SOLUTIONS

PROGRESS TEST-5

CD-1802

(JEE MAIN PATTERN)

Test Date: 07-10-2017

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PHYSICS

1. In first case $\frac{1}{15}=(1.5-1)(\frac{1}{R_1}-\frac{1}{R_2})$

In second case, $\frac{1}{f}=\left(\frac{1.5}{4/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

dividing, we get f = 60 cm

∴ **(B)**

2. Before collision

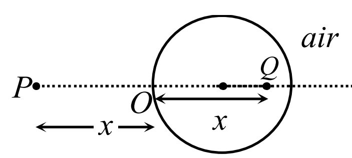
$\vec{V}_{\text{image}}=\vec{V}_{\text{object}}=-\vec{\mathbf{u}}$

After collision

$\vec{V}_{\text{image}}=2\vec{V}_{\text{object}}=\mathbf{2\vec{u}}$

∴ **(C)**

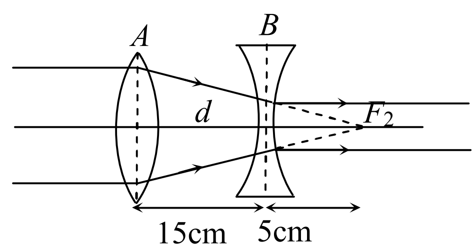
3. $\frac{1.5}{x}+\frac{1}{x}=\frac{1.5-1}{R}$, $\frac{2.5}{x}=\frac{0.5}{R}$



$x=5R$

∴ **(A)**

4. In the absence of concave lens the parallel beam will be focused at F_2 i.e. at a distance 20 cm from lens A. The focal length of concave lens is 5 cm i.e. if this lens is placed at 15 cm from A, then beam will become parallel. So, d = 15 cm



∴ **(B)**

5. **(D)**

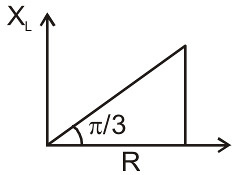
6. For arrangement P,

$$\frac{1}{f_{eq}}=\frac{1}{2R}+\frac{1}{2R}-\frac{2}{3R}=\frac{1}{3R}$$ (R = radius of curvature)

For arrangement Q, $\frac{1}{f_{eq}}=\frac{1}{2R}+\frac{1}{2R}-\frac{1}{3R}=\frac{2}{3R}$

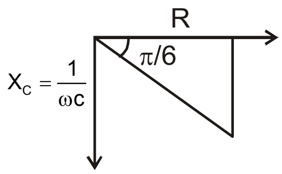
∴ **(C)**

7. **(A)**



$\tan\frac{\pi}{3} = \frac{\omega L}{R} = \sqrt{3}$

$\omega L = 10\sqrt{3}$



$\tan\frac{\pi}{6} = \frac{1}{\omega L.R} = \frac{1}{\sqrt{3}}$

$\omega C = \frac{\sqrt{3}}{10}$

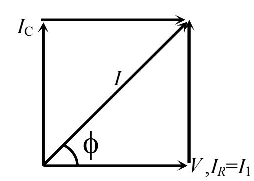
$\frac{\omega L}{\omega C} = \frac{10\sqrt{3}}{\sqrt{3}/10} = 100$

8. $I_{max} = Q.\omega = C.V.\ \frac{1}{\sqrt{LC}} = V_0\sqrt{\frac{C}{L}}$

maximum energy stored = $\frac{1}{2}LI_{max}^2 = \frac{1}{2}CV_0^2$

$\therefore$ **(D)**

9. Using method of phasors, $\tan\phi = \frac{I_C}{I_1} = \frac{4}{3}$



$\phi = 53^0$

so phase difference between I and I_1 is 53^0

$\therefore$ **(C)**

10. For maximum current $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$

$\therefore\ L = \frac{1}{\omega^2 C} = \frac{1}{(100\pi)^2 \times 1 \times 10^{-6}} = \frac{100}{\pi^2}$ H

$\therefore$ **(B)**

11. $\frac{v_1^2}{P_1} = \frac{v_2^2}{P_2} = \text{Resistance}$ $\Rightarrow$ $P_2 = 25\ W$

$\therefore$ **(D)**

12. Comparison of two equations show that

$\phi = \frac{\pi}{2}$ $\therefore$ $\mathrm{P = E_t\, I_t\ \cos\varphi = zero.}$

$\therefore$ **(D)**

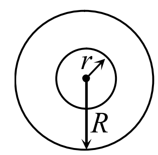
13. Due to the time varying magnetic field induced electric field will be set-up and its lines are in clockwise sense, so force on stationary charge *q* is along (4)

$\therefore$ **(D)**

14. $e = L\frac{dI}{dt}$ $\Rightarrow$ $L = 0.04\,\mathrm{H}$

$\therefore$ **(C)**

15. No current is enclosed in the circle, so from Ampere's circuital law, the magnetic induction at any point inside the infinitely long straight thin walled tube (cylindrical) is zero.



$\therefore$ **(B)**

16. **(A)**

17. **(A)**

$$e = \left|\frac{d\phi}{dt}\right| = \frac{4B_0A - B_0A}{t} = \frac{3A_0B_0}{t}$$

18. **(D)**

19. The Magnitude of magnetic field due to circular loop at the center *C* is

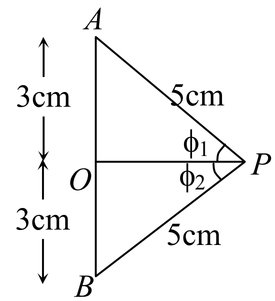
$$B = \frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ **(D)**

20. $R = \frac{mV}{qB} = \frac{\sqrt{2mq\Delta V}}{qB}$ So, $\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$

$\therefore$ **(C)**

21. Now $B = \frac{\mu_0 I}{4\pi r}(\sin\phi_1 + \sin\phi_2)$



Here $r = OP$

Now, $AO = OB = \frac{6}{2}\text{cm} = 3\,\text{cm}$ and

$PB = PA = 5\,\text{cm}$

$\therefore \quad OP = \sqrt{(PB)^2 - (OB)^2}$ = 4 cm

$\therefore \quad r = 4\,\text{cm}$ = 4 × 10^{-2} m

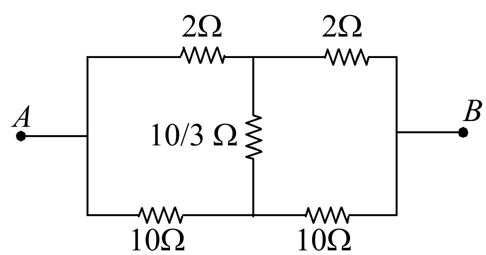
$\sin\phi_1 = \frac{3}{5}$ and $\sin\phi_2 = \frac{3}{5}$

$\therefore$ **(C)**

22. Potential difference between *A* and *B* is zero the current through *R* is zero.

$\therefore$ **(D)**

23. Equivalent ciruit is balanced Wheat- stone bridge as shown



$R_{AB} = \frac{10}{3}\Omega$

$\therefore$ **(A)**

24. $V_{AB} = \frac{6 \times 30}{50}$ = 3.6 V

Terminal voltage of cell $= \frac{2 \times 1.5}{2}$ = 1.5 V

Using $V = kl \Rightarrow 1.5 = \frac{3.6}{300} l$ or l = 125 cm

$\therefore$ **(B)**

25. As usual method of electric field

$\therefore$ **(C)**

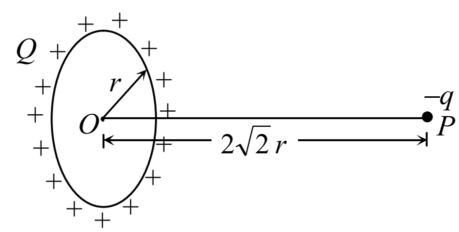
26. $T = \dfrac{2\pi R}{\sqrt{\dfrac{GM}{R}}}$, $T \propto R^{3/2}$

Radius of 2nd satellite is 1% greater

Hence time period is $1 \times \dfrac{3}{2}$ = 1.5 % larger

$\therefore$ **(C)**

27. K.E. of –*q* at *O* = $q(V_O - V_P)$



$= q\left[\dfrac{Q}{4\pi\varepsilon_0 r} - \dfrac{Q}{4\pi\varepsilon_0\sqrt{r^2 + (2\sqrt{2}\,r)^2}}\right]$

$= \dfrac{2}{3}\dfrac{qQ}{4\pi\varepsilon_0 r} = \dfrac{qQ}{6\pi\varepsilon_0 r}$

$\therefore$ **(C)**

28. $V' = \dfrac{Q}{C^1} = \dfrac{Q\left[d' - t\left(1 - \dfrac{1}{K}\right)\right]}{\varepsilon_0 A}$, $V = \dfrac{Q}{C} = \dfrac{Qd}{\varepsilon_0 A}$

Now, $V' = V \Rightarrow d = d' - t\left(1 - \dfrac{1}{K}\right)$

$d' - d = 2\,\text{mm},\ t = 3\,\text{mm} \Rightarrow$ $2 = 3\left(1 - \dfrac{1}{K}\right) \Rightarrow K = 3$

$\therefore$ **(B)**

29. **(D)**

30. **(D)**

CHEMISTRY

31. (A)

$$\frac{P_o - P_s}{P_s} = \frac{n}{N}$$

or, $$\frac{P_o - P_s}{P_s} = \frac{w_B}{M_B} \times \frac{M_A}{w_A} = \frac{185 - 183}{183} = \frac{1.2}{M_B} \times \frac{58}{100}$$

$$\frac{185 - 183}{183} = \frac{1.2 / M_B}{100 / 58}$$

M_B = 64 g/mol

32. (B)

$$\Delta T_f = K_f \frac{w_B}{M_B \times w_A} \times 1000$$

$$\therefore \quad \Delta T_b = k_b \frac{w_B}{M_B \times w_A} \times 1000$$

$$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b}{K_f} \Rightarrow \frac{\Delta T_B}{+0.186} = \frac{0.512}{1.86}$$

$$\Delta T_b = 0.0512^o C$$

33. (A)

$$P_T = P_A^0 + \left(P_B^0 - P_A^0\right) x_B$$

$$120 = 150 + (50 - 150) x_B$$

$$-30 = -100\ x_B$$

$$x_B = \frac{3}{10}\ ; \quad x_A = \frac{7}{10}$$

$$y_A = \frac{P_A^0\, x_A}{P_T} = \frac{150x}{120x} \frac{7}{10}$$

$$y_B = \frac{P_B^0 x_B}{P_T} = \frac{50}{120} \times \frac{3}{10}$$

$$\frac{y_A}{y_B} = \frac{150 \times 7}{50 \times 3} = \frac{7}{1}$$

34. (C)

$$\Delta T_f = K_f \times ix \frac{nNaCl}{1}$$

$$nNaCl = \frac{\Delta T_f}{K_f \times 2} \times 1$$

$= \frac{1}{500 \times 2} = .001$

$wt = .001 \times 58.5 = .0585\,g$

35. (B)

$\frac{1}{2}A \longrightarrow 2B$

$rate = \frac{-1}{½}\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$

or, $\frac{-d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$

36. (B)

$kt_{½} = 2.303 \log_{10}\frac{100}{50}$ ....(i)

$kt_{99\%} = 2.303 \log_{10}\frac{100}{1}$ ....(ii)

$(ii) \div (i)$

$\frac{t_{99\%}}{t_{½}} = \frac{\log_{10}^{10^2}}{\log_{10}^{2}} = \frac{2}{\log_{10}^{2}}$

or, $t_{99\%} = \frac{2}{0.301} \times t_{½} = \frac{2}{0.301} \times 6.93 = 46.06$ min.

or, $k = \frac{0.693}{t_{y_2}} = \frac{0.693}{6.93} = 0.1$

also, $kt_{99\%} = 2.303 \log_{10}\frac{100}{10}$

or 0.10 $t_{99\%}$ =2.303 × 2

or, $t_{99\%} = \frac{4.606}{0.1} = 46.06$ min.

37. (C)

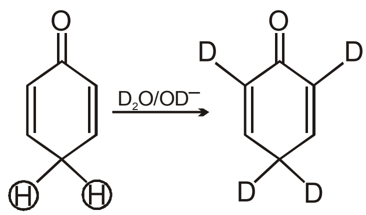
38. (D)

39. (A)

40. (C)

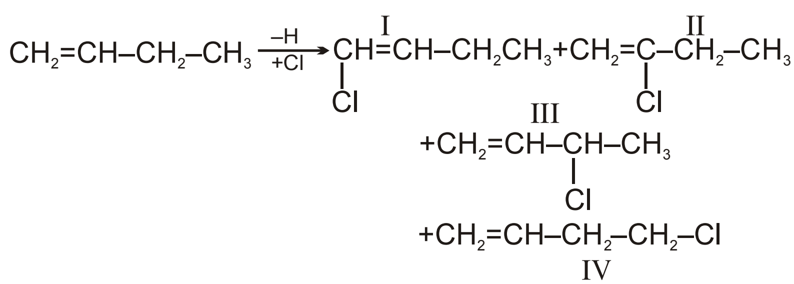
41. **(A)**

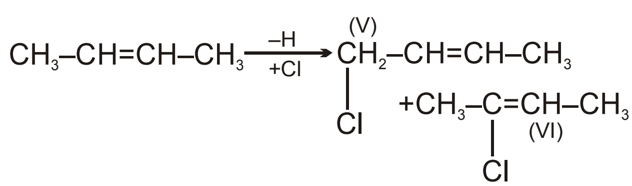
Whenever-ve charge can be created and -v charge goes after resonance from that sites H-atom will be replaced by D.

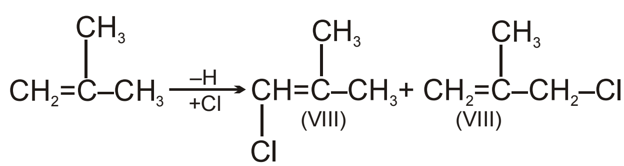


42. **(A)**

Corresponding alkene of C_4H_7Cl formula is C_4H_8 which shows three structural isomers from which H-atoms can be replaced by Cl atoms.



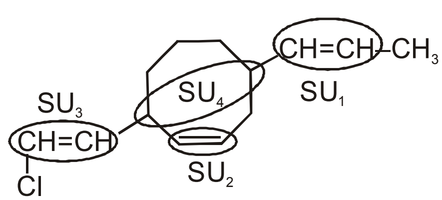




Total acydic structural isomers = (8)

43. **(C)**

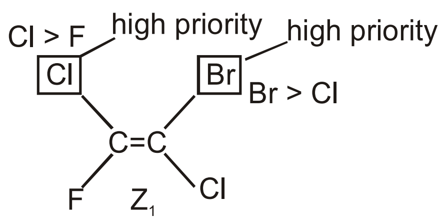
Those unit which is responsible to generate geometrical isomer is stereogenic unit for G.I

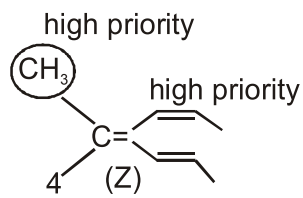


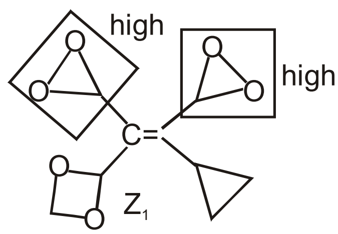
Total stereogenic unit for GI = 4

SO = Stereogenic unit

44. (D)

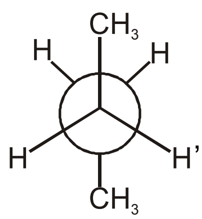
(A) 

(B) 

(C) 

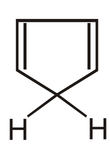
45. (D)

Anti staggered is most stable because two bulkier gr CH_3 is far from eachother.

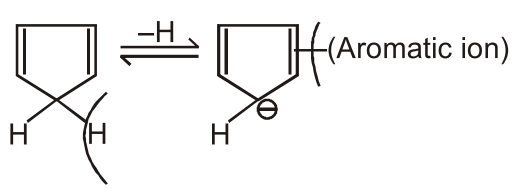


, fully eclipsed and partially eclipsed forms are less stable due to high magnitude of Vander Wall's strain and torsional strain.

46. (A)

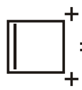


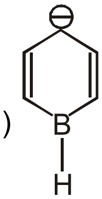
This molecule belongs to carbon acid categeries. Which after lose of H^+ ion achieves aromatic structure

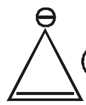


Due to formation of aromatic ion reaction shifts towards forward direction which makes Ka value more.

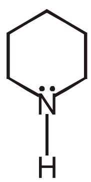
47. (D)

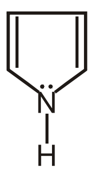
(A)  = Aromatic
(i) cyclic
(ii) planar
(iii) $(4n + z)\ \pi = 2\pi = n = 0$

(B)  Aromatic

(C)  (ϕ) Anti Aromatic

48. (D)

 is most basic due to presence of lone pair in sp^3 hybride orbital

 Due to involvement of lone pair in aromatic sextet it is least basic

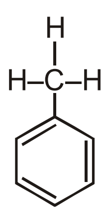
49. (C)

Due to SIR on NO_2 duet presence of two me gr with drawing effect of NO_2 B legs.

50. (D)

Electron releasing power of alkyl gr by H.C effect

α no. of $\alpha(C-H)$ bonds



electron density of benzene ring is high.

No. of $\alpha C-H = 3$

51. (A) 52. (C) 53. (C) 54. (C) 55. (A) 56. (B)
57. (D) 58. (A) 59. (A) 60. (B)

MATHEMATICS

61. (A)

Given $\sqrt{9x^2+6x+1}<(2-x)$

$\Rightarrow \sqrt{(3x+1)^2}<(2-x) \quad \Rightarrow |3x+1|<(2-x) \quad \Rightarrow -(2-x)<3x+1<2-x,\ 2-x>0$

When $-(2-x)<(3x+1)$

$\Rightarrow x>-\frac{3}{2}$

When $3x+1<2-x$

$\Rightarrow x<\frac{1}{4} \qquad \therefore \ x\in\left(-\frac{3}{2},\frac{1}{4}\right)$

62. (A)

$\tan 27x-\tan x=(\tan 27x-\tan 9x)+(\tan 9x-\tan 3x)+(\tan 3x-\tan x)$

$=\left(\frac{\sin 27x}{\cos 27x}-\frac{\sin 9x}{\cos 9x}\right)+\left(\frac{\sin 9x}{\cos 9x}-\frac{\sin 3x}{\cos 3x}\right)+\left(\frac{\sin 3x}{\cos 3x}-\frac{\sin x}{\cos x}\right)$

$=\frac{\sin 18x}{\cos 27x.\cos 9x}+\frac{\sin 6x}{\cos 9x.\cos 3x}+\frac{\sin 2x}{\cos 3x.\cos x}$

$=2\left[\frac{\sin 9x}{\cos 27x}+\frac{\sin 3x}{\cos 9x}+\frac{\sin x}{\cos 3x}\right]$

63. (A)

$\left(\sqrt{1+\frac{3}{2\log_3 x}}\right)\log_3 x+1=0$, Let $\log_3 x=y$

$\Rightarrow 2y^2+3y-2=0 \Rightarrow y=\frac{1}{2},-2 \Rightarrow y=\frac{1}{2}$ not satisfy the equation

So $y=-2 \Rightarrow x=\frac{1}{9}$ solution of given equation

64. (D)

(1,1) belongs to none of R, S and T.

Hence none is reflexive.

R is transitive, since x > y and y > z, then x > z.

Clearly, T = { (6,1),(2,2)} is transitive

Also R is not symmetric.

Since 6 > 3, but $3 \not> 6$. S is symmetric, since x + y = y + x.

65. (B)

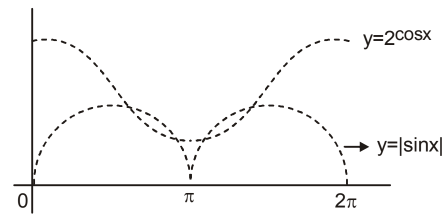
$\because$ Range of $\dfrac{x^2}{1+x^2}$ is [0, 1)

$\therefore$ Range of f(x) $= \left[0, \dfrac{\pi}{2}\right)$

66. (B)

Graph of y = $2^{\cos x}$ and y = |sinx| meet four times in [0, 2π]

Thus, total number of solutions = 4 + 4 + 4 + 2 = 14.



67. (B)

$$\lim_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t\log(1+t)}{4+t^4}\,dt$$

Using L' Hospital's rule,

$$= \lim_{x\to 0} \frac{\dfrac{x\log(1+x)}{4+x^4}}{3x^2} = \lim_{x\to 0} \frac{\log(1+x)}{3x}\cdot\frac{1}{4+x^4}$$

$$= \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12} \qquad \left[\text{Using } \lim_{x\to 0}\frac{\log(1+x)}{x} = 1\right]$$

68. (A)

$$\sin^{-1}\sin\left(\frac{22\pi}{7}\right) = \sin^{-1}\sin\left(3\pi + \frac{\pi}{7}\right) = -\frac{\pi}{7}$$

$$\cos^{-1}\cos\left(\frac{5\pi}{3}\right) = \cos^{-1}\cos\left(2\pi - \frac{\pi}{3}\right) = \frac{\pi}{3}$$

$$\tan^{-1}\tan\left(\frac{5\pi}{7}\right) = \tan^{-1}\tan\left(\pi - \frac{2\pi}{7}\right) = -\frac{2\pi}{7}$$

$$\sin^{-1}\cos(2) = \frac{\pi}{2} - \cos^{-1}\cos 2 = \frac{\pi}{2} - 2$$

$$\therefore \text{Required Value} = -\frac{\pi}{7} + \frac{\pi}{3} - \frac{2\pi}{7} + \frac{\pi}{2} - 2 = \frac{(-18+35)\pi}{42} - 2 = \frac{17\pi}{42} - 2$$

69. (A)

t = $\sin^2 x$.

$I=\frac{1}{2}\int e^t(2-t)dt=\frac{3}{2}e^t-t\frac{e^t}{2}+C=\frac{1}{2}e^{\sin^2 x}(3-\sin^2 x)+C$

70. (A)

$$\int e^x\left(\ln x+\frac{2}{x}-\frac{1}{x^2}\right)dx=\int e^x\left(\ln x+\frac{1}{x}+\frac{1}{x}-\frac{1}{x^2}\right)dx=e^x\left(\ln x+\frac{1}{x}\right)+C_1$$

$$\int\left(e^x\left(\ln x+\frac{1}{x}\right)+C_1\right)dx=e^x\ln x+C_1x+C_2$$

71. (A)

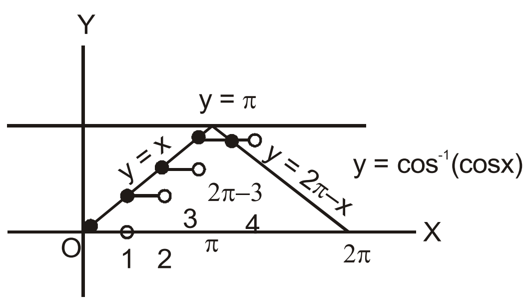
$f'(x)=3kx^2-18x+6$

For monotonic increasing function

$f'(x)\ge 0\ \forall x\in R$

$\Rightarrow 3kx^2-18x+6\ge 0 \Rightarrow kx^2-6x+2\ge 0 \Rightarrow D\le 0 \Rightarrow 36-8k\le 0\Rightarrow k\ge\frac{18}{4} \Rightarrow k\ge 4.5$

72. (B)



at x = 0, 1, 2, 3, 2π –3.

i.e. five solutions

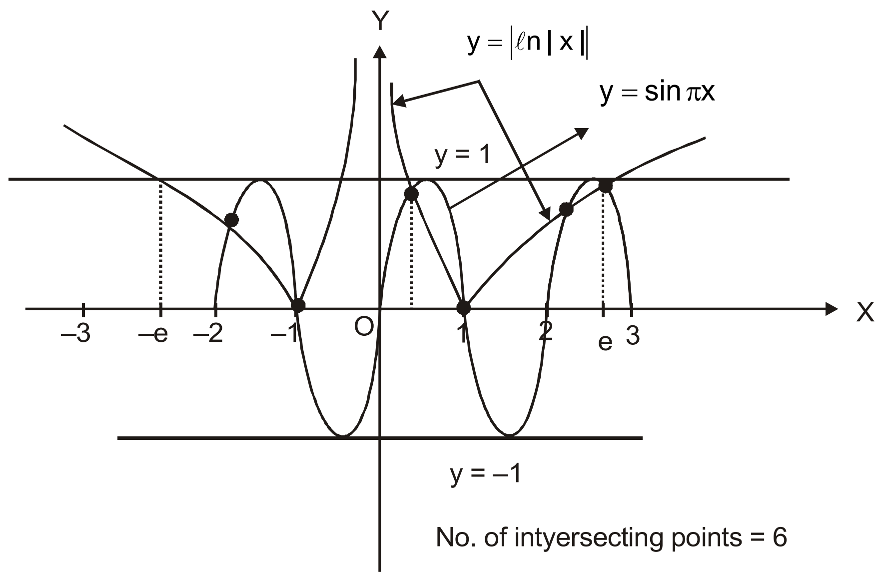
73. (A)

$y=x^3 \Rightarrow \frac{dy}{dx}=3x^2 \Rightarrow \left.\frac{dy}{dx}\right|_{x=-1}=3$

and $\left.\frac{dy}{dx}\right|_{x=1}=3$. Hence tangents are parallel.

74. (C)

from x = –2 to x = e there are 6 intersecting points of $y=\sin\pi x$ and $y=|\ell n|x||$



No. of intyersecting points = 6

75. (C)

Given $f(x) = \sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{\ldots\ \infty}}}}$

$\Rightarrow \quad f(x) = (1-\cos x)^{\frac{1}{2}}(1-\cos x)^{\frac{1}{4}}(1-\cos x)^{\frac{1}{8}}\ldots\infty$

$\Rightarrow \quad f(x) = (1-\cos x)^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\infty} \Rightarrow f(x) = (1-\cos x)^{\frac{\frac{1}{2}}{1-\frac{1}{2}}} \Rightarrow f(x) = 1 - \cos x$

The range of f(x) is [0, 2].

76. (B)

$$\int \frac{2x+3}{(x^2+3x)(x^2+3x+2)+1}dx$$

put $x^2 + 3x = t$ $\quad \Rightarrow \quad (2x + 3)\, dx = dt$

$$\int \frac{dt}{t(t+2)+1} ; \int \frac{dt}{(t+1)^2} = C - \frac{1}{t+1} = C - \frac{1}{x^2+3x+1}$$

$\Rightarrow$ a = 1, b = 3, c = 1 $\Rightarrow$ a + b + c = 5 Ans.

77. (D)

$$\int_0^1 \frac{\pi}{4} e^{\tan\frac{\pi x}{4}} dx$$

Put $\tan\frac{\pi x}{4} = t$ $\quad\therefore\quad \frac{\pi}{4}\sec^2\frac{\pi x}{4}dx = dt \quad \Rightarrow dx = \frac{4dt}{\pi(1+t^2)}$

$$\int_0^1 \frac{\pi}{4} e^{\tan\frac{\pi x}{4}} dx = \int_0^1 e^t \frac{dt}{(1+t^2)}$$

$$= \left[e^t \tan^{-1} t\right]_0^1 - \int_0^1 e^t \tan^{-1} t\, dt = e.\frac{\pi}{4} - \int_0^1 e^x \tan^{-1} x\, dx$$

$$\Rightarrow \int_0^1 \left(\frac{\pi}{4} e^{\tan\frac{\pi x}{4}} + e^x \tan^{-1} x\right) dx = \frac{\pi e}{4}$$

78. (B)

$$\text{Given integral} = \int_0^{\pi} \tan^{-1}\left(\frac{\cos\frac{x}{2} + \sin\frac{x}{2} - \left|\cos\frac{x}{2} - \sin\frac{x}{2}\right|}{\cos\frac{x}{2} + \sin\frac{x}{2} + \left|\cos\frac{x}{2} - \sin\frac{x}{2}\right|}\right) dx$$

$$= \int_0^{\pi/2} \tan^{-1}\tan\frac{x}{2}dx + \int_{\pi/2}^{\pi} \tan^{-1}\cot\frac{x}{2}dx$$

$$= \int_0^{\pi/2} \frac{x}{2}dx + \int_{\pi/2}^{\pi}\left(\frac{\pi}{2} - \frac{x}{2}\right)dx = \frac{\pi^2}{8}$$

79. (A)

$$\int_0^1 [kx]dx \quad (k \in N)$$

$$= \int_0^1 (kx - \{kx\})dx = \frac{k}{2} - k\int_0^{1/k} kx\, dx = \frac{k}{2} - \frac{k^2}{2}.\frac{1}{k^2} = \frac{k-1}{2}$$

$$\therefore \int_0^1 ([x] + [2x] + ... + [20x])dx = \frac{1}{2}(0 + 1 + ... + 19) = 95$$

80. (B)

$$\lim_{x\to\frac{\pi}{4}} \frac{2\sqrt{2}-(\cos x+\sin x)^3}{1-\sin 2x} = \lim_{x\to\frac{\pi}{4}} \frac{2\sqrt{2}-2\sqrt{2}\sin^3\left(\frac{\pi}{4}+x\right)}{1-\sin 2x}$$

put $x=\frac{\pi}{4}+h$

$$\therefore \lim_{h\to 0}\frac{2\sqrt{2}(1-\cos^3 h)}{(1-\cos 2h)} = \lim_{h\to 0}\frac{2\sqrt{2}(1-\cos h)(1+\cos h+\cos^2 h)}{2\sin^2 h} = \frac{3}{\sqrt{2}}$$

81. (D)

Use $\frac{x^2-5x+6}{x^2+x+1}>0 \Rightarrow x\in(-\infty,2)\cup(3,\infty)$ and $[x^2-1]>0 \Rightarrow x\in(-\infty,-\sqrt{2}]\cup[\sqrt{2},\infty]$

$\Rightarrow x\in(-\infty,-\sqrt{2}]\cup[\sqrt{2},2)\cup(3,\infty)$

82. (A)

Required sum = $\int_0^1 \frac{1+x}{1+x^2}dx$

$= \left[\tan^{-1}x\right]_0^1 + \left[\frac{1}{2}\ell n(1+x^2)\right]_0^1 = \frac{\pi}{4}+\frac{1}{2}\ell n2$

83. (C)

LHS = secx + cosecx = $2\sqrt{2} \Rightarrow = \frac{\pi}{4}$ or $\frac{11\pi}{12}$

Note: Let f(x) = secx + cosecx

f(x) decreases in $\left(0,\frac{\pi}{4}\right)$ & then increases $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

at $x=\frac{\pi}{4},\quad f(x)=2\sqrt{2}$

f(x) increases in $\left(\frac{\pi}{2},\pi\right)$ from $(-\infty,\infty)$ so it attains $2\sqrt{2}$ at exactly one value of x

84. (B)

We have $I = \int_2^4 \left(x(3-x)(4+x)(6-x)(10-x) + \sin x \right) dx$ ....(1)

Now $I = \int_2^4 \left((6-x)(3-(6-x))(4+(6-x))(6-(6-x))(10-(6-x)) + \sin(6-x) \right) dx$

$= \int_2^4 \left((6-x)(x-3)(10-x)x(4+x) + \sin(6-x) \right) dx$ ....(2)

$\therefore$ On adding (1) and (2), we get

$2I = \int_2^4 \left(\sin x + \sin(6-x) \right) dx = \left(-\cos x + \cos(6-x)\right)_2^4$ = – cos 4 + cos 2 + cos 2 – cos 4

= 2(cos 2 – cos 4)

Hence I = cos 2 – cos 4

85. (C)

$\lim_{x\to 0} \left(\cos x + a^3 \sin\left(b^6 x\right)\right)^{\frac{1}{x}} \quad \left(1^{\infty} \text{ form}\right) = e^{\lim_{x\to 0} \frac{\cos x + a^3 \sin\left(b^6 x\right) - 1}{x}}$

$= e^{\lim_{x\to 0}\left(\frac{-(1-\cos x)}{x} + \frac{a^3 \sin\left(b^6 x\right)}{b^6 x} \times b^6\right)} = e^{a^3 b^6} = e^{(ab^2)^3} = e^{512}$

(Given) Hence $ab^2 = 8$

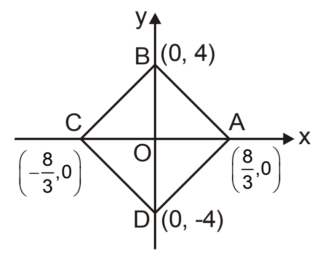
86. (A)

[x] is discontinuous at all integral points, sinx is continuous for all x,

87. (C)

$3|x| + 2|y| \le 8 \Rightarrow \begin{cases} 3x + 2y \le 8 & x \ge 0, \quad y \ge 0 \\ 3x - 2y \le 8 & x \ge 0, \quad y \le 0 \\ -3x + 2y \le 8 & x \le 0, \quad y \ge 0 \\ -3x - 2y \le 8 & x \le 0, \quad y \le 0 \end{cases}$

$\therefore$ Enclosed area ABCD



$= 4 \cdot$ Area of ΔAOB

$= 4 \cdot \frac{1}{2} \cdot \frac{8}{3} \cdot 4 = \frac{64}{3}$ sq. units.

Now, $f(x + y) = f(x) \cdot f(y)$

and $f(1) = 2$

$\therefore$ $f(2) = 2^2$ and $f(3) = 2^3$

Similarly $f(n) = 2^n$, where $n \in N$.

88. (D)

Here $x \in (0, 1)$

$\therefore$ $(\sin x)^x < 1^x = 1 \Rightarrow I_1 < 1$

$x^{\sin x} > x^1 \Rightarrow I_2 > \frac{1}{2}$

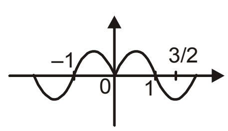
$(\sin x)^x < x^x < x^{\sin x} \Rightarrow I_1 < I_2$

89. (D)

$$\int_{-2014}^{2014} \left[\sin x^7 + x^{13} \cos x\right] dx + 2 \int_{-2014}^{2014} dx = 8056$$

90. (C)

$y = \sin |\pi x|$



$$I = \int_{-1}^{0} x \sin \pi x \, dx + \int_{0}^{1} x \sin \pi x \, dx - \int_{1}^{3/2} x \sin \pi x \, dx$$

$$= 2\int_{0}^{1} x \sin \pi x \, dx - \int_{1}^{3/2} x \sin \pi x \, dx$$