10^5 = \frac{q}{\epsilon_0}$$

$$q = 8.85\times10^{-12}\times2\times10^5.$$

51. (B)

$$\frac{q+q'}{\epsilon_0} = -4\times10^5.$$

52. (C) **53. (A)** **54. (A)** **55. (D)**

56. (A - q,t) ; (B - p) ; (C - r) ; (D- s,t)

$$i = neAv_d$$

$$v_d = \frac{i}{neA} = \frac{jA}{neA} = \frac{j}{ne} = \frac{\sigma E}{ne}\left[\text{where } j=\sigma E=\left(\frac{ne^2\tau}{m}\right)E\right]$$

$$\therefore \quad v_d = \frac{\sigma E}{ne} = \frac{\sigma V}{ne\,l} = \frac{eE}{m}\tau$$

(A) As l doubled up $\rightarrow v_d$ reduces to half.

(B) As V doubled up $\rightarrow v_d$ increases twice.

(C) As d doubles up $\rightarrow v_d$ remains same.

(D) As temperature increases, relaxation time decreases and ultimately drift velocity also decreases.

57. $A\rightarrow P,S;\ B\rightarrow P,Q,S;\ C\rightarrow R;\ D\rightarrow T$

Charge distribution on inner surface is governed only by presence of inner charge, wheras charge distribution on outer surface is governed by outside charges.