SOLUTIONS

PHASE TEST-1

RB-1810 TO 1812

RBK-1805

JEE MAIN PATTERN

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Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS

1. **(A)**

$(\vec{A}_1 + 2\vec{A}_2) . (3\vec{A}_1 - 4\vec{A}_2)$

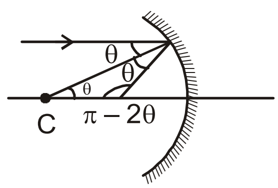
$= 3|\vec{A}_1|^2 + 2|\vec{A}_1||\vec{A}_2|\cos\theta - 8|\vec{A}_2|^2$

$= (|\vec{A}_1|^2 + 2|A_1||\vec{A}_2|\cos\theta + |\vec{A}_2|^2) + 2|\vec{A}_1|^2 - 9|\vec{A}_2|^2$

$= 3^2 + 2 \times 2^2 - 9 \times 3^2 \qquad = 9 + 8 - 81 = -64$

2. **(C)**

$$\cos(\pi - 2\theta) = \frac{d^2 + d^2 - R^2}{2d^2} = 1 - \frac{R^2}{2d^2}$$



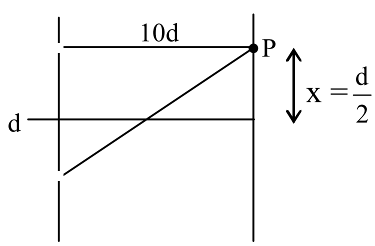
$$\Rightarrow -\cos 2\theta = 1 - \frac{R^2}{2d^2}$$

$$\Rightarrow \frac{R^2}{2d^2} = 1 + \cos 2\theta = 2\cos^2\theta$$

$$\Rightarrow d = \frac{R}{2\cos\theta}$$

3. **(B)**

4. **(A)**



Δx at P = $\frac{dx}{D}$ = $\frac{d^2}{2D}$ = $\frac{(5\lambda)^2}{2 \times 10 \times d}$

Δx = $\frac{(5\lambda)^2}{2 \times 10 \times 5\lambda}$ = $\frac{\lambda}{4}$

$\Delta\phi$ = $\frac{2\pi}{\lambda} \times \Delta x$ = $\frac{\pi}{2}$

$I_0 = 4I \Rightarrow I = \frac{I_0}{4}$

$I_{net} = I + I + 2\sqrt{I}\sqrt{I}\cos\frac{\pi}{2} = 2I = \frac{I_0}{2}$

5. **(D)**

6. **(C)**

$\Delta = (n + 4)\lambda - n\lambda = 4\lambda$

at Y point, forms $\boxed{\text{Fourth Bright Fringe}}$

7. **(D)**

$f_o = 1.5$ cm, $f_e = 6.25$ cm, $u_o = -2$ cm, $v_e = -D = -25$cm

By objective lens $\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

$\frac{1}{1.5} = \frac{1}{v_o} - \frac{1}{-2} \Rightarrow \frac{1}{v_o} = \frac{1}{1.5} - \frac{1}{2}$ or $v_o = 6$ cm

By eye piece $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$

$\frac{1}{6.25} = \frac{1}{-25} - \frac{1}{-u_e} \Rightarrow \frac{1}{u_e} = \frac{1}{6.25} + \frac{1}{25} = \frac{4}{25} + \frac{1}{25} = \frac{1}{5}$

$u_e = 5$ cm, Length of tube = $L = v_o + u_e = 6.0$ cm + 5.0 cm, $L = 11$ cm

8. **(C)**

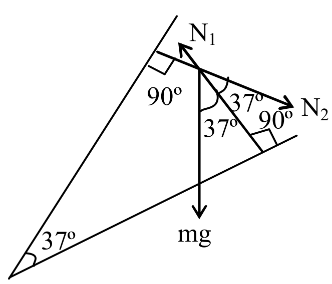
9. **(D)**

10. **(A)**

$2kx\cos 30^\circ = \left(\frac{4m_1m_2}{m_1 + m_2}\right)g$

11. **(A)**

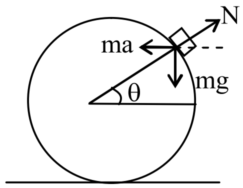
Using lami's theorem



$\frac{mg}{\sin(180^\circ - 37^\circ)} = \frac{N_2}{\sin(180^\circ - 37^\circ)}$

$\therefore N_2 = mg$

12. (C)



By F.B.D of m

N sinθ = mg and N cosθ = ma

$\therefore$ tanθ = $\frac{g}{a}$ $\Rightarrow$ a = g cotθ

$\therefore$ F = (m + M)g cot θ

13. (A)

As there is no friction, horizontal force on B is therefore F = 100 N

$\therefore$ a = $\frac{100}{20}$ = 5m/s^2

but no horizontal force on A acts therefore T = 0

14. (C)

T = m(g – a) $\Rightarrow$ 360 = 60(10 – a) $\Rightarrow$ a = 4m/s^2

15. (C)

When all are pulling

$\vec{F}_{net} = 100 \times 3\hat{i}$ ……(1)

When 'A' stops

$\vec{F}_{net} - \vec{F}_A = 100 \times 1(-\hat{i})$ ……(2)

When 'B' stops

$\vec{F}_{net} - \vec{F}_B = 100 \times 24\hat{j}$

from these three get

$\vec{F}_A + \vec{F}_B$ and solve

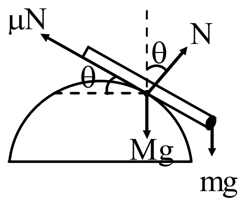
16. (A)

$\frac{3}{4}$ th energy is lost i.e., $\frac{1}{4}$ th kinetic energy is left. Hence, its velocity becomes $\frac{v_0}{2}$ under a retardation of μg in time t_0.

$\therefore \ \frac{v_0}{2} = v_0 - \mu g\, t_0$

or $\mu g\, t_0 = \frac{v_0}{2}$ or $\mu = \frac{v_0}{2g\, t_0}$

17. **(A)**



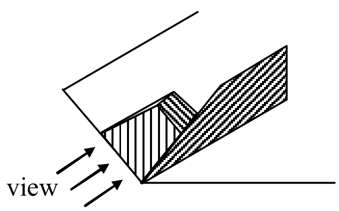
$\mu N \cos\theta = N \sin\theta$

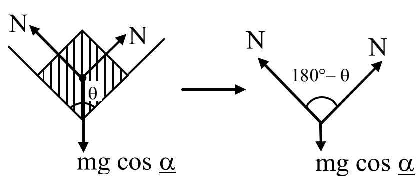
$\mu = \tan\theta$

18. **(B)**

19. **(B)**

Force diagram of block for the view shown





$\Rightarrow N = \frac{mg\cos\alpha}{2\sin(\theta/2)}$

$\therefore$ Net friction up the plane = $2\,\mu N$

$= \mu mg\, \frac{\cos\alpha}{\sin(\theta/2)}$

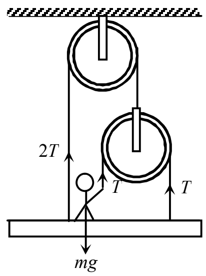
$\therefore \ a = g\left\{\sin\alpha - \mu\frac{\cos\alpha}{\sin(\theta/2)}\right\}$

20. **(C)**

Net force on $m_3 = \sqrt{(30)^2 + (40)^2} = 50$ N and limiting friction on $m_3 = \mu m_3 g = 60N$

$\therefore$ System remain in equilibrium and friction on m_3 = 50 N

21. $4T = mg$



$\therefore \quad T = \dfrac{60 \times 10}{4}$ = 150 N

$\therefore$ **(A)**

22. Applying Snell's law between the points *O* and *P*, we have

$$2 \times \sin 60^0 = (\sin 90^0) \times \frac{2}{(1+H^2)}, 2 \times \frac{\sqrt{3}}{2} = 1 \times \frac{2}{(1+H^2)}$$

$$(1+H^2) = \frac{2}{\sqrt{3}}, \qquad H = \sqrt{\left(\frac{2}{\sqrt{3}} - 1\right)}$$

(A)

23. If angle made by the incident ray with the normal (i.e. *y* axis) is θ. Then

$$\tan\theta = \frac{1}{2} \Rightarrow \sin\theta = \frac{1}{\sqrt{5}}$$

If the refracted ray makes angle θ' with y-axis then from Snell's Law

$$2 \times \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{2} \times \sin\theta' \Rightarrow \sin\theta' = \frac{4}{5}$$

The unit vector along with the refracted ray moves is given by

$$-1 \times \sin\theta' \hat{i} - 1 \times \cos\theta' \hat{j} = -\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$$

$\therefore$ **(B)**

24. x is distance of object from surface.

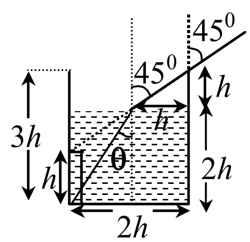
Apparent depth of object from surface = $\frac{x}{\mu}$

Apparent depth of image from surface = $\frac{x+2h}{\mu}$

Distance between the apparent depths of object and image = $\frac{2h}{\mu}$

$\therefore$ **(B)**

25. $\mu \sin\theta = \sin 45^0$



$\frac{\mu h}{h\sqrt{5}} = \frac{1}{\sqrt{2}}$; $\mu = \sqrt{\frac{5}{2}}$

$\therefore$ **(B)**

26. (A)

Resolve the applied force and get normal reaction and limiting friction

27. (C)

Common acceleration $a = \frac{KA}{2m}$

$\therefore f_r = ma = \frac{KA}{2}$

28. (C)

1.6×10^{-20} C is not possible because it does not obey quantization of charge

29. (C)

30. (D)

CHEMISTRY

31. (B)

1 mole $(NH_4)_3PO_4$

= 12 mole H–atoms = 4 mole O–atom

6 mole H–atoms = 2 mole O–atoms

32. (C)

33. (A)

L.R. is D in reaction (I)

$$(1)\quad \underset{9\,\text{mole}}{3D} + \underset{14\,\text{mole}}{4E} \xrightarrow{80\%} \underset{\frac{5}{3}\times 9\times 0.8 = 12\,\text{mole}}{5C} + A$$

$$(2)\quad \underset{12\,\text{mole}}{3C} + \underset{4\,\text{mole}}{5G} \xrightarrow{50\%} 6B + F$$

L.R. is G in reaction (2)

Moles of B formed = $\frac{6}{5}\times 4\times 0.5 = 2.4$

34. (A)

Balancing the equation, we have

$$K_2Cr_2O_7 + H_2SO_4 + 3SO_2 \longrightarrow K_2SO_4 + Cr_2(SO_4)_3 + H_2O$$

35. (A)

$$\underset{xg}{NaOH} + \underset{yg}{Na_2CO_3}$$

In presence of HPh,

Eq. of NaOH + $\frac{1}{2}\times$ eq. of Na_2CO_3

= Eq. of HCl

$$\frac{x}{40}\times 1 + \frac{1}{2}\times\frac{y}{106}\times 2 = \frac{17.5}{1000}\times\frac{1}{10} \qquad \text{................(1)}$$

After this MeOH is added

$$\frac{1}{2}\times \text{eq. of } Na_2CO_3 = \text{eq. of HCl}$$

$$\frac{1}{2}\times\frac{y}{106}\times 2 = 2.5\times\frac{1}{10}\times\frac{1}{1000} \qquad ....(2)$$

Placing the value of Eqn.........(1)

$$\frac{x}{40}+\frac{2.5}{10000}=\frac{17.5}{10000}$$

$$\frac{x}{40}=15\times 10^{-4} \Rightarrow x = 15\times 40\times 10^{-4}$$

x = 0.06 g

36. (A)

$$I_2 + AsO_2^- + 2H_2O \longrightarrow HAsO_4^{2-} + 2I^- + 3H^+$$

m–eq. of $HAsO_2$ (in 50 mL) = m–eq. of $I_2 = 35\times 0.05\times 2 = 3.5$

m–eq. of $HAsO_2$ in 250 mL = $3.5\times\frac{250}{50}$ = 17.5

mass of $HAsO_2$ in sample $=\frac{17.5}{2}\times(108)\times 10^{-3} = 0.945\,g$

% of $HAsO_2$ in the sample = $=\frac{0.945}{3.78}\times 100$ = 25%

37. (A)

$$V=\frac{nRT}{P}=\frac{10\times 0.0821\times T}{0.821}$$

V = T

$\log V = \log T$

slope = 1

$\theta = 45°$

38. (B)

h × d (glycerine) = h × d (mercury)

$5\times 2.75 = h\times 13.6$

h = 1 m

$P_{gas} = 1760\,mm$ Or (1000 + 760) mm Hg

PV = nRT

$$\frac{1760}{760}\times 10 = n\times 0.082\times 300$$

n = 0.94 mol

39. (C)

$$n_{N_2} = \frac{77}{28} = 275$$

$$n_{O_2} = \frac{23}{32} = 0.72$$

% by volume of O_2 = % by mol of O_2

$$= \frac{n_{O_2}}{\text{Total moles}} \times 100 = \frac{0.72}{2.75 + 0.72} \times 100$$

= 20.8

40. (C)

Total no. of moles n = $n_1 + n_2$

$$\frac{P(V_1 + V_2)}{RT} = \frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2}$$

$$T = \frac{P(V_1 + V_2) T_1 T_2}{P_1 V_1 T_2 + P_2 V_2 T_1}$$

According to Boyle's law :

$$p_1 V_1 + p_2 V_2 = P(V_1 + V_2)$$

From eqns. (1) and (2).

$$T = \frac{(p_1 V_1 + p_2 V_2) T_1 T_2}{(p_1 V_1 T_2 + p_2 V_2 T_1)}$$

41. (D)

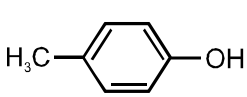
42. (A)

43. (D)

Salicylic acid is more acidic than p-hydroxy benzoic acid.

44. (A)

45. (C)



is lowest K_a value so this compound has largest pK_a value.

46. (B)

CH_3–CH(Cl)–COOH is strongest acid due to –I of –Cl atom.

$(CH_3)_2$CH–COOH is weakest acid due to greater +I effect of –R.

47. (A)

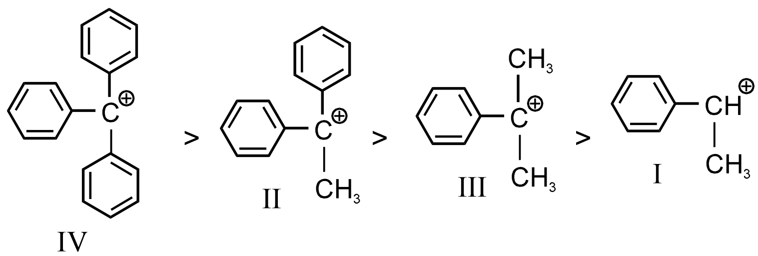
1 is least stable since charge separation is done and +ve charge is towards –M group. 4 is most stable because nonpolar.

48. (A)

Carbocation stablized by + M effect.

49. (B)

The correct stability order of the following carbocations is $\mathrm{IV} > \mathrm{II} > \mathrm{III} > \mathrm{I}$



Stability of carbocation depend upon conjugation > Hyperconjugation

50. (D)

(D) is not havig α–H.

51. (B)

$NaOH \rightarrow Na + :\ddot{\underset{\cdot\cdot}{O}}^{\ominus}–H$

52. (C)

positive radius $\alpha \dfrac{1}{+\text{ve O.S.}}$

(A) $\overset{+4}{Mn}O_2^{-2}$ (B) $\overset{+1}{K}\overset{+7}{Mn}O_4^{-8}$ (C) $\overset{+2}{Mn}\overset{-2}{O}$ (D) $\underset{x=+3}{K_3[Mn(CN)_6]}$

53. (B)

$19K^+ = 1.34\,\text{Å}$ (Cationic radius)

$9^F = 1.34\,\text{Å}$ (Anionic radius)

54. (D)

All are iso-electronic species.

55. (D)

A Gives equeous solution [PH < 7]

B Reacts with strong acid and alkalis respectively.

C Gives an aqueous solution which is strongly alkaline

A - Acidic – P $(OH)_3$ or H_3PO_4

B - Amphoteric – Al $(OH)_3$, H_3AlO_3

C - Basic – NaOH

x = Phousphorous – Non metal

y = Aluminium - Metal

c = Sodium - Metal

56. (A)

(A) Lattic energy depend upon :

(i) Size of cation and anion both

(ii) Product of charges at cation & anion

(B) $CdCl_2 > CaCl_2$–Both Hydration & Lattice is high than $CaCl_2$

As per (born haber cycle)

(C) $F^- > Cl^- > Br^- > I^-$ (Hydration energy)

so, AgF > AgCl > AgBr > Ag I (Solubility in water)

(D) $Be_3N_2 > Mg_3N_2 > Ca_3N_2$ (Thermal stability)

57. (A)

Lattice α Hardness

(A) Ti > ScN > MgO > NaF – order of lattic energy

(B) NaCl < CsCl – Co-ordinate no. NaCl = 6

CsCl = 8

(C) $BeCl_2 < MgCl_2 < CaCl_2$ – Melting point

58. (B)

$Cs^+I_3^-$ (large cation stabilises by large anion)

59. (D)

$\underset{2S'}{Li} + e^- \xrightarrow{Ea} \underset{2S^2}{Li^-}$ exothermic

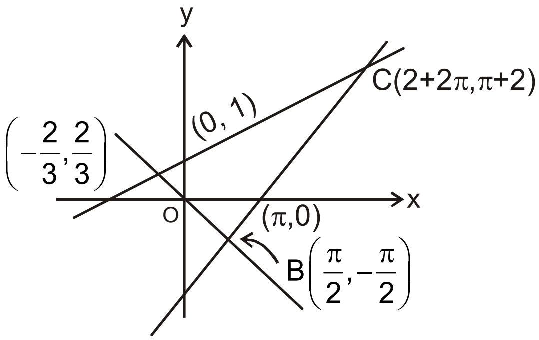
60. (C)

(I) $HClO_4 > H_2SO_4 > HNO_3 > H_3PO_4$

(II) $HClO_3 > HBrO_3 > HIO_3$

MATHEMATICS

61. (C)



if (a, sina) lie inside the tringle, then $a \in (0, \pi)$

62. (D)

f(– x) = f(x)

An even function hence neither one-one nor onto

63. (C)

g(x) = ax + b

g(1) = 2

$\Rightarrow$ a + b = 2

g(3) = 0

$\Rightarrow$ 2a = -2

a = -1

b = 3

g(x) = -x + 3

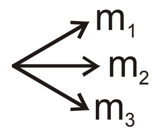
$\cot\left[\cos^{-1}[|\sin x| + |\cos x|] - \sin^{-1}[|\sin x| + |\cos x|]\right]$

$|\sin x| + |\cos x| \in \left[1, \sqrt{2}\right]$

$\Rightarrow \cot\left[\cos^{-1} 1 - \sin^{-1} 1\right] = 0 = g(3)$

64. (B)

$4m^3 - 3am^2 - 8a^2m + 8 = 0$



$m_1m_2m_3 = -2$

$\Rightarrow \quad m_3 = 2 \quad (\because m_1m_2 = -1)$

65. (A)

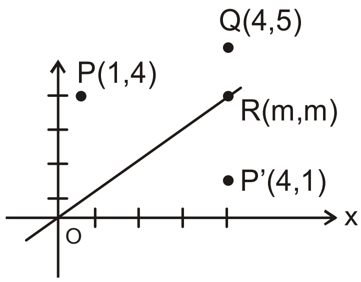


Image of (1, 4) about the line y = x is (4, 1) $\Rightarrow P'(4,1)\,Q(4,5)$ and R(m, m) are collinear.

$\Rightarrow\ m = 4$

66. (A)

$1 - 3x \geq 0$

$x \leq \frac{1}{3}$

And

$-x^2 + x + 6 \geq 0$

$x^2 - x - 6 \leq 0$

$x^2 - 3x + 2x - 6 \leq 0$

$(x-3)(x+2) \leq 0$

$x \in [-2, 3]$

The answer will be $\left[-2, \frac{1}{3}\right]$

67. (C)

Given equation $\sec^2 (a + 2)\, x + a^2 - 1 = 0$

$\Rightarrow\ \tan^2 (a + 2)x + a^2 = 0 \Rightarrow \tan^2 (a + 2)x = 0$ and $a = 0$

$\Rightarrow\ \tan^2 2x = 0 \qquad \Rightarrow\ \tan^2 2x = 0 \qquad \Rightarrow\ x = 0, \frac{\pi}{2}, \frac{\pi}{2}$

$\therefore$ (0, 0), (0, $\pi/2$), (0, $-\pi/2$) are ordered pairs satisfying the equation.

68. (D)

$f(x) = \log_{\sqrt{2}} \left(2 - \log_2\left(16\sin^2 x + 1\right)\right)$

$1 \leq 16\sin^2 x + 1 \leq 17$

$\therefore \quad 0 \le \log_2 (16 \sin^2 x + 1) \le \log_2 17$

$\therefore \quad 2 - \log_2 17 \le 2 - \log_2 (16 \sin^2 x + 1) \le 2$

Now consider

$0 < 2 - \log_2 (16 \sin^2 x + 1) \le 2$

$\therefore \quad -\infty < \log_{\sqrt{2}} [2 - \log_2 (16 \sin^2 x + 1)] \le \log_{\sqrt{2}} 2 = 2$

$\therefore$ the range is $(-\infty, 2]$

69. (B)

$$(\cot^{-1} x)\left(\frac{\pi}{2} - \cot^{-1} x\right) + 2\cot^{-1} x - \frac{\pi}{2}\cot^{-1} x + 3\left(\frac{\pi}{2} - \tan^{-1} x\right) - 6 > 0$$

$$-(\cot^{-1} x)^2 + 5\cot^{-1} x - 6 > 0$$

$$(\cot^{-1} x)^2 - 5(\cot^{-1} x) + 6 < 0$$

$$(\cot^{-1} x - 3)(\cot^{-1} x - 2) < 0$$

$$2 < \cot^{-1} x < 3$$

$\cot 3 < x < \cot 2 \qquad (\because \cot^{-1} x$ is decreasing)

70. (B)

$$1 + \tan^2(\tan^{-1} 2) + 1 + \cot^2(\cot^{-1} 3) = 1 + 2^2 + 1 + 3^2 = 15$$

71. (A)

$$\cos^{-1}(1 - x) + m\cos^{-1} x = \frac{n\pi}{2}$$

Domain $x \in [0,1]$

$\cos^{-1}(1 - x) + m\cos^{-1} x > 0 \qquad (\because m > 0)$

There is no solution

72. (D)

$$f(x) = \frac{\pi}{2} - 3\tan^{-1} x$$

$g(x) = 2\tan^{-1} x$

$$\lim_{x \to 0} \frac{f(x) - f(a)}{g(x) - g(a)} = \frac{f'(a)}{g'(a)} = -\frac{3}{2}$$

73. (A)

$$\underset{x\to 0}{\text{Lt}}\frac{\sin(\pi-\pi\sin^2(\tan(\sin x))}{x^2}$$

$$\underset{x\to 0}{\text{Lt}}\frac{\sin(\pi\sin^2(\tan(\sin x)]}{\pi\sin^2(\tan(\sin x)}\times\pi\left(\frac{\sin(\tan(\sin x)}{x}\right)^2=\pi$$

74. (A)

$$\underset{x\to\frac{\pi}{2}}{\text{Lt}}\ x\left[x^{5c-1}\left(1-\frac{7}{x}+\frac{2}{x^5}\right)^c-1\right]=\ell$$

case - I 5c – 1 > 0, then $\ell\to\infty$

case- II 5c – 1 < 0 then $\ell\to-\infty$

since limit is finite and non-zero so 5c – 1 = 0 $\Rightarrow c=\frac{1}{5}$

$$\therefore\quad \lambda=\underset{x\to\infty}{\text{Lt}}\ x\left[\left(1+\frac{7}{x}+\frac{2}{x^5}\right)^{\frac{1}{5}}-1\right]$$

$$=\underset{x\to\infty}{\text{Lt}}\ x\left[1+\left(\frac{1}{5}\right)\left(\frac{7}{x}+\frac{2}{x^5}\right)+..........-1\right]$$ (by binomial approximation)

$$=\frac{7}{5}$$

75. (C)

$$\lim_{x\to 0}\frac{\cos x-1}{x^2}\left(\frac{\cos x-1}{x^{n-2}}-\frac{(e^x-1)}{x^{n-2}}\right)\Rightarrow n-2=1:n=3$$

76. (A)

$$\underset{x\to\infty}{\text{Lt}}\left[\sqrt{x^2-x+1}-(ax+b)\right]=0$$

so a > 0, on rationalizing

$$\underset{x\to\infty}{\text{Lt}}\left[\frac{(x^2-x+1-\left[a^2x^2+b^2+(2ab)x\right]}{\sqrt{x^2-x+1}\ +ax+b}\right]=0$$

so $1-a^2=0$ $\qquad$ $-1-2ab=0$

a=1,

$$\underset{x\to\infty}{\text{Lt}}\ \sec^2\left[k!\,\pi(-1/2)\right]=1 \qquad =a$$

77. (C)

f(x + T) = f(x + 2T) = ... =f (x + nT) = f (x))

$$\underset{n\to\infty}{\text{Lt}}\frac{nf(x)(1+2+3+.....+n)}{(f(x)(1+2^2+3^2+.....+n^2)}=\underset{n\to\infty}{\text{Lt}}\frac{n\left(\frac{n(n+1)}{2}\right)}{\frac{n(n+1)(2n+1)}{6}}=\frac{3}{2}$$

78 (C)

$$-265\left[\underset{h\to0}{\text{Lt}}\frac{h^2+3}{\left(\frac{f(1-h)-f(1)}{-h}\right)\left(\frac{\sin 5h}{h}\right)}\right]$$

$$=-265\times\frac{3}{f'(1).5}$$

$$=-\frac{53\times3}{f'(1)}$$

$$=-\frac{53\times3}{-53}\quad(\because f'(1)=-53)$$

= 3

79. (A)

$(x^2-a^2)^2(x^2-b^2)^2=0$ and $(y^2 - a^2)^2 = 0$

$(x=\pm a \text{ or } x=\pm b)$ and $y=\pm a$

$\therefore\ (x,y)=(\pm a,\pm a)$ or $(\pm b,\pm a)$

80. (A)

$\frac{1}{2}ab=11 \ \Rightarrow ab=22$

Also $\frac{2}{a}+\frac{3}{b}=1 \Rightarrow 2b+3a=ab$

$\Rightarrow 4b^2+9a^2+12ab=a^2b^2$

$\therefore\ 4b^2+9a^2=220$

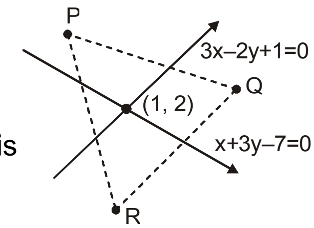
81. **(B)**

PA+PC is minimum when P is collinear with A and C. PB+PD is minimum when P is collinear with B&D.

$\therefore PA + PB + PC + PD$ is minimum when P is the point of intersection of diagonals AC & BD and its minimum value is $AC + BD$

82. **(C)**

Circumcentre of ΔPQR is (1, 2).



Straight line through it of slope 'm' is y – 2 = m(x – 1)

intersect axes at $A\left(1-\frac{2}{m},0\right)$ and $B(0,2-m)$. Now area of ΔOAB is

$$= \frac{1}{2}\left(1-\frac{2}{m}\right)(2-m) = \frac{1}{2}\left(4-m-\frac{4}{m}\right) \geq 4.$$

83. **(B)**

m = 3 and n = 4

$\Rightarrow$ $x^2 - 4x + 3 < 0$ $\Rightarrow$ $(x-1)(x-3) < 0$ $\Rightarrow$ $x \in (1, 3)$

$\Rightarrow$ x = 2 is only integer solution.

84. **(C)**

Let perpendicular bisector of AB is 3x + 4y – 20 = 0

and perpendicular bisector of AC is 8x + 6y – 65 = 0.

Image of A w.r.t. 3x + 4y – 20 = 0 is B

and image of A w.r.t. 8x + 6y – 65 = 0 is C.

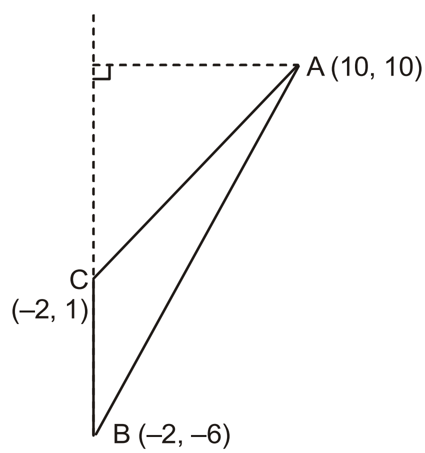
For B, $\frac{x-10}{3} = \frac{y-10}{4} = -2\left(\frac{30+40-20}{25}\right)$

$\Rightarrow$ B = (–2, –6)

For C, $\frac{x-10}{8} = \frac{y-10}{6} = -2\left(\frac{80+60-65}{100}\right)$

$\Rightarrow$ C = (–2, 1)

Area of $\Delta ABC = \frac{1}{2}(10 + 2)(1 + 6) = 42.$



85. **(B)**

$S_1 - S_2 = 0 \Rightarrow 5ax + (c-d)y + a + 1 = 0$

and 5x + by – a = 0 represents the same line

$\therefore \quad \frac{a}{1} = \frac{c-d}{b} = \frac{a+1}{-a}$

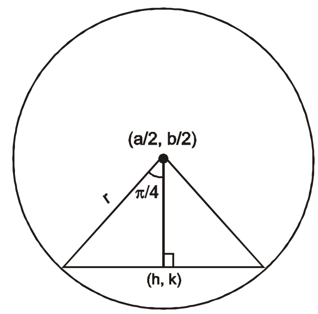
$\Rightarrow$ ab = c – d and $a^2 + a + 1 = 0$

$\Rightarrow$ no real value of a.

86. (C)

$r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2}$

$\cos 45^0 = \dfrac{\sqrt{\left(h - \frac{a}{2}\right)^2 + \left(k - \frac{b}{2}\right)^2}}{\frac{\sqrt{a^2+b^2}}{2}}$

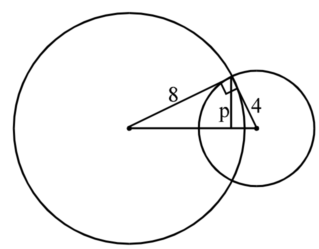


87. (A)

Distance 'd' between the centres is = $\sqrt{8^2+4^2} = 4\sqrt{5}$

Also $4\sqrt{5} \cdot p = 8 \cdot 4 \Rightarrow p = \frac{8}{\sqrt{5}}$

$\Rightarrow$ length of common chord is $\frac{16}{\sqrt{5}}$



88. (C)

Radius of circles are r_1, r_2, 1

line y = x +1

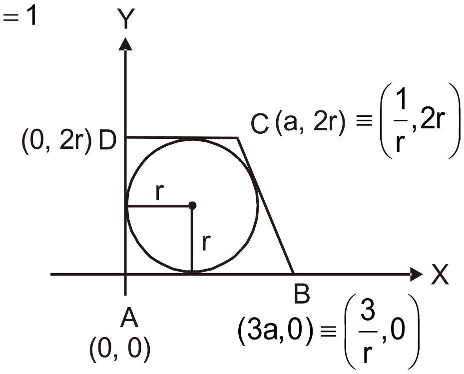
Perpendicular from (0, 0) = $\frac{1}{\sqrt{2}}$

$r_1 > \frac{1}{\sqrt{2}} \quad \Rightarrow \quad r_1 = 1 - 2d > \frac{1}{\sqrt{2}}$

$\Rightarrow \quad d < \frac{\sqrt{2}-1}{2\sqrt{2}}$

89. (B)

Area of trapezium ABCD $= \frac{1}{2}(a+3a)(2r) = 4 \Rightarrow ar = 1$



Equation of line BC is $y = -r^2\left(x - \frac{3}{r}\right)$

or, $y + r^2x - 3r = 0$

$\because$ BC is the tangent to the circle

$\Rightarrow \frac{|r + r^3 - 3r|}{\sqrt{1+r^4}} = r \Rightarrow r^4 + 4 - 4r^2 = 1 + r^4 \Rightarrow r = \frac{\sqrt{3}}{2}$

90. (C)

Centre of the circles lies on $x + y = 3a$.

let centres are $(\alpha, 3a-\alpha)$ and $(\beta, 3a-\beta)$

$\Rightarrow \alpha, \beta$ be the roots of equation

$(x-a)^2 + (2a-x)^2 = x^2$

$\Rightarrow x^2 - 6ax + 5a^2 = 0$

$\alpha + \beta = 6a, \ \alpha\beta = 5a^2$

$|\alpha - \beta| = 4a$

$\Rightarrow C_1C_2 =$ distance between centres $= 4\sqrt{2}\,a$