SOLUTIONS

PHASE TEST-2

GZR-1901 TO 1907

GZRK-1901-1902

GZBS-1901

JEE MAIN PATTERN

Test Date: 15-10-2017

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PHYSICS

1. $\frac{\Delta T}{T}\times 100=\frac{\frac{1}{5}}{25}\times 100=0.8\%$

$\therefore$ **(B)**

2. (B) = L

$ML^2T^{-2}=\frac{\alpha[L]^{1/2}}{[L]}$

$\alpha=[M][L^{5/2}][T^{-2}]$

$\therefore$ **(D)**

3. p = 1mm, N = 100

Least count, $C=\frac{P}{N}=\frac{1\text{mm}}{100}=0.01\text{mm}$

The instrument has a positive zero error $e=+NC=+4\times 0.01=+0.04\text{mm}$

Main scale reading is 2 × (1 mm) = 2 mm

Circular scale reading is 67 (0.01) = 0.67 mm

$\therefore$ observed reading is $R_0=2+0.67=2.67\text{ mm}$

So true reading = R_0-e = 2.63 mm

$\therefore$ **(C)**

4. $\vec{r}=(2t-3t^2)\hat{i}+2t\hat{j}-t^2\hat{k}$, $\vec{v}=(2-6t)\hat{i}+2\hat{j}-2t\hat{k}$, $\vec{a}=-6\hat{i}-2\hat{k}$

If $\vec{v}\perp\vec{a}$, $\vec{v}\cdot\vec{a}=0$ $\therefore -6(2-6t)+4t=0$, $40t=12$

$t=\frac{3}{10}=0.3\text{ s}$

$\therefore$ **(C)**

5. The maximum distance covered by the vehicle before coming to rest $=\frac{v^2}{2a}=\frac{(15)^2}{2\times 0.3}=375\text{ m}$.

The corresponding time = $t=\frac{v}{a}=\frac{15}{0.3}=50$ sec.

Therefore after 50 sec, the distance covered by the vehicle = 375 m, from the instant of beginning of braking.

The distance of the vehicle from the traffic signal after one minute = (400 – 375) m = 25 m.

$\therefore$ **(A)**

6. $R = \frac{u^2}{g}\sin 2\theta = \frac{u^2}{g}$

Velocity of take off at *P* or $u = \sqrt{Rg} = \sqrt{90 \times 10} = 30\text{m/s}$

$v = \sqrt{u^2 + 2g\sin\theta S}$ [v → velocity at point *O*]

$= \sqrt{(30)^2 + 2 \times 10 \times \frac{1}{\sqrt{2}} \times 80\sqrt{2}} = 50\text{m/s}$

∴ **(C)**

7. If *u* is the initial speed of the second stone, then

$0 = u^2 - 2g(4h)$

or $u = \sqrt{8gh}$

If they meet at the height *x* from ground,

For A, $h - x = \frac{1}{2}gt^2$

For B, $x = (\sqrt{8gh})\,t - \frac{1}{2}gt^2$

∴ $h = \sqrt{8gh}\ t$

or $t = \sqrt{\frac{h}{8g}}$

∴ **(B)**

8. As $F_1 - F_2 < 2\mu Mg$, so system will not accelerate. Again here $F_1 > F_2$, so block A is the driving block and block *B* is driven block. So friction on block *A* acts towards left but in the block B it may act left or right.

∴ **(B)**

9. Distance travelled along *OE* in 2s = 4 × 2 = 8 m

Distance travelled perpendicular to *OE* in 2s = $\frac{1}{2}at^2 = \frac{1}{2}\left(\frac{6}{2}\right)2^2$ = 6 m

Displacement = $\sqrt{6^2 + 8^2}$ = 10 m

∴ **(D)**

10. Normal ≤ contact force ≤ $\sqrt{(\text{normal})^2 + (\text{maximum friction})^2}$

$Mg \le F \le \sqrt{(Mg)^2 + (\mu Mg)^2}$

∴ **(C)**

11. **(D)**

$f = \mu R = \mu mg$, where m is mass of the combination, $f = 0.5 \times 10 \times 10$ N = 50 N.

So, a force of 10 N is unable to start the motion of the system. There is no relative motion between *A* and *B*.

12. Let x be the extension in the spring when 2 kg block leaves the contact with ground. Then,

$kx = 2g$

or $x = \frac{2g}{k} = \frac{2\times10}{40} = \frac{1}{2}\text{m}$

Now, from conservation of mechanical energy

$$mgx = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \qquad (m = 5 \text{ kg})$$

or $v = \sqrt{2gx - \frac{kx^2}{m}}$

Substituting the values $v = \sqrt{2\times10\times\frac{1}{2} - \frac{(40)}{4\times5}} = 2\sqrt{2}$ m/s

∴ **(B)**

13. **(D)**

Block will return after maximum elongation.

i.e. $F.x_{max} - ½ Kx^2_{max} - \mu\, mgx_{max} = 0$

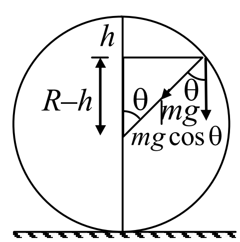
$x_{max} = \frac{2(F - \mu mg)}{k} = \frac{8\mu mg}{k}$

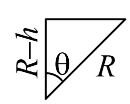
14. $\frac{v^2}{r} = \frac{4}{r^2}$ i.e. $v = \frac{2}{\sqrt{r}}$

hence $p = \frac{2m}{\sqrt{r}}$

∴ **(D)**

15. $mg\cos\theta = \frac{mv^2}{R}$





or $g\cos\theta = \frac{v^2}{R} = \frac{2gh}{R}$

or $\cos\theta = \frac{2h}{R}$ and $R\cos\theta + h = R$

or $3h = R$ or $h = \frac{R}{3}$

∴ **(D)**

16. $a_c = k^2rt^2$ or $\frac{v^2}{r} = k^2rt^2$ or $v = krt$

Therefore, tangential acceleration, $a = \frac{dv}{dt} = kr$

or tangential force, $F_t = ma_t = mkr$

Only tangential force does work,

Power = $F_t v = (mkr)(krt)$ or Power = mk^2r^2t

∴ **(B)**

17. **(C)**

P = Fv or P = mav or p = m $\left(\frac{v\,dv}{dx}\right)$v

or P = mv^2 $\frac{dv}{dx}$ or dx = $\frac{m}{P}$v^2 dv

or $\int_0^x dx = \frac{m}{p}\int_{v_1}^{v_2} v^2 dv$

or x = $\frac{m}{P}\frac{(v_2^3 - v_1^3)}{3}$

or x = $\frac{m}{3P}(v_2^3 - v_1^3)$

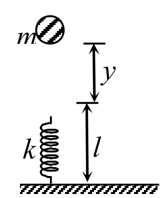
18. Because the efficiency of machine is 90%.

Hence potential energy gained by mass = $\frac{90}{100}$×energy spent = $\frac{90}{100}\times 5000$ J = 4500 J

When the mass is released now, gain in KE on reaching the ground = KE on hitting the ground = loss of potential energy = 4500 J

∴ **(B)**

19. Let compression in string is x_0 when net force is zero,

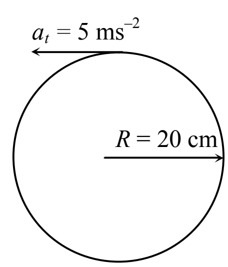


$mg = k\, x_0$

$x_0 = mg/k$ ∴ $h = l - \frac{mg}{k}$

∴ **(B)**

20. $a_c = a_t$ ⇒ $\frac{v^2}{r} = 5$



$v = \sqrt{5 \times 20}$ = 10 cm/s

$v = at$

$t = \frac{10}{5} = 2s$

∴ **(B)**

21. Kinetic energy at top = $\frac{1}{2}m(V\cos\theta)^2 = 5$ J.

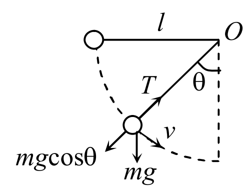
$\therefore$ **(C)**

22. Work done by friction = $\int \vec{F}\cdot d\vec{s}$

$$= \int_0^x \mu mg\cos\theta \frac{dx}{\cos\theta} = \mu mgx = 20 \text{ J}$$

$\therefore$ **(C)**

23. $T = 2mg$



$$T - mg\cos\theta = \frac{mv^2}{l}$$

$$2mg - mg\cos\theta = \frac{mv^2}{l} \quad \ldots\text{(i)}$$

By conservation of energy $mgl\cos\theta = \frac{1}{2}mv^2$

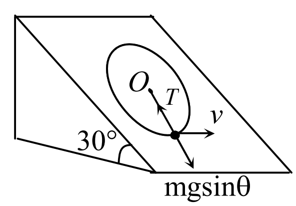
$$\frac{mv^2}{l} = 2mg\cos\theta \quad \ldots\text{(ii)}$$

From (i) and (ii) $2mg - mg\cos\theta = 2mg\cos\theta$

$$\cos\theta = \frac{2}{3}$$

$\therefore$ **(C)**

24. $T - mg\sin\theta = \frac{mv^2}{r}$



$T = mg\sin\theta + \frac{mv^2}{r}$

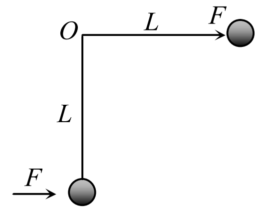
$= 2 \times 10 \times \frac{1}{2} + \frac{2\times 36}{0.5}$

= 10 + 144

= 154 N

$\therefore$ **(B)**

25. By work energy theorem,



$$W_{mg} + W_F = \Delta K.E.$$

$$-mgL + FL = K_f - K_i$$

But $F = mg$

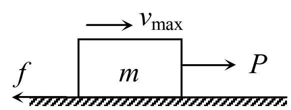
$\therefore\ -mgL + mgL = K_f$

$\Rightarrow\ K_f = 0$

$\therefore$ **(D)**

26. $P = Fv$

For maximum velocity,



$F = f = \mu mg$

$$v_{max} = \frac{P}{\mu mg}$$

$\therefore$ **(B)**

27. On applying work energy theorem in the frame of wedge.

$ma_0R - mgR = 0 \Rightarrow a_0 = g$

$\therefore$ **(C)**

28. $\text{mgh} = \frac{80}{100}(\text{mg})(100)$

$\text{h} = 80\,\text{m}$

$\therefore$ **(A)**

29. **(D)**

Since $v_{1Y} = v_{2Y} = 0$

And $Y_1 = Y_2 = -Y$

$(a_{1Y} = a_{2Y} = -g\cos\theta)$

Hence from, $y = vt + \frac{1}{2}at^2$

Time taken for both the bullets will be same.

30. $\frac{1}{2}gt^2 = H$ … (i)

$gt = v_y$ …(ii)

$v_x = v_y$

Range = $u_xt = v_yt = gt^2 = 2H$

$\therefore$ **(B)**

CHEMISTRY

31. (D)

32. (B)

K.E. = $\frac{3}{2}$ RT

$\therefore$ $(K.E.)_1 = \frac{3}{2} \times R \times 400 \Rightarrow (K.E.)_2 = \frac{3}{2} \times R \times 800$

$\therefore \frac{(KE)_2}{(KE)_1} = 2$ or $(KE)_2 = 2\ (KE)_1$

33. (D)

34. (C)

$2\pi r = n\lambda$

or $\lambda = \frac{2\pi r}{n} = \frac{2\pi a n^2}{n}$ [z = 1 for H]

or $\lambda = 2\pi a n = 4\pi a$

35. (B)

36. (D)

$Na_2SO_4.xH_2O \xrightarrow[-xH_2O]{\Delta} Na_2SO_4$

Let the total molecular weight of the compound be y

Then, $y - \frac{55.9}{100} y = 142$ [$\because$ M.W. of Na_2SO_4 = 142]

$\frac{44.1}{100} y = 142$

$y = \frac{142 \times 100}{44.1} = 321.99$

Now, M.W. of $Na_2SO_4 . xH_2O$ = 142+18x

142+18x = 321.99

$x = \frac{321.99 - 142}{18} = 10$

37. (B)

Molality = $\frac{\frac{20 \times 0.75}{60}}{\frac{50}{1000}} = 5$

38. (C)

$$2NH_3 \longrightarrow N_2 + 3H_2$$

$$M_{mix} = \frac{17}{1+\alpha}$$

$$\frac{r_{mix}}{r_{SO_2}} = 2 = \sqrt{\frac{64}{\frac{17}{1+\alpha}}}$$

$$68 = 64 + 64\alpha \Rightarrow \alpha = \frac{1}{16}$$

% dissociation = 6.25%

39. (A)

$$\left(\underset{x}{C_2}H_2 + \underset{20-x}{CO}\right) : 20 \text{ ml}$$

$$C_2H_5 + \frac{5}{2}O_2 \longrightarrow 2CO_2 + H_2O$$

Initial :

$$x \longrightarrow \frac{5}{2}x \longrightarrow 2x$$

$$CO + \frac{1}{2}O_2 \longrightarrow CO_2$$

$$(20-x) \longrightarrow \left(\frac{20-x}{2}\right) \longrightarrow (20-x)$$

Volume after reaction = 34 ml

V_{CO_2} formed + V_{O_2} remained = 34 ml

$$2x + (20-x) + \left\{30 - \left(\frac{5}{2}x + \frac{20-x}{2}\right)\right\} = 34$$

20 + x + 30 – 2x – 10 = 34

40 – x = 34

x = 6 ml

or

After passing KOH,

8 ml of O_2 remained

$V_{absorbed}$ = 34 – 8 = 26

$V_{CO_2} = 26$

$20 + x = 26$

$x = 6$ ml

40. (D)

$$\text{At } 27^\circ C;\ 1V = n_{He} R(300) \Rightarrow n_{He} = \frac{V}{300R}$$

$$\text{at } 127^\circ C;\ 2V = (n_{He} + n_P) R(400) \Rightarrow n_{He} + n_P = \frac{V}{200R}$$

$$\therefore \quad n_P = \frac{V}{600R}$$

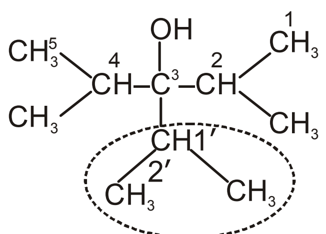
$$\text{at } 327^\circ C;\ PV = \left(\frac{V}{300R} + \frac{2V}{600R}\right) R(600)$$

P = 4 atm

41. (B)

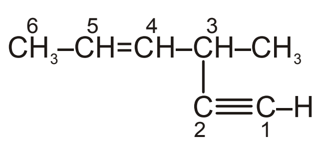
Those atoms which attached with sp hybridized carbon then it is present linearly.

42. (A)

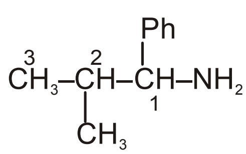


1′-methylethyl

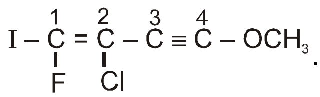
43. (C)



44. (B)



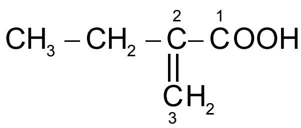
45. (D)



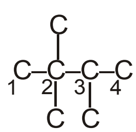
46. (C)

S.F. = HOOC – COOH, M.F. = $C_2H_2O_4$, M.W. = 90

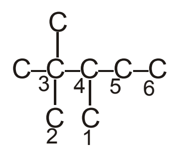
47. (B)

 2-Ethylprop-2-enoic acid

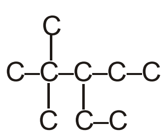
48. (B)

(A) 

2, 2, 3-Trimethylbutane

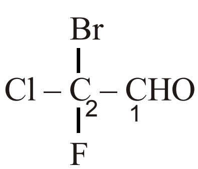
(C) 

3, 3-Dimethylhexane

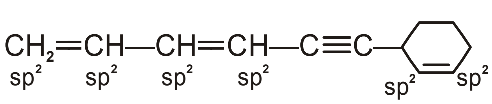
(D) 

3-Ethyl-2,2-dimethyl pentane

49. (A)



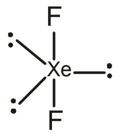
50. (A)



51. (B)

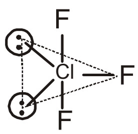
$NaOH \rightarrow Na^{+} + :\ddot{O}^{\ominus}-H$

52. (B)



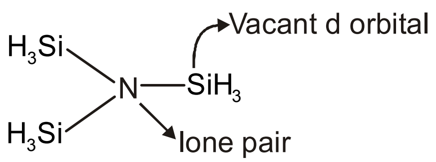
, $:\ddot{O} = C = \ddot{O}:$

53. (D)

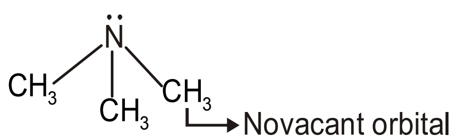


T-shape

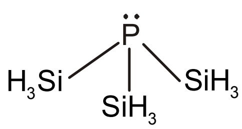
54. (B)



(due to Back Bonding)



(No back bonding)



(No back bonding due to large size of atoms)

55. (D)

As electronegativity of halogen attached with sulphur increases, suphur becomes more electron deficient and hence its tendency of get electrons from oxygen through $p\pi - d\pi$ bonding also increases i.e. extent of $p\pi - d\pi$ bonding increases and hence, bond order also increases.

56. (A)

(A) Lattic energy depend upon :

(i) Size of cation and anion both

(ii) Product of charges at cation & anion

(B) $CdCl_2 > CaCl_2$–Both Hydration & Lattice is high than $CaCl_2$

As per (born haber cycle)

(C) $F^- > Cl^- > Br^- > I^-$ (Hydration energy)

so, AgF > AgCl > AgBr > Ag I (Solubility in water)

(D) $Be_3N_2 > Mg_3N_2 > Ca_3N_2$ (Thermal stability)

57. (A)

Lattice α Hardness

(A) Ti > ScN > MgO > NaF – order of lattic energy

(B) NaCl < CsCl – Co-ordinate no. NaCl = 6

CsCl = 8

(C) $BeCl_2$ < $MgCl_2$ < $CaCl_2$ – Melting point

58. (B)

$Cs^{+}I_3^{-}$ (large cation stabilises by large anion)

59. (D)

$\underset{2S'}{Li} + e^{-} \xrightarrow{Ea} \underset{2S^2}{Li^{-}}$ exothermic

60. (C)

(I) $HClO_4 > H_2SO_4 > HNO_3 > H_3PO_4$

(II) $HClO_3 > HBrO_3 > HIO_3$

MATHEMATICS

61. (A)

Using $\dfrac{3\sin 76^0 .\sin 16^0 + \cos 76^0 \cos 16^0}{\cos 76^0 \sin 16^0 + \sin 76^0 \cos 16^0}$

$$= \frac{2\sin 76^0 \sin 16^0 + [\sin 76^0 \sin 16^0 + \cos 76^0 \cos 16^0]}{\sin 92^0} = \frac{\cos 60° - \cos 92° + \cos 60°}{\sin 92°}$$

$$= \frac{1-\cos 92^0}{\sin 92^0} = \frac{2\sin^2 46^0}{2\sin 46^0 \cos 46^0} = \tan 46^0 = \cot 44^0$$

62. (C)

$$\frac{a_1 + a_2 + a_3 .....a_p}{a_1 + a_2 + a_3 .....a_q} = \frac{p^2}{q^2}$$

$$\Rightarrow \frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$$

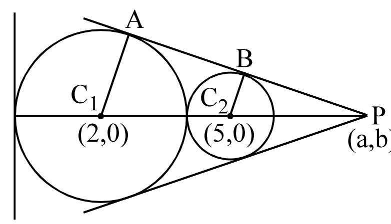
$$\frac{a_1+\left(\frac{p-1}{2}\right)d}{a_1+\left(\frac{q-1}{2}\right)d}=\frac{p}{q} \qquad \text{(i)}$$

for a_6 put $\frac{p-1}{2}=5$ and for a_{21} put $\frac{p-1}{2}=20$

$\Rightarrow p=11, q=41$

$\Rightarrow \frac{p}{q}=\frac{11}{41}$

63. (D)



$\frac{C_1P}{C_2P} = \frac{2}{1}$

∴ C_2 is the midpoint of C_1 and P

∴ P(8, 0)

equation of line through P

$y - 0 = m(x - 8)$

$mx - y - 8m = 0$

perpendicular from (2, 0) = radius i.e. 2

$\left|\frac{2m-8m}{\sqrt{1+m^2}}\right| = 2 \quad \Rightarrow \quad 9m^2 = 1 + m^2 \Rightarrow \quad m = -\frac{1}{2\sqrt{2}}$ or $\frac{1}{2\sqrt{2}}$ (rejected)

∴ $y = -\frac{1}{2\sqrt{2}}(x-8)$

for y-intercept put x = 0

$y = \frac{8}{2\sqrt{2}} = 2\sqrt{2}$

64. (A)

Let equation of line $\frac{x}{\cos\theta}=\frac{y}{\sin\theta}=r$

$(OA\cos\theta, OA\sin\theta)$, and $(OB\cos\theta, OB\sin\theta)$

Will satisfy $y - x - 10 = 0$ and $y - x - 20 = 0$

respectively

If $P(r\cos\theta, r\sin\theta)$ then $\frac{1}{r^2} = \left(\frac{\sin\theta - \cos\theta}{10}\right)^2 + \left(\frac{\sin\theta - \cos\theta}{20}\right)^2$

$\Rightarrow (r\cos\theta - r\sin\theta)^2 = 80 \Rightarrow$ locus of P is $(y-x)^2 = 80$

65. (D)

Here ax + by = 20 is a chord with (2, 3) as its mid-point.

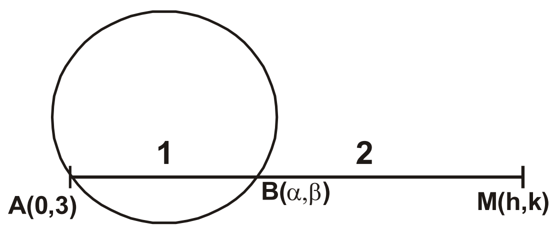
$\Rightarrow -\frac{a}{b} = -1 \qquad \Rightarrow a = b$

Now, $2a + 3b = 20$

$\Rightarrow 5a = 20 \Rightarrow a = b = 4$

Hence $a^{103} + b^{103} = 2^{207}$

66. (D)

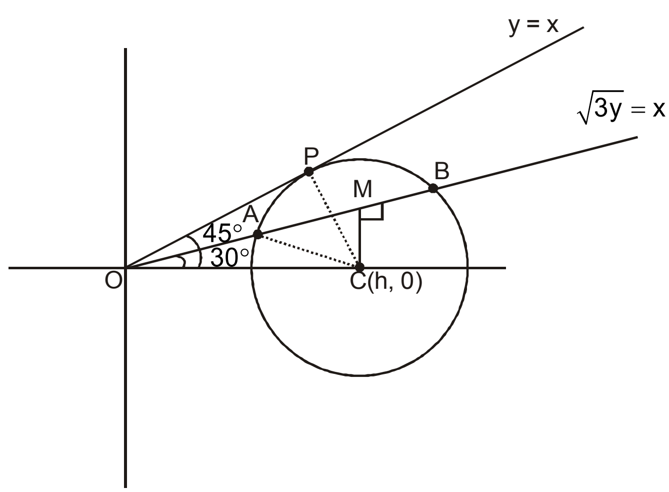


$\alpha = \frac{h}{3} \quad \beta = \frac{k+6}{3}$

Hence $\frac{h^2}{9} + \frac{(k+6)^2}{9} + 4 \times \frac{h}{3} - 6 \times \frac{k+6}{3} + 9 = 0 \Rightarrow h^2 + k^2 + 12h - 6k + 9 = 0$

$\Rightarrow x^2 + y^2 + 12x - 6y + 9 = 0$

67. (A)



$OP = CP = \frac{h}{\sqrt{2}} = AC$

$CM = h\sin 30° = \frac{h}{2}$

From ΔACM

$AC^2 = CM^2 + AM^2$

$\frac{h^2}{2} = 1 + \frac{h^2}{4} \Rightarrow h^2 = 4$

h = 2

$\text{radius} = \frac{h}{\sqrt{2}} = \sqrt{2}$

equation of circle

$(x-2)^2 + y^2 = 2$

68. (C)

a, ar, ar^2, ar^3 (G.P.)

a – 2, ar – 7, ar^2 – 9, ar^3 – 5 (A.P.)

$\therefore$ 2(ar – 7) =(a – 2) + (ar^2 – 9)

$\Rightarrow$ 2ar – 14 = a(1+ r^2) – 11

$\Rightarrow$ a(1 – r) (r – 1) = 3 ....(i)

Also 2(ar^2 – 9) = (ar – 7) + (ar^3 – 5)

$\Rightarrow$ 2ar^2 – 18 = ar (1 + r^2) – 12

$\Rightarrow$ a.r(r – 1)(1 – r) = 6 ....(ii)

From (i) & (ii), r = 2 and a = – 3

$\therefore$ third term of A. P. = ar^2 – 9 = (– 3).$(2)^2$ – 9 = – 12 – 9 = – 21

69. (B)

A.M $\geq$ G.M

$\frac{a+b+c}{3} \geq (abc)^{\frac{1}{3}}$; for (a, b, c > 0)

$\Rightarrow a+b+c \geq 3(abc)^{\frac{1}{3}}$

but given ab^2c^3, $a^2b^3c^4$, $a^3b^4c^5$ are in A.P

Hence 2abc = 1 + $a^2b^2c^2$ $\Rightarrow$ $(abc - 1)^2 = 0$ $\Rightarrow$ abc = 1

hence minimum value of

$a+b+c = 3(abc)^{\frac{1}{3}} = 3.(1)^{\frac{1}{3}} = 3$

70. (C)

Let T_r be the r^{th} term of given series, $T_r = \dfrac{2r+1}{\dfrac{r(r+1)(2r+1)}{6}} = \dfrac{6}{r(r+1)} = 6\left[\dfrac{1}{r} - \dfrac{1}{r+1}\right]$

$$\sum_{r=1}^{35} T_r = 6\left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + ... + \frac{1}{35} - \frac{1}{36}\right] = 6\left[1 - \frac{1}{36}\right] = \frac{35}{6}$$

71. (C)

$$S = \frac{5}{13} + \frac{55}{13^2} + \frac{555}{13^3} + ... \qquad \text{----(i)}$$

$$\frac{S}{13} = \frac{5}{13^2} + \frac{55}{13^3} + ...\infty \qquad \text{----(ii)}$$

(i) – (ii)

$$\frac{12}{13}S = \frac{5}{13} + \frac{50}{13^2} + \frac{500}{13^3} + ... \Rightarrow S = \frac{13}{12} \times \left[\frac{\frac{5}{13}}{1 - \frac{10}{13}}\right] = \frac{65}{36}$$

72. (A)

$$\frac{x}{-1/\lambda} + \frac{y}{+1} = 1 \qquad \frac{x}{-3} + \frac{y}{3/2} = 1$$

$$\left|\left(-\frac{1}{\lambda}\right)\right| \left|(-3)\right| = 1.\frac{3}{2}$$

$\lambda = 2$ (Here λ can't be negative)

73. (C)

$$P \equiv \frac{x}{\cos\frac{\pi}{4}} = \frac{y}{\sin\frac{\pi}{4}} = 6\sqrt{2} \quad \Rightarrow x = 6,\ y = 6$$

Since P(6,6) lie on circle

72 + 12 (g + f) + c = 0 ......(i)

Since y = x touches the circle, then

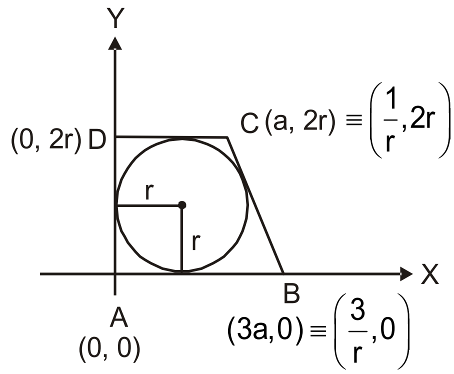
$2x^2 + 2x(g+f) + c = 0$ has equal roots D = 0

$4(g+f)^2 = 8c \Rightarrow (g+f)^2 = 2c$ ......(ii)

From equation (i), we get

$(12(g+f))^2 = [-(c+72)]^2 \Rightarrow 144(2c) = (c+72)^2 \Rightarrow (c-72)^2 = 0 \Rightarrow c = 72$

74. (B)



Area of trapezium ABCD $= \frac{1}{2}(a+3a)(2r) = 4 \Rightarrow ar = 1$

Equation of line BC is $y = -r^2\left(x - \frac{3}{r}\right)$

or, $y + r^2x - 3r = 0$

$\because$ BC is the tangent to the circle

$\Rightarrow \frac{|r + r^3 - 3r|}{\sqrt{1+r^4}} = r \Rightarrow r^4 + 4 - 4r^2 = 1 + r^4 \Rightarrow r = \frac{\sqrt{3}}{2}$

75. (C)

76. (A)

77. (C)

$\left.\begin{matrix} \_\ \_\ 20 \\ \_\ \_\ 40 \\ \_\ \_\ 60 \\ \_\ \_\ 04 \end{matrix}\right\} \underline{|3} \times 4 = 24$ $\qquad$ $\left.\begin{matrix} \_\ \_\ 12 \\ \_\ \_\ 16 \\ \_\ \_\ 24 \\ \_\ \_\ 64 \end{matrix}\right\} 2 \times 2 \times 4 = 16$

Total number of numbers = 24+16= 40

78. (A)

Alphabetical order of letters is B, E, K, R, U

words with 'B' = 4! = 24

words with 'E' = 4! = 24

words with 'KB' = 3! = 6

Words with 'KE' = 3! = 6

Words with 'KR' = 3! = 6

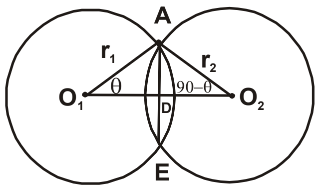
Next word will be KUBER

Whose rank is = 24 + 24 + 18 + 1 = 67

79. (A)

The circumcentre of ΔPQR will be orthocentre of ΔABC which is at (1, 1).

80. (A)

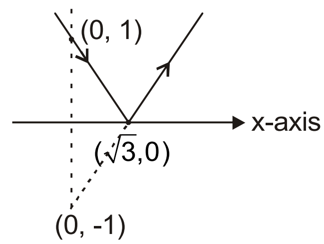


Let O_1 and O_2 are the centre of circles with radii r_1 and r_2 respectively and $\angle AO_1O_2 = \theta$

$AD = r_1 \sin\theta$; $AD = r_2 \cos\theta$

$$AD^2\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}\right)=1 \Rightarrow \quad AD=\frac{r_1r_2}{\sqrt{r_1^2+r_2^2}} \text{ so } AE = 2AD$$

81. (A)



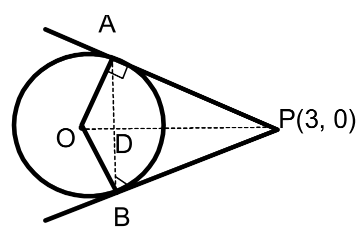
$x+\sqrt{3}y=\sqrt{3}$

$(y+1)=\frac{1}{\sqrt{3}}x$

$\sqrt{3}y = x-\sqrt{3}$

82. (A)

Equation of AB is T = 0 i.e. $x=\frac{4}{3}$, $OD=\frac{4}{3}$



$\Rightarrow AD^2 = OA^2 - OD^2$

$\Rightarrow AD^2 = 4-\frac{16}{9}=\frac{20}{9}$

$\Rightarrow AD^2 = \frac{2\sqrt{5}}{3} \Rightarrow AB = \frac{4\sqrt{5}}{3} \Rightarrow$ Area of triangle

$$PAB = \frac{1}{2}.\frac{4\sqrt{5}}{3}.\left(3-\frac{4}{3}\right)=\frac{10\sqrt{5}}{9}\text{sq.units}$$

83. (B)

Let $y = m_1x$ and $y = m_2x$ be the two lines represented by $ax^2 + 2hxy + by^2 = 0$ so that

$m_1+m_2=-\frac{2h}{b}$ and $m_1m_2=\frac{a}{b}$ ...(1)

Given $m_2 = m_1^2$

$\therefore$ From (1), $m_1 + m_1^2 = -\frac{2h}{b}$ ...(2)

and $m_1.m_1^2 = \frac{a}{b}$ i.e., $m_1^3 = \frac{a}{b}$ ...(3)

The required condition is obtained by eliminating m_1 between (2) and (3).

Cubing (2), we get $(m_1 + m_1^2)^3 = \left(-\frac{2h}{b}\right)^3$

$\Rightarrow$ $m_1^3 + m_1^6 + 3m_1^3(m_1 + m_1^2) = -\frac{8h^2}{b^3}$

$\Rightarrow$ $\frac{a}{b} + \frac{a^2}{b^2} + 3\frac{a}{b}\left(-\frac{2h}{b}\right) = -\frac{8h^3}{b^3}$ [Using (2) and (3)]

$\Rightarrow$ $ab^2 + a^2b - 6abh = -8h^3$ or $ab(a + b) - 6abh + 8h^3 = 0$.

84. (B)

The equation of the bisectors of the angles between the lines $x^2 - 2pxy - y^2 = 0$ is

$$\frac{x^2 - y^2}{1-(-1)} = \frac{xy}{-p} \quad \text{or} \quad \frac{x^2 - y^2}{2} = -\frac{xy}{p}$$

i.e. $x^2 + \frac{2}{p}xy - y^2 = 0$ ...(1)

Also, $x^2 - 2qxy - y^2 = 0$ ...(2)

is the equation of the bisectors of the angles between the same lines (given).

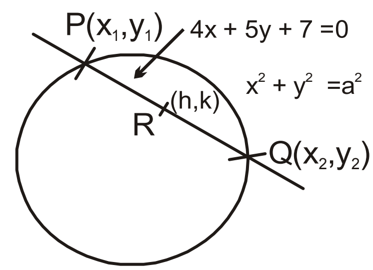
From (1) and (2), by comparing coefficients, we get $\frac{1}{1} = \frac{2/p}{-2q} = \frac{-1}{-1}$

i.e. $1 = -\frac{1}{pq}$ or $pq = -1$.

85. (A)

86. (C)

$$h=\frac{x_1+x_2}{2}, K=\frac{y_1+y_2}{2}$$



$$x^2+\left(\frac{4x+7}{-5}\right)^2=a^2$$

$$\Rightarrow 41x^2+56x+49-a^2.25=0$$

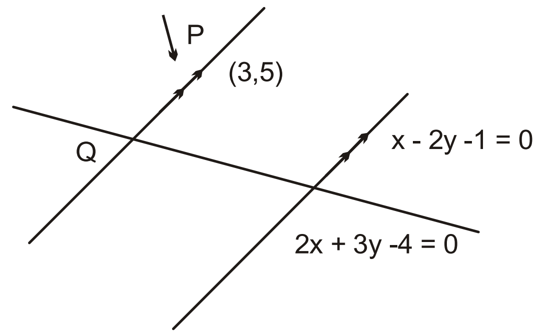
$$\therefore \quad \frac{x_1+x_2}{2}=\frac{-56}{41\times 2}=\frac{-28}{41}$$

Similarly, $\frac{y_1+y_2}{2}=\frac{-70}{41\times 2}=\frac{-35}{41}$

87. (B)

If θ be the angle that the line $x-2y-1=0$ makes with the positive x - axis, measured in the anti-clockwise sense, then $\tan\theta=\frac{1}{2}$ $\therefore \cos\theta=\frac{2}{\sqrt{5}}, \ \sin\theta=\frac{1}{\sqrt{5}}$

$$Q\equiv(3+r\cos\theta, 5+r\sin\theta)$$



$$\therefore 2\left(3+r.\frac{2}{\sqrt{5}}\right)+3\left(5+\frac{r}{\sqrt{5}}\right)-4=0$$

$$\Rightarrow \ r=\frac{-17.\sqrt{5}}{7}$$

$\therefore$ distance = $\frac{17\sqrt{5}}{7}$ unit

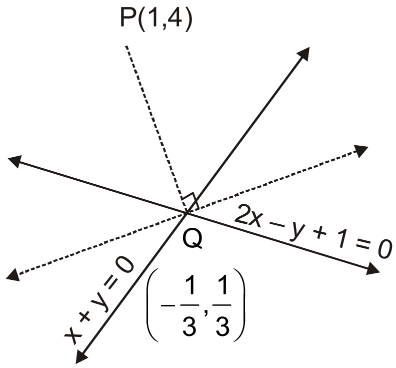
88. (C)

We have, $x+y+\lambda(2x-y+1)=0$

Clearly, it represents a family of line passing through the intersection of the lines x + y = 0 and 2x – y + 1 = 0 i.e. the point (– 1/3, 1/3)

The required line passes through (–1/3, 1/3) and is perependicular to the line joining (1, 4) and (-1/3, 1/3). So, its equation is

$$y-\frac{1}{3}=-\frac{4}{11}\left(x+\frac{1}{3}\right) \Rightarrow \ 12x+33y=7$$



89. (B)

The given circle $x^2 + y^2 - 4x - 6y - 12 = 0$ has its centre at (2, 3) and radius equal to 5.

Let (h, k) be the coordinates of the centre of the required circle. Then, the point (h, k) divides the line joining (– 1, – 1) to (2, 3) in the ratio 3 : 2, where 3 is the radius of the required circle. Thus, we have

$$h = \frac{3\times 2 + 2(-1)}{3+2} = \frac{4}{5} \text{ and } k = \frac{3\times 3 + 2(-1)}{3+2} = \frac{7}{5}$$

Hence, the equation of the required circle is

$$\left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = 3^2 \Rightarrow 5x^2 + 5y^2 - 8x - 14y - 32 = 0.$$

90. (A)

According to question

A(0,0) and B(1,1) are the end points of the diameter of the circle.