SOLUTIONS

WEEKLY TEST-12

GZRA-1901, GZR-1901(A)

GZRS-1901

(JEE MAIN PATTERN)

Test Date: 09-09-2017

Mentors® Eduserv™

Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd.,
Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS

1. (A)

$$\langle \vec{v} \rangle = \frac{x(10)-x(0)}{10-0} = \frac{(2.4\times100)-(0.12\times1000)}{10} = \frac{240-120}{10} = 12\ m/s$$

2. (B)

$$V_x = \frac{dx}{dt} = 2bt - 3ct^2$$

at t = 5 s, $v_x = (2\times2.4\times5)-(3\times0.12\times25) = 24-9 = 15\ m/s$

at t = 10 s, $v_x = (2\times2.4\times10)-(3\times0.12\times100) = 48-36 = 12\ m/s$

$$\therefore \frac{v_x|_{t=5s}}{v_x|_{t=10s}} = \frac{15}{12} = 1.25$$

3. (C)

$$V_x = 0 \Rightarrow 2bt - 3ct^2 = 0$$

$$\Rightarrow t = \frac{2b}{3c} = \frac{2\times2.4}{3\times0.12} = \frac{40}{3}\text{sec}$$

4. (C)

As, $v = \frac{dx}{dt} = 4t - 3t^2 = 0 \Rightarrow t = 0$ and $\frac{4}{3}$ seconds

$\therefore$ distance travelled in 3 seconds,

$$|S| = |\underset{0\to\frac{4}{3}s}{\vec{S}_1}| + |\underset{\frac{4}{3}s\to3s}{\vec{S}_2}| = |x(4/3)-x(0)| + |x(3)-x(4/3)| = \frac{307}{27}m$$

$$\therefore \langle v \rangle = \frac{307}{27\times3} = \frac{307}{81}\ m/s \text{ and } \langle \vec{v} \rangle = \frac{x(3)-x(0)}{3}$$

$$= -3\ ms^{-1}$$

5. (D)

From relation,

$$s_{nth} = u + (2n-1)\frac{a}{2}$$

$$10 = u + \frac{a}{2}(2\times2-1) \text{ and } 25 = u + \frac{a}{2}(2\times5-1)$$

on solving, we get, $u = \frac{5}{2}ms^{-1}$ and $a = 5ms^{-2}$

$$\therefore\ s_7 = \frac{5}{2} + \frac{5}{2}(2\times7-1) = 35m$$

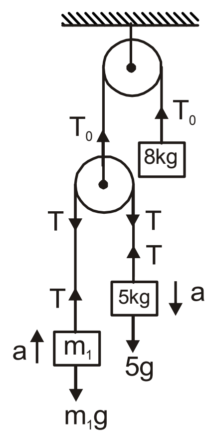
6. (B)

Here $T_0 = 8g = 80N$

Also, $T_0 = 2T$

$\therefore T = 40N$

From force diagram of 5 kg block,



$50 - T = 5a$

$50 - 40 = 5a$

$10 = 5a \quad \therefore a = 2ms^{-2}$

from force diagram of m_1,

$T - m_1g = m_1a$

$\Rightarrow\ 40 - m_1g = m_1\times2$

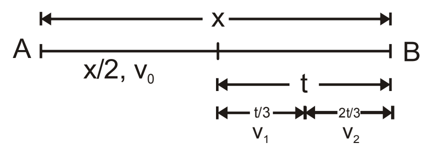
$\Rightarrow\ m_1 = \frac{40}{12} = \frac{10}{3}kg$

7. (A)

At point 'IV' graph is parallel to time axis, so velocity is zero. At point 'I', velocity is constant and positive. At point 'V' the velocity is constant and negative and at point 'II' velocity is increasing in magnitude.

8. (C)

$$< v > = \frac{x}{\frac{x}{2v_0} + \frac{3x}{2(v_1 + 2v_2)}}$$



$$< v > = \frac{2v_0(v_1 + 2v_0)}{v_1 + 2v_2 + 3v_0}$$

9. (A)

$$x^2 + y^2 = 1 \quad \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \quad \Rightarrow v_x = -\frac{y}{x}v_y$$

$$v^2 = v_x^2 + v_y^2 \Rightarrow \left(-\frac{y}{x}v_y\right)^2 + \left(v_y\right)^2 = v^2$$

$$\Rightarrow v_y^2 = \frac{v^2x^2}{x^2 + y^2}; \; v_y = \frac{\pm vx}{\sqrt{x^2 + y^2}}$$

and so, $v_x = \mp\frac{vy}{\sqrt{x^2 + y^2}}$ so velocity is,

$$\vec{v} = \frac{\left(\mp y\,\hat{i} \pm x\hat{j}\right)v}{\sqrt{x^2 + y^2}}$$

10. (A)

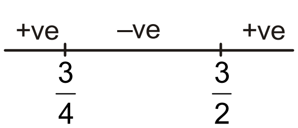
$a = \frac{dv}{dt} = (4t - 3)$. The particle retards when acceleration is opposite to velocity.

so; $\vec{a} \cdot \vec{v} = 0$

$$(4t - 3)(2t^2 - 3t) < 0$$

$(4t - 3)t(2t - 3) < 0$; As t > 0 so

$$(4t - 3)(2t - 3) < 0$$



So required time interval is

$$\frac{3}{4}s < t < \frac{3}{2}\text{second}$$

11. (A)

From constraint relation;

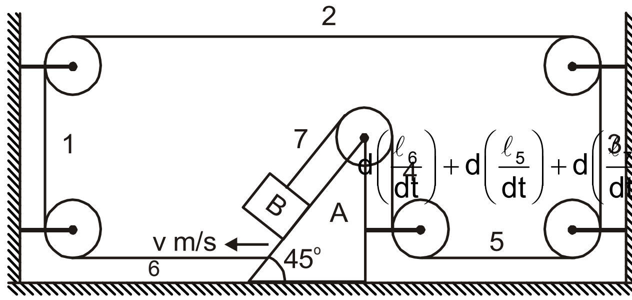
$$a_A + 2a_B + a_C = 0$$

$\therefore\ 5 + 2(-2) + a_C = 0 \qquad \therefore\ a_C = -1\ ms^{-2}$

so, $a_C = 1\ ms^{-2} \downarrow$ (downward)

12. (C)

All the strings in sections 1,2,3,4 does not change. Only 5, 6 and 7 changes. So from constraint relation; $\ell_6 + \ell_5 + \ell_7 = L$



$$\frac{d\ell_6}{dt} + \frac{d\ell_5}{dt} + \frac{d\ell_7}{dt} = 0$$

$-v + v + v_7\,(\text{block}) = 0$

$\therefore\ V_{block}$ with respect to wedge = 0

Hence velocity of block B is v m/s wrt ground.

13. (B)

From constraint relation;

$$-2v_A + 2v_B + v_C = 0$$

$2(8-5) + v_C = 0 \qquad \Rightarrow v_C = -6\ ms^{-1}$

So block C goes down with $6\ ms^{-1}$. And block C would have velocity $8\,ms^{-1}$ towards left.

Hence net velocity of block $C = \sqrt{6^2 + 8^2} = 10\ m/s$.

14. (C)

$$a = \frac{(50-30)g}{80} = \frac{g}{4}ms^{-2}$$

so,

tension developed in string connected to spring s_2 is,

$$T=\frac{2\times30\times50\times g}{(30+50)}=\frac{2\times30\times50}{80}g=37.5g$$

So, Reading of $S_2=37.5\text{kg}$ and Thus reading of S_1 = 75 kg

Reading of weighing machine $=\left(\dfrac{30g-30\frac{g}{4}}{g}\right)=\dfrac{90}{4}=22.5\text{kg}$

15. (D)

The block can be moved with least effort on a rough surface (μ) if the force is applied at an angle of friction. $\lambda=\theta=\tan^{-1}(0.75)=37^\circ$

The magnitude of external force

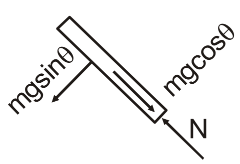
$F=mg\sin\theta=10\times10\sin37^\circ=60\text{N}$

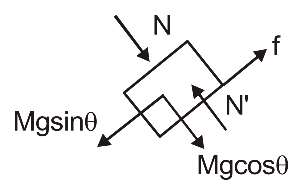
16. (A)

Angle of repose, $\phi=\tan^{-1}\left(\dfrac{3}{4}\right)=37^\circ$

here $\theta>\phi$, hence, the block has a tendency to slide down.

F.B.D of rod;





$Mg\cos\theta+N=N',\ N=mg\cos\theta$

$N'=Mg\cos\theta+mg\cos\theta$

For the block to remain stationary,

$f=Mg\sin\theta$

If f is static in nature, $f\le f_{max}$

$Mg\sin\theta\le\mu N'\Rightarrow Mg\sin\theta-\mu Mg\cos\theta\le\mu mg\cos\theta$

$\therefore\ m\ge M\left(\dfrac{\tan\theta}{\mu}-1\right)\ \Rightarrow\ m\ge12\left(\dfrac{1}{3/4}-1\right)$

$$m \geq 12\left(\frac{4}{3}-1\right) \Rightarrow m \geq 4\,kg$$

hence minimum mass of rod is 4 kg.

17. (C)

$0-t_1 \rightarrow$ uniformly retarded motion

$t_1-t_2 \rightarrow$ particle at rest

$t_2-t_3 \rightarrow$ uniform negative velocity

$t_3-t_4 \rightarrow$ particle at rest

$t_4-t_5 \rightarrow$ uniform negative velocity

18. (C)

Height of first body after time *t*, $h_1 = v_0 t - \frac{1}{2}gt^2$

Height of second body after time $(t-\tau)$, $h_2 = v_0(t-\tau) - \frac{1}{2}g(t-\tau)^2$

If they meet after time *t*, $h_1 = h_2$ $\Rightarrow t = \frac{v_0}{g} + \frac{\tau}{2}$

19. (A)

$a = v\frac{dv}{dx} = \frac{25}{(x+2)^3}$, $\frac{v^2}{2} = 25 \times \left[-\frac{1}{2(x+2)^2}\right]_o^x$, $v^2 = 25\left[\frac{1}{4} - \frac{1}{(x+2)^2}\right]$

$v = \sqrt{25\left[\frac{1}{4} - \frac{1}{(x+2)^2}\right]}$, $v_{max} = \frac{5}{2} = 2.5$ m/s (at $x = \infty$)

20. (C)

$mg - B = mf$

$B - (m-m')g = (m-m')f$

$\Rightarrow m'g = (2m-m')f \Rightarrow m' = \frac{2mf}{g+f} \Rightarrow w' = \frac{2wf}{g+f}$

21. (A)

we have a = 3 – 2t

$$\int_u^v dv = \int_0^t (3-2t)dt = 3t - \frac{2t^2}{2} = 3t - t^2$$

$$\Rightarrow \quad v = u + 3t - t^2$$

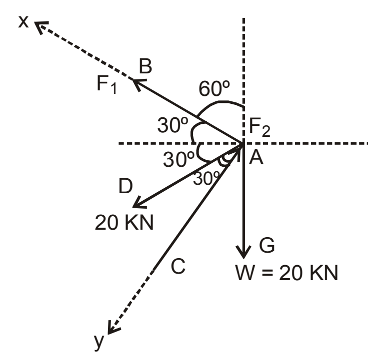
$$\Rightarrow \quad \int_{x_0}^{x_0} dx = ut + \frac{3t^2}{2} - \frac{t^3}{3} \quad \Rightarrow \quad 0 = ut + \frac{3}{2}t^2 - \frac{t^3}{3}$$

$$\Rightarrow \quad u = -\frac{3}{2}t + \frac{t^2}{3} = -\frac{3}{2}.5 + \frac{25}{3} = \frac{25}{3} - \frac{15}{2}$$

$$\Rightarrow \quad u = \frac{50-45}{6} = \frac{5}{6} \text{m/s}$$

22. (D)

Since pulley is frictionless, same force exists throughout in the flexible cable. Hence force in AD is also 20 KN as shown in figure. Also we have $AC \perp AB$ (see fig.). Selecting AB and AC as cartesian X- and Y-axis.



$\Sigma F_x = 0 \quad \Rightarrow F_1 + 20\sin 30^o - 20\sin 30^o = 0$

$\therefore \qquad F_1 = 0$

$\Sigma F_y = 0 \quad \Rightarrow -F_2 + 20\cos 30^o + 20\cos 30^o = 0$

$\Rightarrow F_2 = 40\cos 30^o \approx 20\sqrt{3}$ kN.

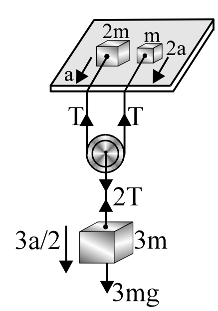
$\therefore \quad F_2 = (20\sqrt{3})\text{kN} \approx (34.6)\text{kN}$

23. (A)

Let T be the tension in the string; a be the acceleration of the mass 2m; 2a be the acceleration of mass m.

T = m.2a

The mass 3m will come down with an acceleration $\frac{a+2a}{2} = \frac{3a}{2}$



$\therefore \quad 3mg - 2T = 3m.\frac{3a}{2}$

or $\quad 3mg - 4ma = \frac{9ma}{2}$

or $\quad \frac{17a}{2} = 3g$

or $a = \frac{6}{17}g$

$\therefore$ the acceleration of 3m mass $\frac{3}{2}a = \mathbf{\frac{9}{17}g}$

24. (A)

Velocity of upper block when lower block hits obstacle.

$$u = \sqrt{2a\ell} = \sqrt{2 \times \frac{F}{2M} \times \ell} = \sqrt{\frac{F\ell}{M}}$$

Now, after collision,

retardation of upper block w.r.t. earth,

$$a = \frac{\mu Mg}{M} = \mu g$$

$$\therefore \quad V^2 = u^2 + 2as$$

$$\therefore \quad 0 = \frac{F\ell}{M} - 2\mu g \ell / 2 \quad \Rightarrow \quad \mu = \frac{F}{Mg}.$$

25. (A)

$x_A = x_B$

$10.5 + 10t = \frac{1}{2}at^2$

$a = \tan 45° = 1$

$t^2 - 20t - 21 = 0$

$t = \frac{20 \pm \sqrt{400 + 84}}{2}$

t = 21 sec

26. (D)

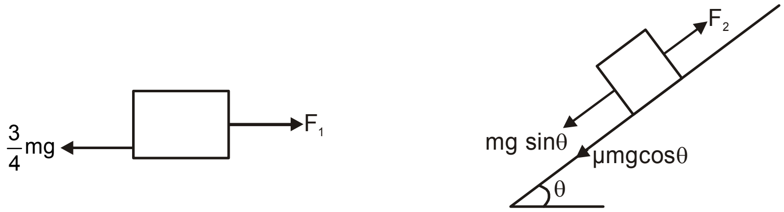
Coefficient of static friction $\mu_s = \frac{F_1}{R} = \frac{75}{mg} = \frac{75}{20 \times 10} = 0.375$

Coefficient of kinetic friction $\mu_k = \frac{60}{mg} = \frac{60}{20 \times 10} = 0.3$

27. (D)

Conceptual

28. (A)



$F_1 = \frac{3}{4}mg$ and $F_2 = mg\sin\theta + \frac{3}{4}mg\cos\theta$

$$\therefore \quad \frac{F_2}{F_1} = \frac{\sin\theta + \frac{3}{4}\cos\theta}{\frac{3}{4}} = \frac{4\sin\theta + 3\cos\theta}{3} = \frac{5}{3}\sin\left[\theta + \tan^{-1}\left(\frac{3}{4}\right)\right]$$

$$\therefore \quad \frac{F_2}{F_1} = \frac{5}{3}\sin(\theta + 37^\circ)$$

Hence for $\theta \in (0, 90^\circ)$; value of $\frac{5}{3}\sin(\theta + 37^\circ) > 1$

$\therefore \quad F_2 > F_1$ for $\theta \in (0^\circ, 90^\circ)$

29. (D)

30. (C)

During retardation, acceleration opposes velocity.

Velocity implies the direction of motion of a body.

CHEMISTRY

31. (B)

W_0 will remain same

$$\left.\begin{aligned} h\nu_1 &= W_0 + K.E_1 \\ \text{and } h\nu_2 &= W_0 + K.E_2 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} hx &= W_0 + y \\ \text{and } h(2x) &= W_0 + 3y \end{aligned}\right\}$$

Solve for $W_0 = h\nu_0 = \frac{hx}{2} \Rightarrow \nu_0 = \frac{x}{2}$

32. (A)

Frequency of revolution means number of revolution per sec $\left[= \frac{1}{\text{Time period per revolution}}\right]$

$\Rightarrow$ Frequency in nth orbit = $\frac{v_n}{2\pi r_n} \propto \frac{Z/n}{n^2/Z} = \frac{Z^2}{n^3}$ $\left[\because v_n \propto \frac{Z}{n} \text{ and } r \propto \frac{n^2}{Z}\right]$

$$\Rightarrow \frac{(\text{Freq. of revolution of } e^- \text{ in } He^+(Z=2))_{n=3}}{(\text{Freq. of revolution of } e^- \text{ in } H(Z=1))_{n=2}} = \frac{\frac{2^2}{3^3}}{\frac{1^2}{2^3}} = \frac{32}{27}$$ [2nd Excited state means n = 3]

33. (C)

First line in Lyman series correspond to transition $2 \to 1 \Rightarrow \frac{1}{\lambda} = R \times 1^2 \times \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4} \times R$

and 2nd line in Balmer series corresponds to transition $4 \to 2 \Rightarrow \frac{1}{\lambda} = R \times Z^2 \times \left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3}{16} \times RZ^2$

$\Rightarrow \frac{3}{4}R = \frac{3}{16}R \times Z^2 \Rightarrow Z = 2$

Thus, $E_2 = -13.6\frac{Z^2}{n^2}\text{eV} = -13.6 \times \frac{2^2}{2^2}\text{eV} = -13.6 \text{ eV}$

34. (D)

Ground state e^- in H atom can only be excited by energy greater than 10.2 eV. Thus, 15 eV photon energy will ionize the atom and 8.4 eV photon will not be able to excite the electron at all.

Only 12.75 eV excite the to higher states.

Now, Thus, will be excited to n = 3

–0.85 eV ———————— n = 4
–1.51 eV ———————— n = 3
–3.4 eV ———————— n = 2
–13.6 eV ———————— n = 1

During de-excitation, corresponding to six transitions : wavelength will be emitted.

35. (A)

As e^- moves from higher to lower orbit : $E_n \downarrow \Rightarrow K.E._n \uparrow$ $[\because K.E._n = -E_n]$

Similarly, $P.E._n \downarrow$ $[\because P.E._n = 2E_n]$

Angular momentum(L) = $\frac{nh}{2\pi} \Rightarrow L \downarrow$ as $n \downarrow$

$\lambda_c = \frac{h}{mv_n} [\because K.E_n \uparrow \Rightarrow v_n \uparrow] \Rightarrow \lambda_c \downarrow$

36. (B)

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{\frac{h}{\sqrt{2m_p K.E_p}}}{\frac{h}{\sqrt{2m_\alpha K.E_\alpha}}} = \sqrt{\frac{m_\alpha K.E_\alpha}{m_p K.E_p}} = \sqrt{4 \cdot \frac{K.E_\alpha}{K.E_p}} = 4 \cdot \frac{V_\alpha}{V_p} \quad [\because m_\alpha = 4m_p]$$

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \frac{1}{2} \text{ if } \frac{V_\alpha}{V_p} = \frac{1}{8} \Rightarrow V_p : V_\alpha \equiv 8:1 \text{ or } \frac{K.E._\alpha}{K.E._p} = \frac{1}{16}$$

37. (D)

Probability $\propto 4\pi r^2 \Psi^2 dr$

38. (B)

$P_1 = \frac{5 \times RT}{V}$; p.pr. of He $= \frac{2}{5} \times \left(\frac{5RT}{V}\right)$

$= \frac{2 \times 0.0821 \times 400}{8.21}$

$= 8$ atm

39. (B)

$P_{gas} = P_{dry gas} + P_{moisture}$ at T K

or $P_{dry} = 830 - 30 = 800$

Now at $T_2 = 0.99\ T_1$;

at constant volume $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$P_{dry} = \frac{800 \times 0.99T}{T} = 792$ mm

$\therefore\ P_{gas} = P_{dry} + P_{moisture}$

$= 792 + 25 = 817$ mm

40. (C)

$$P \propto d;\ P = kd \text{ and } k = \frac{1\,\text{atm}}{1\,\text{metre}}$$

$$PV = nRT;\ kd\left(\frac{1}{6}\pi d^3\right) = nRT;$$

$$\frac{d_1^4}{d_2^4} = \frac{n_1}{n_2};\frac{1}{4^4} = n_1/n_2; n_2 = 256$$

no. of moles added = 256 –1 = 255

41. (B)

$$2P + 3Q \longrightarrow P_2Q_3$$

$$P + 2Q \longrightarrow PQ_2$$

$$M_{P_2Q_3} = \frac{15.9}{0.15} = 2P + 3Q$$

and $M_{PQ_2} = \frac{9.3}{0.15} = P + 2Q$

$\Rightarrow P = 26 \text{ and } Q = 18$

42. (B)

	HCl	$+ KOH \longrightarrow$	KCl	$+ H_2O$
Mmoles	$0.2V$	$0.4V$		
MMoles	$-$	$0.2V$	$0.2V$	

Mmoles $K^+_{Total} = 0.2V + 0.2V$

$\Rightarrow$ MmoleK$K^+_{Total} = 0.2V + 0.2V$

Mmoles OH^- = 0.2 V and Mmoles Cl^- = 0.2 V

$\Rightarrow [K^+] = 0.2M; [OH^-] = 0.1M, [Cl^-] = 0.1M$

43. (D)

Clearly,

2 moles NH_3 production $\equiv$ 1 mole Ca required (theoretically)

Yield overall = 0.5 × 1 × 0.5 = 0.25

$\Rightarrow$ Actual Ca required $= \frac{1}{0.25} = 4$ moles

44. (C)

$\Delta X = 2\times10^{-6}$ m

$\Delta X \times \Delta V \geq \frac{h}{4\pi m}$

$\Delta V = 28.5$ m/s

45. (D)

$NH_3(g)$	+	$HCl(g) \longrightarrow$	$NH_4Cl(s)$
$\frac{2PV}{RT}$ mole		$\frac{5PV}{RT}$ mole	0
0		$\frac{3PV}{RT}$ mole	–

$P_f = \frac{nRT}{2V} = \frac{3PVRT}{2VRT} = 1.5P$

46. (A)

$\Rightarrow N < S < F < Cl.$

47. (B)

$\Delta H_{hyd}\ \alpha\ \frac{1}{\text{size of ion}}$

48. (D)

All are iso-electronic species.

49. (A)

$\Delta H_{ion.} = -\Delta Heg$

50. (C)

Maximum Jump = IP_7

$\therefore$ No. of V.S.E. = 6

Group - No = ViA/ 16th

51. (B)

F > Cl > Cl^- > F^-

|

due to high electron gain enthalpy

$iP\,\alpha\,\frac{1}{\text{size}}$

52. (A)

Cr = 3d

Mo = 4d

W = 5d

$1P_1$ = 3d < 4d < 5d

53. (C)

54. (B)

55. (A)

56. (B)

57. (A)

Allred-Roschew's scale

$$En = 0.359\frac{Zeff}{r^2} + 0.744$$

Zeff = Z – σ

r = 1.175 Å

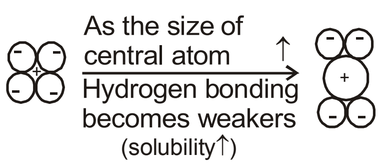
58. (A)

As the lattice energy increases solubility will decreases.

(i) BeF_2 > Ca F_2 > Mg F_2 – solubility

(ii) LiH CO_3 < $NaHCO_3$ < $KHCO_3$ – solublity

Through Hydrogen bonding Bi-Carbonate ions (Na^+ is best fitted in the void) and an moving down the group with inc. Size of central atom Hydrogen bonding becomes weaker and hence solubility inc. down the group.



$$H.B.\,\alpha\frac{1}{solubility}[H.B.\,\alpha\ small\,size]$$ (H.B – Hydrogen bond)

(iii) Li_2SO_4 < Na_2SO_4 < K_2SO_4 – solubility

(iv) LiOH < NaOH < KOH – order of solubility

59. (A)

(i) LiF > NaF > KF > RbF : Lattice energy

(iii) Li^+ < Mg^{2+} < Al^{3+} :Hydration energy

60. (D)

MATHEMATICS

61. (D)

orthocentre of triangle BCH is the vertex A(–1, 0)

62. (A)

The circumcentre of ΔPQR will be orthocentre of ΔABC which is at (1, 1).

63. (A)

ar (quad. ABDC) = ar (ΔABC) + ar (ΔCBD) $= \frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 8 & 1 & 1 \\ 4 & 3 & 1 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} 4 & 3 & 1 \\ 8 & 1 & 1 \\ 6 & 6 & 1 \end{vmatrix} = 14$

64. (D)

The point Q is (–b, –a) and the point R is (–a, –b)

$\therefore$ mid point of PR is (0, 0)

65. (A)

Circumcentre O $\equiv \left(-\frac{1}{3}, \frac{2}{3}\right)$ and orthocentre H $\equiv \left(\frac{11}{3}, \frac{4}{3}\right)$

$\therefore$ coordinate of centroid G is (1, $\left(1, \frac{8}{9}\right)$

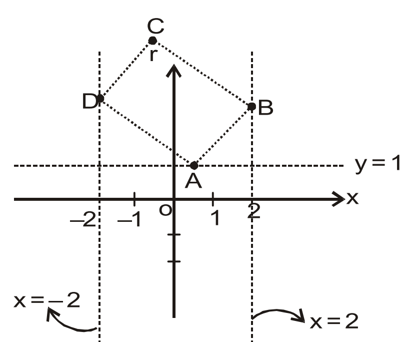
A(1, 10), G $\left(1, \frac{8}{9}\right)$

AG : GD = 2 : 1

$\therefore$ D = $\left(1, -\frac{11}{3}\right)$

$\therefore$ coordinate of the mid point of BC is $\left(1, -\frac{11}{3}\right)$

66. (D)



Let equation of AB be y = x + a

$\therefore$ A(1 – a, 1) and B(2, 2 + a)

$\therefore$ equation of AD is

y – 1 = – 1 (x – 1 + a)

$\therefore$ D (– 2, 4 – a)

Let C(h,k)

$\Rightarrow$ h + 1 – a = 2 – 2 $\Rightarrow$ h = a – 1 and $k + 1 = 2 + a + 4 - a \Rightarrow k = 5$

$\therefore$ Locus of C(h,k) is $y = 5$

67. (C)

68. (D)

$\tan(180° - \theta) = \text{slope of } AB = -3$

$\therefore \tan\theta = 3$

$\therefore \frac{OC}{AC} = \tan\theta, \frac{OC}{BC} = \cot\theta$

$\Rightarrow \frac{BC}{AC} = \frac{\tan\theta}{\cot\theta} = \tan^2\theta = 9$

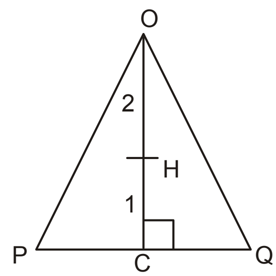
69. (A)

In an equilateral triangle the orthocentre and the centroid are the same. OPQ is the equilateral triangle in which $OC \perp PQ$.

Clearly, the point H which divides OC internally in the ratio 2:1 is the orthocentre.

Clearly, $OC = \frac{1}{\sqrt{2}}$. So, $OH = \frac{2}{3} \times \frac{1}{\sqrt{2}}$

$\therefore H = \left(\frac{2}{3\sqrt{2}}\cos 45°, \frac{2}{3\sqrt{2}}\sin 45°\right)$



70. (A)

$$\begin{vmatrix} 1 & -2 & 3 \\ k & 3 & 1 \\ 4 & -k & 2 \end{vmatrix} = 0 \Rightarrow 1(6+k) - k(3k-4) + 4(-2-9) = 0$$

$-3k^2 + 5k - 38 = 0 \Rightarrow \text{Discriminant} < 0$

Thus no real value of k

71. (C)

As we know that diagonals of a square are perpendicular to each other.

Let the equation of other diagonal is

$x + 7y = k.$

Also, passes through (–4, 5).

$\therefore -4 + 35 = k$

$\Rightarrow k = 31$

$\therefore$ Required equation is $x + 7y - 31 = 0$

72. (D)

Let the point be P(h,h). Given that, PA= PB

$\Rightarrow \sqrt{(h-5)^2 + h^2} = \sqrt{h^2 + (h-3)^2}$

$\Rightarrow 4h = 16$

$\Rightarrow$ h = 4

$\therefore$ Coordinates of P be (4,4)

73. (C)

The given points are collinear, if Area of Δ = 0

$$\Rightarrow \begin{vmatrix} k & 2-2k & 1 \\ -k+1 & 2k & 1 \\ -4-k & 6-2k & 1 \end{vmatrix} = 0$$

Appling $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} k & 2-2k & 1 \\ -2k+1 & 4k-2 & 0 \\ -4-2k & 4 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -2k+1 & 4k-2 \\ -4-2k & 4 \end{vmatrix} = 0$$

$\Rightarrow$ k= – 1 or k= 1/2, neglecting 1/2, as when k = 1/2, points are same.

74. (C)

Let the line 3x + y – 9 = 0 divides the line segment joining A (1,3) and B (2,7) in the ratio k : 1 at point C

then the coordinates of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

$\because$ C lie on line 3x + y – 9 = 0, so it satisfies the equation of line.

$$3\left(\frac{2k+1}{k+1}\right) + \left(\frac{7k+3}{k+1}\right) - 9 = 0 \qquad \therefore \quad k = \frac{3}{4}$$

So, the required ratio is 3 : 4 (internally)

75. (B)

Let the pair of straight lines cut the x-axis at $(x_1, 0)$ and $(x_2, 0)$ and y-axis at $(0, y_1)$ and $(0, y_2)$

$$x^2 + \alpha xy + 3y^2 - 5x - 9y + \beta = 0$$

Putting y = 0, $x^2 - 5x + \beta = 0$

$$\therefore \quad (x_2 - x_1)^2 = (x_1 + x_2)^2 - 4x_1x_2 = \left(\frac{5}{1}\right)^2 - 4\beta = 25 - 4\beta$$

Again putting x = 0, $3y^2 - 9y + \beta = 0$

$$\therefore \quad (y_2 - y_1)^2 = \left(\frac{9}{3}\right)^2 - 4\frac{\beta}{3} = 9 - \frac{4\beta}{3}$$

Now, $(x_2 - x_1)^2 = (y_2 - y_1)^2 \Rightarrow 25 - 4\beta = 9 - \frac{4\beta}{3} \Rightarrow \beta = 6$.

Also, $\Delta = 0 \Rightarrow 3\beta + \frac{45}{4}\alpha - \frac{81}{4} - \frac{75}{4} - \frac{\beta.\alpha^2}{4} = 0 \Rightarrow \alpha = \frac{7}{2}$ or 4

76. (D)

Bisectors of the lines in both the cases will remain unchanged

$\therefore$ Required equation is $\frac{x^2 - y^2}{1-(-1)} = \frac{xy}{-p} \Rightarrow px^2 + 2xy - py^2 = 0$.

77. (A)

This can happen, if three lines are real and distinct as well as angle between any two adjacent sides is $\frac{2\pi}{3}$.

$\Rightarrow f(m) = bm^3 + dm^2 + cm + a = 0$ has 3 distinct real roots

and $\frac{m_1 - m_2}{1+m_1m_2} = \frac{m_2 - m_3}{1+m_2m_3} = \frac{m_3 - m_1}{1+m_3m_1} = \pm\sqrt{3} \Rightarrow 3 + m_1m_2 + m_2m_3 + m_3m_1 = 0$

and $3bm^2 + 2dm + c = 0$ has roots α, β with $f(\alpha)f(\beta) < 0 \Rightarrow 3b + c = 0$.

78. (A)

For $\lambda = 0, \frac{x}{1} + \frac{y}{3} - 1 = 0$ and $\frac{x}{2} + \frac{y}{4} - 1 = 0$ are one pair of possible lines

$\frac{x}{1} + \frac{y}{4} = 1, \frac{x}{2} + \frac{y}{3} = 1$ are the other pair of possible lines.

$\therefore 2\left(1 - \frac{y}{3}\right) + \frac{y}{4} = 1 \Rightarrow y = \frac{12}{5}; x = \frac{2}{5}$

Also $\left(\frac{2}{5} + \frac{4}{5} - 1\right)\left(\frac{1}{5} + \frac{3}{5} - 1\right) + \lambda\frac{2}{5}\cdot\frac{12}{5} = 0 \Rightarrow \lambda = \frac{1}{24}$

79. (D)

Equation of the lines joining the origin to the points of intersection of the given curves is

$$3x^2 + pxy - 4x(y + 2x) + 1.(y + 2x)^2 = 0 \quad \Rightarrow \quad x^2 - pxy - y^2 = 0$$

which are perpendicular for all values of p.

80. (D)

Equation of lines joining the origin to the Points of intersection of given lines is

$3x^2 + mxy - 4x(2x + y) + 1(2x + y)^2 = 0 \Rightarrow -x^2 + (m-4)xy + y^2 = 0$

which are perpendicular for all value of m

81. (D)

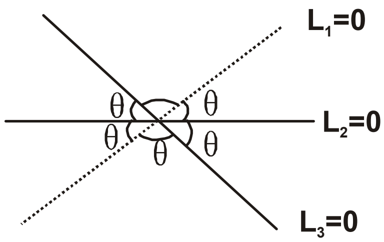
$$\frac{a}{\sqrt{cb}} - 2 = \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{b}} \Rightarrow a = b + c + 2\sqrt{bc} \Rightarrow a = \left(\sqrt{b} + \sqrt{c}\right)^2$$

$$\Rightarrow (\sqrt{a} - \sqrt{b} - \sqrt{c})(\sqrt{a} + \sqrt{b} + \sqrt{c}) = 0 \Rightarrow \sqrt{a} + \sqrt{b} + \sqrt{c} = 0 \text{ (not valid)}$$

$$-\sqrt{a} + \sqrt{b} + \sqrt{c} = 0$$

hence x = – 1, y = 1

82. (A)



$$3\theta = 180^\circ \Rightarrow \theta = 60^\circ$$

83. (D)

Since points AB = AC = 1 and triangle is right angle at A. We have

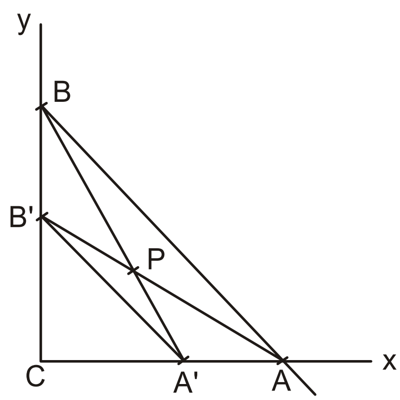
$$\tan\alpha . \tan\beta = -1$$

$$\Rightarrow \cos(\alpha - \beta) = 0 \Rightarrow \alpha - \beta = \frac{\pi}{2}$$

84. (B)

Take the co-ordinate axes along CA and CB.

Let CA = a and CB = b.



Equation to AB' is $\frac{x}{a} + \frac{y}{kb} = 1$

Equation to $A'B$ is $\frac{x}{ka} + \frac{y}{b} = 1$

$$\left[\text{since } \frac{CA'}{CA} = \frac{CB'}{CB} = k(\text{say})\right]$$

Let $P \equiv (\alpha, \beta)$ be the point of intersection of AB' and $A'B$, then

$$\alpha = \frac{k}{k+1}a, \qquad \beta = \frac{k}{k+1}b$$

$\therefore$ Locus of (α, β) is $\frac{x}{a} = \frac{y}{b}$

85. (A)

P lies on perpendicular bisector of BC and at a distance of $\frac{\sqrt{3}}{2}BC = \sqrt{15}$ units from the mid-point of BC.

$\therefore$ P can be $\left(\sqrt{3}, 2-\sqrt{3}\right)$ or $\left(-\sqrt{3}, 2+2\sqrt{3}\right)$ but A and P should lie on the same side of BC.

$\therefore$ P is $\left(-\sqrt{3}, 2+2\sqrt{3}\right)$

86. (C)

Equation of reflected ray passing through (4, 5) and (12, 9) is x – 2y + 6 = 0.

Point of incidence $\equiv (6,6)$

87. (D)

$ar(\Delta PAB) = ar(\Delta PAC)$

$\Rightarrow \frac{1}{2}\times 2\times |y| = \frac{1}{2}|1(y-2)+x(2-0)+0(0-y)| \quad \Rightarrow \quad 2x - y = 2 \text{ or } 2x + 3y = 2$

$$\therefore \quad \tan\theta = \left|\frac{2+\frac{2}{3}}{1-2.\frac{2}{3}}\right| = 8.$$

88. (C)

shifting the origin to (2, 5) the lines are

3 (x + 2) – 2 (y + 5) + 5 = 0

k (x + 2) + 6 (y + 5) –3 = 0

or 3x – 2y + 1 = 0

kx + 6y + 2k –3 = 0

Now, $c_1c_2\left(a_1a_2 + b_1b_2\right) < 0 \qquad \Rightarrow \left(2k-3\right)\left(3k+6\left(-2\right)\right) < 0$

$\left(2k-3\right)\left(3k-12\right) < 0$

$\frac{3}{2} < k < 4$

89. (B)

$\Rightarrow \left(2\alpha - 9 + 4\right)\left(4+3+4\right) < 0 \quad \Rightarrow \left(2\alpha - 5\right) < 0 \ \Rightarrow \alpha < \frac{5}{2}$

90. (C)

Given point can be written as $\left(\frac{(1-k)x_1 + kx_2}{(1-k)+k}, \frac{(1-k)y_1 + ky_2}{(1-k)+k}\right)$

Now this point divides the line segment Joining (x_1, y_1) and (x_2, y_2) internally in the ratio k : 1 – k so we must have

k > 0 & 1 – k > 0

$\Rightarrow 0 < k < 1$

$\Rightarrow k \in (0,1)$