SOLUTIONS

WEEKLY TEST-3

RBPA

(JEE ADVANCED PATTERN)

Test Date: 09-09-2017

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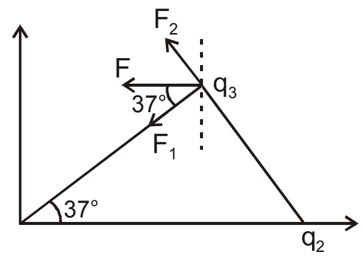
PHYSICS

1. (5)

For F to be along negative x-axis, q_1 has to be negative while q_2 has to be positive.

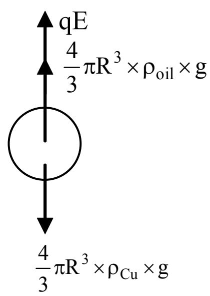
also $F_1 \cos 53 = F_2 \cos 37°$



where $F_1 = \dfrac{K.q_1 q_3}{(4cm)^2}$ and $F_2 = \dfrac{K.q_2 q_3}{(3cm)^2}$

on putting values $q_2 = \dfrac{27}{32}\,\mu C$

2. (2)



For equilibrium

$$qE + \frac{4}{3}\pi R^3 \times \rho_{oil} \times g = \frac{4}{3}\pi R^3 \times \rho_{Cu} \times g$$

$$q = \frac{\frac{4}{3}\pi R^3 (\rho_{Cu} - \rho_{oil})g}{E}$$

$$q = \frac{\frac{4}{3} \times 3.14 \times (0.5 \times 10^{-2})^3 (7.8) \times 10^3 \times 10}{3600}$$

$$q = \frac{\frac{4}{3} \times 3.14 \times 7.8 \times 0.125 \times 10^{-2}}{3600}$$

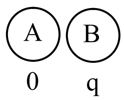
$q = 3.4 \times 10^{-5}$ C

$= 34 \times 10^{-6}$ C

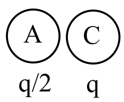
$= 34\ \mu$C

3. **[8]**

Third conductor is A, B & C have equal charge = q



Final charge = $\frac{q+0}{2} = \frac{q}{2}$



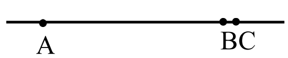
final charge = $\frac{\frac{q}{2}+q}{2} = \frac{3}{4}q$

New force = $k\frac{q}{2} \times \frac{3q}{4r^2} = \frac{3kq^2}{8r^2}$

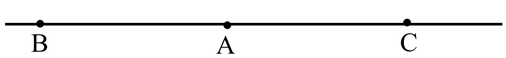
original force = $\frac{kq^2}{r^2}$

$\therefore$ new force = $\frac{3F}{8}$

4. **[3]**



f_{net} on A = $\frac{k[q_B + q_C]q_A}{r^2} = f_1$



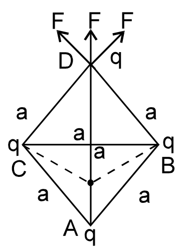
$\frac{k[q_C - q_B]q_A}{r^2} = f_2 \Rightarrow \frac{q_B + q_C}{q_C - q_B} = \frac{f_1}{f_2}$

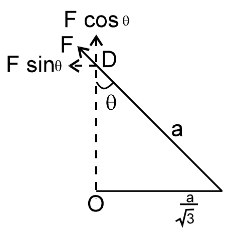
$$\frac{1+\frac{q_C}{q_B}}{\frac{q_C}{q_B}-1} = 7$$

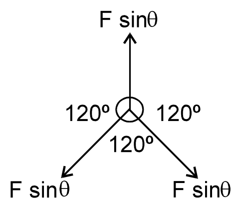
$$\Rightarrow 1+\frac{q_C}{q_B} = \frac{7.q_C}{q_B} - 7$$

$$8 = \frac{6.q_C}{q_B} \qquad \Rightarrow \qquad \frac{q_C}{q_B} = \frac{8}{6} = \frac{4}{3}$$

5. **(6)**



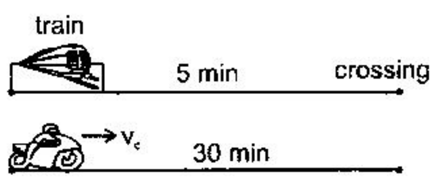




$$\cos\theta = \sqrt{\frac{2}{3}}$$

$$\therefore\ F_{net} = 3F\cos\theta = 3\frac{Kq^2}{a^2}\sqrt{\frac{2}{3}} = \frac{\sqrt{6}\ q^2}{4\pi\epsilon_0\ a^2}$$

6. **(4)**



$$t_{cycle} = \frac{10\text{ km}}{20\text{ kmh}^{-1}} = \frac{1}{2}\text{ h} = 30\text{ min}$$

$$\Delta t = 5\min = \frac{5}{60}\text{ hr}$$

Train running as per schedule

So $V_{train} = \frac{10}{(5/60)} = \frac{10 \times 60}{5}$ = 120 kmh^{-1}

7. (2)

Distance travelled from time 't-1' sec to 't' sec is

$$S = u + \frac{a}{2}(2t-1) \quad ...(1)$$

from given condition S = t ... (2)

(1) & (2) $\Rightarrow t = u + \frac{a}{2}(2t-1) \Rightarrow u = \frac{a}{2} + t(1-a)$.

Since u and a are arbitrary constants, and they must be constant for every time.

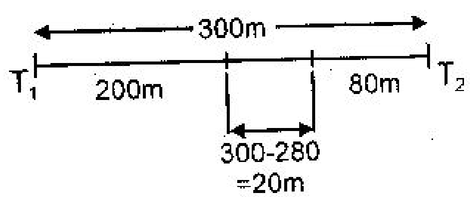
$\Rightarrow$ coefficient of t must be equal to zero.

$\Rightarrow 1 - a = 0 \Rightarrow a = 1$ for $a = 1, u = \frac{1}{2}$ unit

Initial speed is $\frac{1}{2}$ unit.

8. (2)

Initial distance between trains is 300m. Displacement of 1st train is calculated by area under V-t. curve of train $1 = \frac{1}{2} \text{ x } 10 \text{ x } 40 = 200 \text{ m}$



Displacement of train $2 = \frac{1}{2} \text{ x } 8 \text{ x } (-20) = -80 \text{ m}$.

Which means it moves towards left.

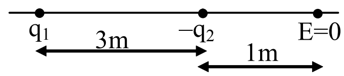
$\therefore$ Distance between the two is 20 m.

9. (A, C)

$E_A = i + 2\hat{j} + 3\hat{k}$, $E_B = \hat{i} + \hat{j} - \hat{k}$, $\vec{E}_A . \vec{E}_B = 0 \Rightarrow \vec{E}_A \perp \vec{E}_B$

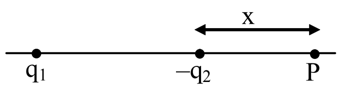
$E_B = \frac{kq}{3}$, $E_C = \frac{kq}{12}$, $|E_B| = 4|E_C|$

10. (B, C)

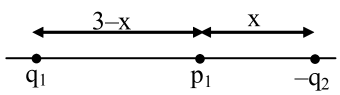


$$\frac{kq_2}{1^2} = \frac{kq_1}{16}$$

$q_1 = 16q_2$ ...(1)



$V_{at\ P} = 0$



$$-\frac{kq_2}{x} + \frac{kq_1}{3+x} = 0$$

$$+\frac{q_2}{x} = \frac{16q_2}{3+x}$$

$3 + x = 16x$

$3 = 15x$

$x = 15x$

$$x = \frac{1}{5} = 0.2 \text{ m}$$

$$\frac{kq_1}{3-x} = \frac{kq_2}{x}$$

$16x = 3 - x$

$$x = \frac{3}{17} \text{ meter}$$

11. (A, D)

12. (A, B, C)

In equilibrium position, if x_0 is stretch of spring then $kx_0 = qE$ or $x_0 = \frac{qE}{k}$

If x_m is maximum stretch of spring , $\frac{1}{2}kx_m^2 = qEx_m$ or $x_m = \frac{2qE}{k}$

Amplitude will be $x_m - x_0 = \frac{2qE}{k} - \frac{qE}{k} = \frac{qE}{k}$

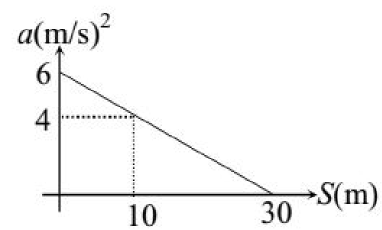
13. (A, B)

q_1 and q_2 both are negative and $\frac{kq_1}{\left(\frac{2a}{3}\right)^2} = \frac{kq_2}{\left(\frac{a}{3}\right)^2} \Rightarrow \frac{q_1}{q_2} = 4$

14. (B, C)

Area $= \frac{1}{2} \times 10 \times (6+4) = \frac{v^2}{2}$

$v =$ 10 m/s



Area upto 30 m $= \frac{1}{2} \times 30 \times 6 = \frac{v^2}{2}$

$v^2 = 180$

$v_{max} = \sqrt{180} < 14$

15. (A,B,C)

|Displacement| $\leq$ Distance. A particle revolving in a circle has zero average velocity everytime it reaches the starting point.

Average speed of a particle in a given time is never less than the magnitude of the average velocity.

It is possible to have a situation in which $\left|\frac{d\bar{v}}{dt}\right| \neq 0$ but $\frac{d}{dt}|\bar{v}| = 0$.

The average velocity of a particle is zero in a time interval. It is possible that the instantaneoous velocity is never zero in the interval.

16. (A,C,D)

$v = \sqrt{x} \Rightarrow \frac{dx}{dt} = \sqrt{x} \quad \Rightarrow \frac{dx}{x^{1/2}} = dt \Rightarrow 2\sqrt{x} = t + C$

but given at t = 0; x = 4 $\Rightarrow$ c = 4

$x = \frac{(t+4)^2}{4} \Rightarrow x = \frac{(6)^2}{4} = \frac{36}{4} = 9$ m (Putting t = 2 sec.)

$a = v\frac{dv}{dx} = \sqrt{x} \times \frac{1}{2\sqrt{x}} = \frac{1}{2}$ m/s²

17. (C)

$v = \cos\left(\frac{\pi}{3}t\right) \Rightarrow \frac{dx}{dt} = \cos\left(\frac{\pi}{3}t\right)$

$\Rightarrow x = \int_0^2 \cos\left(\frac{\pi}{3}t\right)dt = \int_0^{\frac{3}{2}} \cos\left(\frac{\pi}{3}t\right)dt + \left|\int_{\frac{3}{2}}^{2} \cos\left(\frac{\pi}{3}t\right)dt\right| \Rightarrow x = \frac{3}{\pi}\left[2 - \frac{\sqrt{3}}{2}\right]$

18. (A)

19. $mg = qE$

$m = \frac{10 \times 10^{-6} \times 10^5}{10}$ **=** 0.1 kg = 100 gm

$\therefore$ **(B)**

20. Electric field at centre of *B* due to

charge on *A* = $\frac{KQ_AX}{\left(\sqrt{R^2+X^2}\right)^3} = \frac{9 \times 10^9 \times 10^{-5} \times 0.4}{(0.5)^3}$

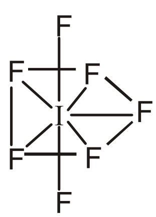
charge on *B* = zero

Net electric field = $\sqrt{\left(10^5\right)^2 + \left(2.88 \times 10^5\right)^2}$ = 3 × 10^5 N/C

$\therefore$ **(C)**

CHEMISTRY

21. (5)



22. (4)

23. (4)

24. (5)

$$\frac{PV}{RT} = 0.5$$

b = 0

$$\left(P + \frac{a}{V^2}\right)(V) = RT$$

$$\Rightarrow \; PV + \frac{a}{V} = RT$$

$$\Rightarrow \; 0.5RT + \frac{a}{V} = RT$$

$\Rightarrow$ a = 1.25

$\Rightarrow$ 4a = 5

25. (1)

60 mL total volume

Let the oxide be Cl_2O_n

$\Rightarrow$ 30 mL Cl_2 and 30 mL Cl_2O_n is present.

$$\underset{\substack{30\text{mL}\\0}}{Cl_2} + \underset{\substack{30\text{mL}\\0}}{Cl_2O_n} \xrightarrow{\Delta} \underset{\substack{0\\60\text{mL}}}{2Cl_2} + \underset{\substack{0\\15n\text{ mL}}}{\frac{n}{2}O_2}$$

$\Rightarrow$ 60 + 15n = 75 $\qquad \Rightarrow$ 15n = 15

$\Rightarrow$ n = 1

Hence the formula is Cl_2O.

$\therefore$ The number of atom is one molecule of oxide is 1.

26. (7)

Factual

27. (5)

(ii), (iv), (vii), (viii), (x) are aromatic.

(i), (iii), (v), (ix) are non aromatic.

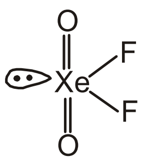
(vi) is antiaromatic.

28. **(3)**

29. **(A), (B), (C), (D)**

30. **(A), (B), (D)**

31. **(D)**



sea saw Sp^3d

32. **(A), (C)**

33. **(B)**

34. **(A), (B), (C)**

(A) Factual

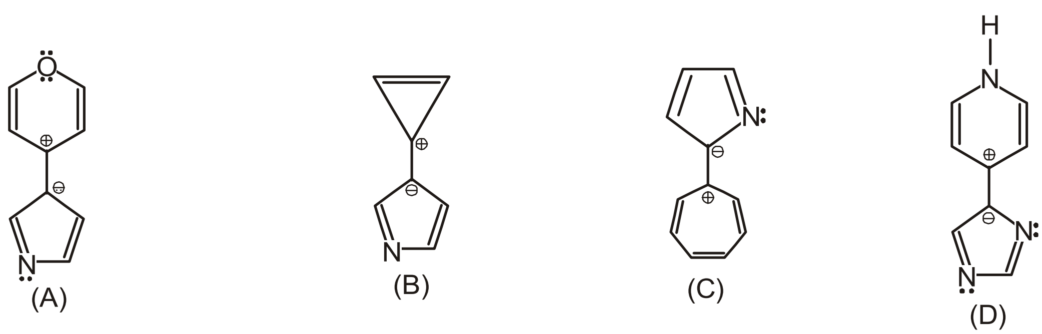
(B) Independent benzenoid rings have higher resonance energy as compared to fused rings.

(C) Independent benzenoid rings have higher resonance energy as compared to fused rings.

(D) Resonance energy increases with increasing no. of fused rings, resonance energy of phenanthrene is greater than that of Anthracene

35. **(B, C, D)**

Polar form of compound I, II, III and IV are respectively



∴ stability of D > A > C > B.

∴ stability of compound IV > I > III > IV

∴ double bond stability between two ring in IV > I > III > II

So, polar nature of indicated bond in IV > I > III > II

Also, rotation barrier along indicated bond in IV < I < III < II

36. (A,B,C,D)

$$T_C = \frac{8a}{27Rb}, P_C = \frac{a}{27b^2}, V_C = 3b$$

$$\Rightarrow \frac{T_C}{P_C} = \frac{8b}{R}$$

If $\frac{T_C}{P_C}$ is high $\Rightarrow$ b($\uparrow$) ; If V_C is high $\Rightarrow$ b($\uparrow$)

$$\frac{T_C}{V_C} = \frac{8a}{81Rb^2}.$$

If $\frac{T_C}{V_C}(\uparrow) \Rightarrow a\uparrow$; If size is small, $b\downarrow$

37. (A)

Using $Z = 1 + \frac{Pb}{RT}$

$$\Rightarrow \frac{b}{RT} = \frac{(Z-1)}{P} = \left(\frac{2-1}{1000}\right)atm^{-1} = 10^{-3}atm^{-1}$$ or $$\frac{b}{RT} = \left(\frac{2.2-1}{1200}\right)atm^{-1} = 10^{-3}atm^{-1}$$

38. (C)

$$V_m = \frac{ZRT}{P} = \left(\frac{2 \times 0.082 \times 200}{500}\right)L = 0.0656\ L$$

39. (A)

Conceptual

40. (C)

Conceptual

MATHEMATICS

41. (4)

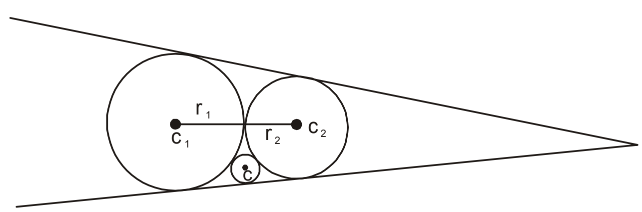
$$\because 2\sqrt{r\,r_1} + 2\sqrt{r\,r_2} = 2\sqrt{r_1\,r_2}$$

or $2\sqrt{r \times 36} + 2\sqrt{r \times 9} = 2\sqrt{9 \times 36}$

or $2 \cdot 6\sqrt{r} + 2.3\sqrt{r} = 2 \times 3 \times 6$

or $12\sqrt{r} + 6\sqrt{r} = 36$

or $18\sqrt{r} = 36$ or $\sqrt{r} = 2 \therefore r = 4$



42. (6)

$C_1 : (x-1)^2 + (y-1)^2 = 1$

$C_2 : (x-8)^2 + (y-1)^2 = 4$

The line L : 3x – 4y + k = 0 will lie between these circles if centre lie on opposite sides of L and lengths of perpendiculars drawn from centre > radius.

$\Rightarrow (3.1 - 4.1 + k)(3.8 - 4.1 + k) < 0$

or $\dfrac{|3.1 - 4.1 + k|}{5} > 1$ or $\dfrac{|3.8 - 4.1 + k|}{5} > 2 \Rightarrow k \in (-10, -4)$.

43. (2)

Equation of required straight line

$\left(x^2 + y^2 + 5x - 8y + 1\right) - \left(x^2 + y^2 - 3x + 7y - 25\right) = 0$

i.e; $8x - 15y + 26 = 0$

$\therefore$ Required distance $= \dfrac{|8(1) - 15(0) + 26|}{\sqrt{64 + 225}}$ = 34/17 = 2

44. (1)

For $0 < x \le 1 \quad a = \pi, \quad b = \pi \Rightarrow a^2 + b^2 = 2\pi^2 < 10\pi$

For $-1 \le x < 0 \quad a = 0, b = 2\pi \qquad \Rightarrow a^2 + b^2 = 4\pi^2 > 10\pi$

Hence x = – 1 is the only integral value of x for which $a^2 + b^2 > 10\pi$

45. (3)

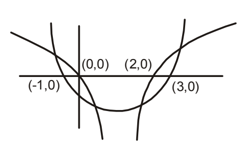
The range of $\cos^{-1} x + \cot^{-1} x - \sin^{-1}(\sin x)$ is $\left[\dfrac{\pi}{4} - 1, \dfrac{7\pi}{4} + 1\right]$

Then $\dfrac{\pi}{4} - 1 \le 2p - 1 \le \dfrac{7\pi}{4} + 1 \Rightarrow \dfrac{\pi}{8} \le p \le \dfrac{7\pi}{8} + 1$

Hence p = 1, 2, 3, three values.

46. (4)

$x^2 - 2x - 3 = \log_2 |1 - x|$



$y = x^2 - 2x - 3$

$y = \log_2 |1 - x|$

47. (2)

$x_{n+1} = \sqrt{2 + x_n}$

or $\lim\limits_{n\to\infty} x_{n+1} = \sqrt{2 + \lim\limits_{n\to\infty} x_n}$

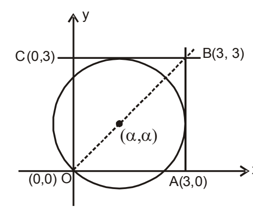
or $t = \sqrt{2+t}$ $\left(\because \lim\limits_{x\to\infty} x_{n+1} = \lim\limits_{x\to\infty} x_n = t\right)$

or $t^2 - t - 2 = 0$

or (t – 2) (t + 1) = 0 or t = 2 $(\because x_n > o \forall n, t > 0)$

48. (1)

Let OABC is the square with O(0, 0), A (3, 0), B (3, 3) and C (0, 3) circle through O touches AB and BC then centre of circle is (α, α)



$\Rightarrow \sqrt{2}\alpha = 3 - \alpha$

$\Rightarrow \alpha = \dfrac{3}{\sqrt{2}+1} = 3(\sqrt{2}-1) = 3\sqrt{2} - 3$

$\Rightarrow$ | m – n | = 1

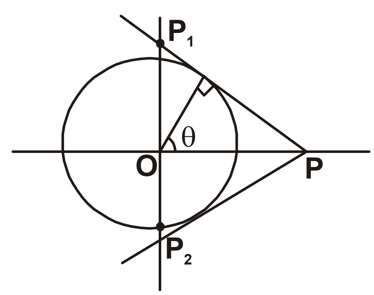
49. (A, B, D)

$9\ell^2 + 6\ell + 1 = 5\left(\ell^2 + m^2\right)$

$\Rightarrow$ perpendicular distance of (3, 0) from $\ell x + my + 1 = 0$ is $\sqrt{5}$

$\Rightarrow$ centre of the circle = (3, 0) $\Rightarrow$ radius = $\sqrt{5}$

50. (A, C)



$OP = 5\sqrt{2}\sec\theta$, $OP_1 = 5\sqrt{2}\operatorname{cosec}\theta$

Area of triangle $PP_1P_2 = \dfrac{100}{\sin 2\theta}$

Area of ΔPP_1P_2 is minimum, if $\theta = \dfrac{\pi}{4}$

OP = 10 $\Rightarrow P = (10,0)$ or $(-10,0)$

51. (A, B, C, D)

Any point on the parabola is $P\left(at^2, 2at\right)$

$\therefore$ midpoint of S (a, 0) and P (at^2, 2at) is

$$R\left(\frac{a+at^2}{2}, at\right) \equiv (h,k) \therefore h = \frac{a+at^2}{2}, k = at$$

Eliminate t,

i.e. $2x = a\left(1+\frac{y^2}{a^2}\right) = a + \frac{y^2}{a}$

i.e. $2ax = a^2 + y^2$

i.e. $y^2 = 2a\left(x - \frac{a}{2}\right)$

Its a parabola with vertex at $\left(\frac{a}{2}, 0\right)$

latus rectum = 2a

Directrix is $x - \frac{a}{2} = -\frac{a}{2}$ i.e. $x = 0$

focus is (a, 0)

52. (A,B,D)

As, L = $\sum_{r=7}^{2400} \log_7\left(\frac{r+1}{r}\right)$ = $\log_7\left(\frac{8}{7}\right)$ + $\log_7\left(\frac{9}{8}\right)$ + $\log_7\left(\frac{10}{9}\right)$ + ... + $\log_7\left(\frac{2401}{2400}\right)$

= $\log_7\left(\frac{8}{7}\times\frac{9}{8}\times\frac{10}{9}\times........\times\frac{2401}{2400}\right)$ = $\log_7\left(\frac{2401}{7}\right)$ = $\log_7(343)$ = 3.

M = $\prod_{r=2}^{1023} \log_r(r+1)$ = ($\log_2 3 \times \log_3 4 \times \log_4 5 \times ....\times \log_{1023} 1024$)

= $\log_2(1024) = \log_2 2^{10} = 10$.

Also, $N = \sum_{r=2}^{2011}\left(\frac{1}{\log_r p}\right)$

$$= \sum_{r=2}^{2011}\left(\log_p r\right) = \log_p 2 + \log_p 3 + \log_p 4 + \ldots\ldots\ldots + \log_p 2011 = \log_p(2 \cdot 3 \cdot 4 \cdot \ldots\ldots\ldots \cdot 2011)$$

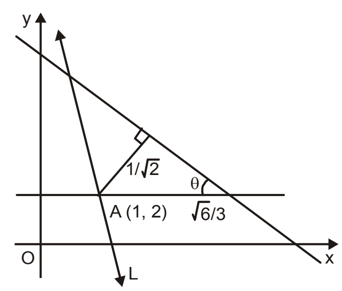
$= \log_p r = 1$

Hence, $(L + M + N) = 3 + 10 + 1 = 14$.

$L^2 + M^2 = 101$, $LMN = 30$

53. (A,D)

$$\sin\theta = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{6}}{3}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$



Angle made by L with positive x - axis can be

$$\frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12},\ \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$$

54. (A,B)

If $|x| \le 1$

then $2\tan^{-1} x = \sin^{-1}\frac{2x}{1+x^2}$

$x \ge 1,\ 2\tan^{-1} x = \pi - \sin^{-1}\frac{2x}{1+x^2}$

$x \le -1,\ 2\tan^{-1} x = -\pi - \sin^{-1}\frac{2x}{1+x^2}$

55. (A,B)

$\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi \Rightarrow \sin^{-1} + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$

$\cos^{-1} x + \cos^{-1} y = \cos^{-1}(-z) \Rightarrow xy - \sqrt{1-x^2}\ \sqrt{1-y^2} = -z \Rightarrow x^2 + y^2 + z^2 + 2xyz = 1$

AD can be true only if $x, y, z > 0$; for c put $x = y = z = 1/2$]

56. (A,D)

The given variable line can be expressed as

$\Rightarrow$ $3x + 4y - 7 + \lambda(x - 2y + 1) = 0$

$\Rightarrow$ $L_1 + \lambda L_2 = 0$

$L_1 \equiv 3x + 4y - 7 = 0$, $L_2 \equiv x - 2y + 1 = 0$

Point of intersection of $L_1 = 0$ and $L_2 = 0$ is (1, 1)

$\therefore$ $a = b = 1 \Rightarrow a + 2b = 3$

Now, $l = \underset{x\to 0^-}{\text{Limit}} \dfrac{[\sin x] - 2 + \{\cos x\}}{\{x\} - 1} = \underset{h\to 0}{\text{Limit}} \dfrac{[\sin(0-h)] - 2 + \{\cos(0-h)\}}{\{0-h\} - 1} = \dfrac{-1-2+1}{1-1} = \dfrac{-2}{0}$.

Hence l does not exist]

57. (C)

$f(-1)\ f\left(-\frac{1}{2}\right) < 0$ (and f(x) is decreasing in [–1, –1/2])

$f\left(-\frac{1}{2}\right) f\left(\frac{1}{2}\right) < 0$ (and f(x) is increasing [–1/2, 1/2])

$f\left(\frac{1}{2}\right) f(1) < 0$ (and f(x) is decreasing [1/2, 1])

58. (D)

R.H.L = $\lim_{\theta\to\frac{\pi^+}{3}} \left[\sin^{-1}(\sin 3\theta)\right]$

$\lim_{\theta\to\frac{\pi^+}{3}} \left[\pi - 3\theta\right] = -1$

L.H.L = $\lim_{\theta\to\frac{\pi^-}{3}} \left[\sin^{-1}(\sin 3\theta)\right] = \lim_{\theta\to\frac{\pi^-}{3}} \left[\pi - 3\theta\right] = 0$

Hence R.H.L = L.H.L

59. (B)

$$\lim_{\theta\to 0^+} \frac{A_1}{A_2} = \lim_{\theta\to 0^+} \frac{\sin^4 \theta/2}{(1-\cos\theta/2)^2} = 4$$

60. (A)

$$\lim_{\theta\to 0^+} \frac{A_1}{A_3} = \lim_{\theta\to 0^+} \frac{\tan(\theta/2)\sin^2 \theta/2}{1/2(\theta - \sin\theta)} = \frac{3}{2}$$