SOLUTIONS

MEAITS 2018

UNIT TEST-1

(MAIN & ADVANCED PATTERN)

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JEE MAIN

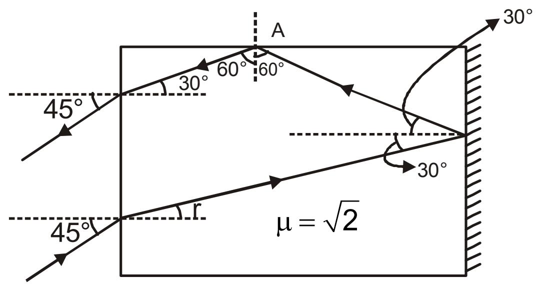
PHYSICS

1. **(C)**

the magnitude will decrease till the direction of the velocity with respect to man becomes vertical. It will increase there after.

2. **(C)**

from snell's law



$$1\sin 45° = \sqrt{2}\sin r$$

$$\sin r = \frac{1}{2}$$

$$r = 30°$$

Critical angle of a glass

$$\sin C = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$$

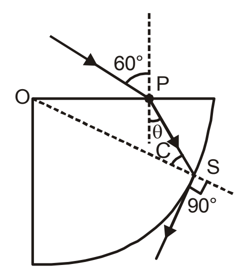
C = 45°

at point A angle of incident is greater than critical angle. So at a point A total internal reflection takes place at point B angle of incident is less than critical angle. So refraction takes place at B. and refracted ray parallel to incident ray.

deviation = 180°

3. **(B)**

At P, $\dfrac{\sin 60°}{\sin\theta} = \sqrt{3}$



$$\sin\theta = \frac{1}{2}$$

$$\theta = 30°$$

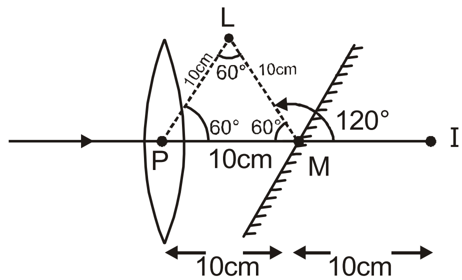
Also $\text{Sin } C = \dfrac{1}{\mu} = \dfrac{1}{\sqrt{3}}$

In triangle OPS

$$\frac{OP}{\sin C} = \frac{R}{\sin(90+\theta)}$$

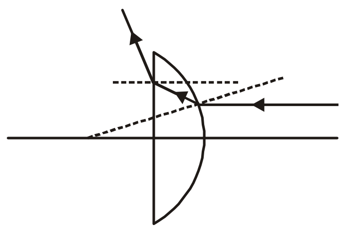
$$OP = \frac{R\sin C}{\sin(90+30)} = \frac{\frac{1}{\sqrt{3}}.R}{\frac{\sqrt{3}}{2}} = \frac{2R}{3}$$

4. (D)



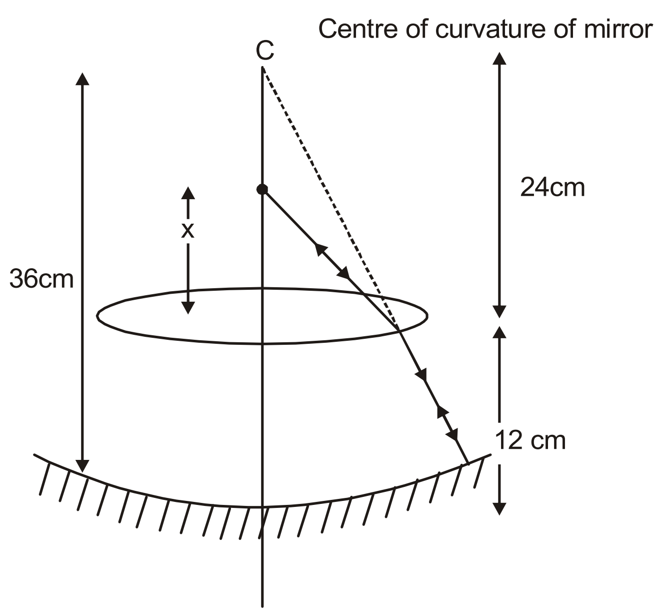
Angular deviation of reflected ray = 120° in anticlockwise direction. so rotation of mirror is equal to 60°

5. (B)



When a ray moves from denser to rarer medium it bend away from normal.

6. (B)



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{-24} - \frac{1}{-x} = \frac{1}{40}$$

$$\frac{1}{x} = \frac{1}{40} + \frac{1}{24} = \frac{3+5}{120} = \frac{8}{120}$$

$x = 15\text{cm}$

7. (C)

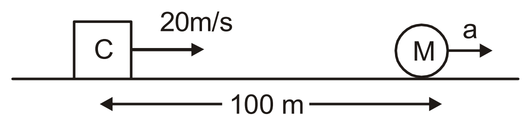
$f_{\text{plano concave}} > f_{\text{bi concave}}$

$f_{\text{plano concave}}\ \alpha$ radius of curvature of surface.

8. (A)

If t is the time in which man can catch his friend then

$$\frac{1}{2}at^2 + 100 = 20t$$



or, $\dfrac{at^2}{2} - 20t + 100 = 0$

for t have real solution

$$(-20)^2 - 4 \times \frac{a}{2} \times 100 \geq 0$$

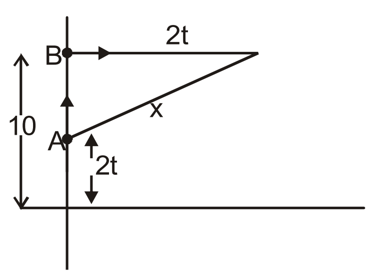
$a \leq 2\ \text{m/s}^2$

9. (B)

The time taken to move net 2 steps is 8s, and so far 8 sleps he takes 32 se in last 5 sleps he will take 5 sec and fall into pit.

10. (A)

At any Instant the distance between them $x = \sqrt{(2t)^2 + (10-2t)^2}$



for closest approach, $\dfrac{dx}{dt} = 0$

after solving, we get t = 2.5 sec.

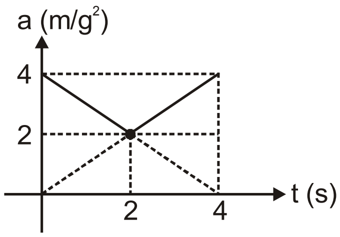
11. (D)

acceleration can be written as

$a = 2 + 2 - t = 4 - t, t \leq 2s$

$a = 2 + t - 2 = t, t \geq 2s$

there for, acceleration time graph of the particle will be as shown below.



Change in velocity = Area enclosed (a – t) graph

$v_f - 0 = 4 \times 4 - \frac{1}{2} \times 4 \times 2$

= 12 m/s

12. (C)

$a = v\left(\frac{dv}{dx}\right)$

st.line (a = v) ; Constant $\left(\frac{dv}{dx}\right)$

13. (C)

$500 \cos\theta = 250$

$\cos\theta = \frac{1}{2}$

$\theta = 60°$

14. (A)

OM comparing with

$\vec{v} = v_x\hat{i} + v_y\hat{j}$, we get

$v_x = \frac{dx}{dt} = ky$ ...(i)

and $v_y = \frac{dy}{dt} = kx$ ...(ii)

Dividing equation (ii) and (i) we get

$y\,dy = x dx$

Now $\int ydy = \int xdx$

$$\frac{y^2}{2} = \frac{x^2}{2} + \text{constant}$$

$$y^2 = x^2 + \text{constant}$$

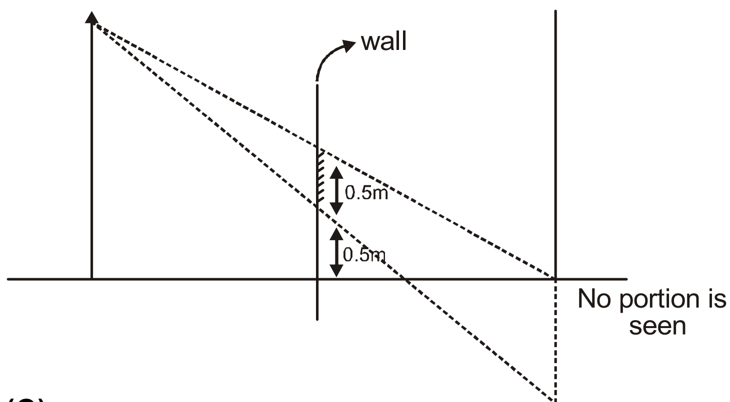
15. (B)

Here x = 1, y = $4t^2$ and z = t

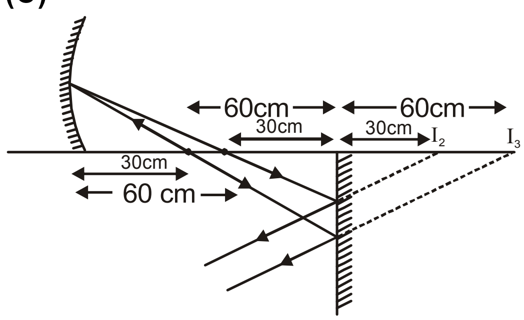
$\therefore$ y = $4z^2$, is represents a parabola

16. (C)

17. (C)



18. (C)



total no. of image = 3

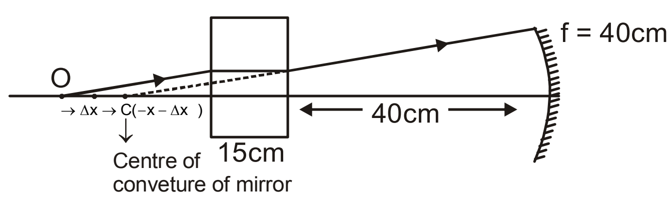
19. (A)

$$\Delta u = t\left(1 - \frac{1}{\mu}\right)$$

$$= 15\left[1 - \frac{2}{3}\right] = 5$$

$$(x - \Delta x) + 15 + 40 = 2f = 80$$

x = 30 cm



20. (A)

$\sec = \frac{1}{\sqrt{2}}$ $\quad$ C = 45

for T.I.R $90 - \phi > 45$

$\phi < 45$

21. (A)

$s = (\mu - 1)\,A$

$\left(\frac{3}{2} - 1\right) \times 4 = 2°$

deviation due to mirror = $\pi - 2i$

= 180 – 4

= 176°

S_{net} = 2° + 176° = 178°

22. (A)

All colours are parallel to incident ray after refraction through slab.

23. (A)

$$-\frac{1}{f_{Meq}} = \frac{2}{f_\ell} - \frac{1}{f_m}$$

$$-\frac{1}{f_{Meq}} = 2\left(\frac{4}{3} - 1\right)\left[\frac{1}{\infty} - \frac{1}{-20}\right] - \frac{1}{-10}$$

$$-\frac{1}{f_{Meq}} = \frac{2}{3} \times \frac{1}{20} + \frac{1}{10} = \frac{1}{30} + \frac{1}{10}$$

$$-\frac{1}{f_{Meq}} = \frac{1+3}{30} = \frac{4}{30}$$

$f_{Meq} = -7.5\ \text{cm}$

for image consider at object = $2f_{Meq} = 15\text{cm}$

24. (C)

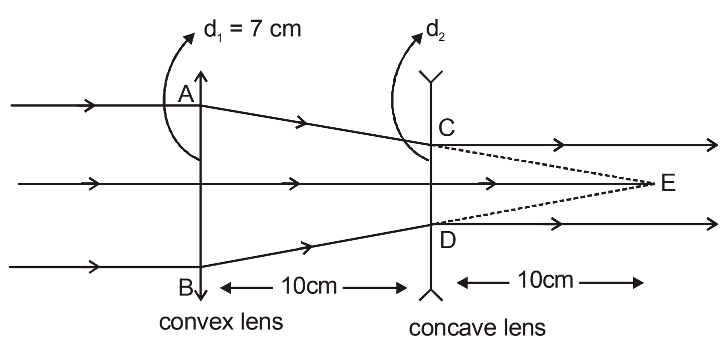
from similar length

ABE and CDE

$$\frac{AB}{CD} = \frac{20}{10} = 2$$

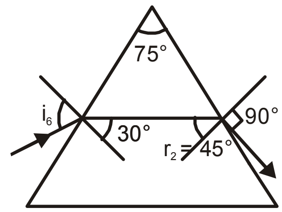
$$\frac{7}{CD} = 2$$

$CD = 3.5\,\text{cm}$



25. (A)

$\sin r_2 = \frac{1}{\alpha} = \frac{1}{\sqrt{2}}$



$r_2 = 45$

Apply smell's law

sin = $\sqrt{2}\,.\sin 30°$

$i_0 = 45$

S = i + e – A
= 45 + 90 – 75
= 60°

26. (A)

$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2 = \vec{g} - \vec{g} = 0$

$\vec{v}_{12} = \text{const}$

velocity of 1 with respect to 2 is const. so trajectory is a straight line

27. (A)

28. (A)

$y = x\tan\theta - \frac{gx^2}{2u^2}\left(1 + \tan^2\theta\right)$

$y = 10m, x = 20m, g = 10m/s^2$ (u = 20 m/s)

then tan θ = 1, 3

at $\tan\theta = 1$ range is maximum

and $\tan\theta = 3$ range is minimum

$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta.\cos\theta}{10}$

$= \frac{2 \times 20 \times 20 \times \frac{3}{\sqrt{10}} \times \frac{1}{\sqrt{10}}}{10}$

= 24m

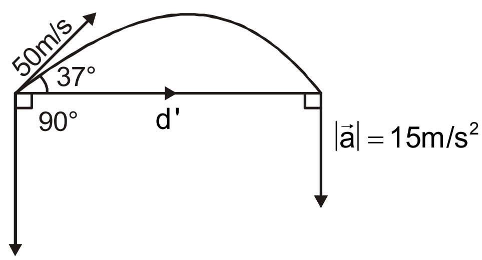
distance AB = 24 – 20 = 4 m

29. (A)

$T_F = \frac{2u\sin\theta}{a}$

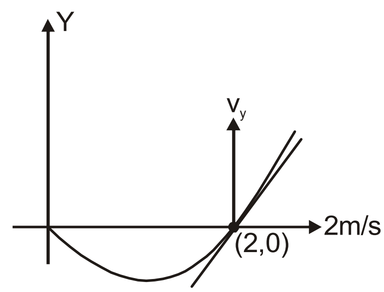
$= \dfrac{2 \times 50 \times \sin 37°}{15}$

= 4 sec



30. (A)

$\tan\theta = \dfrac{v_y}{v_x}$



$\tan\theta = \dfrac{v_y}{2}$

$\dfrac{dx}{dy} = \dfrac{v_y}{2}$ ....(i)

y = $x^2 - 2x$

$\left.\dfrac{dy}{dx}\right| = 2x - 2$

$\left.\dfrac{dy}{dx}\right|_{x=2} = 2 \times 2 - 2 = 2$

From eq. (i)

$\dfrac{dy}{dx} = \dfrac{v_y}{2}$

$2 = \dfrac{v_y}{2}$, $v_y = 4m/s$

$\vec{v} = v_x\hat{i} + v_y\hat{j} = 2\hat{i} + 4\hat{j}$

CHEMISTRY

31. **(C)**

$$\text{mole of x} = \frac{\text{wt of x}}{\text{Mol. wt of x}} = \frac{1\text{kg}}{2.6\times10^{-27}\text{kg}\times6.022\times10^{23}}$$

$$= \frac{1}{2.6\times6.022}\times10^{4}$$

32. **(A)**

24.95 g $CuSO_4.5H_2O$

$$nCuSO_4.5H_2O = \frac{24.95}{249.5} = 0.1$$

$nCuSO_4 = 0.1$

wt $CuSO_4$ = 0.1 × 159.5 = 15.95

wt H_2O from $CuSO_4\cdot5H_2O$ = 24.95 – 15.95 = 9g

If ‘x’ is the wt of H_2O in which $CuSO_4.5H_2O$ is added total wt of solution = (x + 24.95) g

$$\% \text{ by wt } CuSO_4 = \frac{15.95}{(x+24.95)}\times100 = 10$$

$159.5 = x + 24.95$

$x = 134.55$ g

33. **(B)**

1 lit solution has 3 mole $Na_2S_2O_3$ 1000 ml solution has 3 × 158 = 474 g

$m_{solution}$ = v × d = 1250 g

$m_{solvent} = 1250 - 474 = 776$ g

$$\text{molality} = \frac{n_{solute}}{m_{solvent}}\times1000 = \frac{3}{776}\times1000 = 3.866$$

$\text{molality}_{Na+} = 2\times\text{molality}_{Na_2S_2O_3} = 7.732$

34. **(B)**

Let dibasic acid is H_2A

Silver salt : Ag_2A

$$\underset{0.90g}{Ag_2A} \longrightarrow \underset{0.54g}{Ag}$$

By POAC of Ag

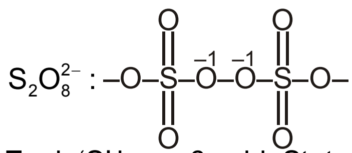
$$2 \times \frac{0.90}{(216 + M_{A^{2-}})} = \frac{0.54}{108}$$

$$(216 + M_{A^{2-}}) = 360$$

$$M_{A^{2-}} = 360 - 216 = 144$$

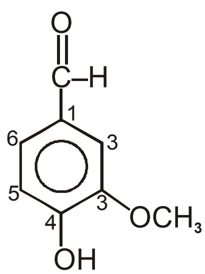
Mol. wt H_2A = 2 + 144 = 146

35. (B)



Each ‘S’ has +6 oxid. State with two ‘O’ atom in ‘–1’ oxidation state

36. (D)



4-Hydroxy-3-methoxybenzaldehyde

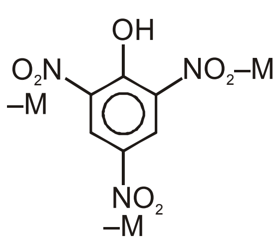
37. (D)

Acceptor group decreases basic strength while donar group increases basic strength

38. (C)

Anti-aromatic compound is paramagnetic.

39. (C)



–M-group increases acidic strength

40. (C)

–M group decreases the stability of carbocation while +M group increases.

41. **(D)**

He is noble gas with smallest size

42. **(B)**

The order of electron gain enthalpy for halogens is Cl > F > Br > I

43. **(D)**

Ion	**No. of atoms**
NH_4^+	5
SO_4^{2-}	5
NO_3^-	4
OH^-	2

More numbers of atoms in polyatomic ion, larger is the size.

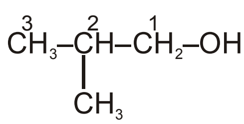
44. **(B)**

Greater the number of shells present in atom, greater is the shielding effect observed for electron present in valancy shell.

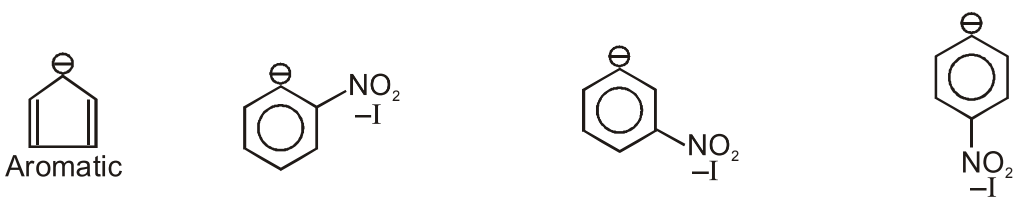
45. **(B)**

Electronegativity does not depend upon stability of electronic configuration.

46. **(D)**

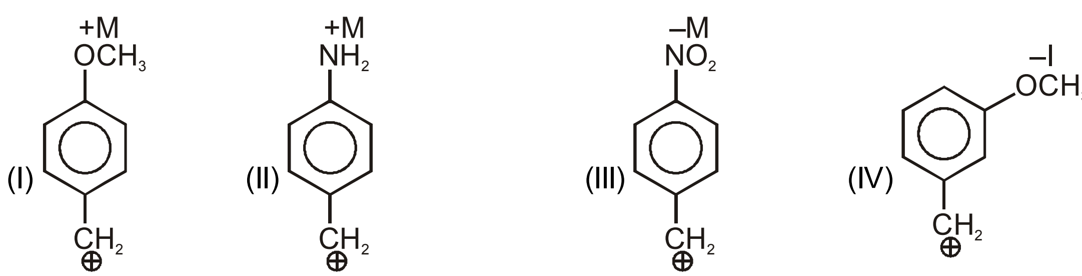


47. **(D)**



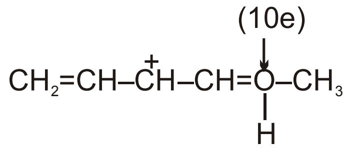
48. **(B)**

$$BDE \propto \frac{1}{\text{stability of carbocation}}$$

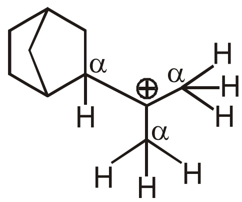


Stability - II > I > IV > III

49. (B)



50. (B)



51. (C)

$$m_{Fe}\Big|_{120\,\ell b\text{ of }Fe_2O_3} = m_{Fe}\Big|_{x\,\ell b\text{ of }Fe_3O_4}$$

$$\frac{112}{160}\times 120\ \ell b = \frac{168}{232}\times x\,\ell b$$

$$x = \frac{112\times 232\times 120}{168\times 160} = 116\ \ell b$$

52. (C)

In 1 lit solution, mole of $[CO(NH_3)_5SO_4]Cl$ = 0.005

mole of AgCl (Y) = 0.005 mole

mole of $[CO(NH_3)_5Cl]SO_4$ = 0.005

mole of $SrSO_4$(Z) = 0.005 mole

53. (C)

The balanced reaction is

$$2KNO_3 + 6FeSO_4 + 5H_2SO_4 \longrightarrow 2KHSO_4 + 3Fe_2(SO_4)_3 + 2NO + 4H_2O$$

$$a + c = 2 + 5 = 7$$

54. (C)

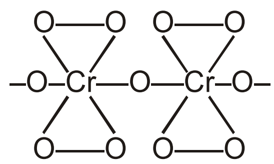
$$Cr_2O_{11}^{2-} : 2x + 11(-2) = -2$$

$$2x = 20 \Rightarrow x = 10$$

$$\text{No. of peroxide bonds} = \frac{\text{Total increase in oxid.no}}{2}$$

$$= \frac{20-12}{2} = 4$$

Also



no. of peroxide bonds = 4

55. **(C)**

Law of multiple proportion is defined for different compounds having same constituent elements.

56. **(C)**

Oxide and hydroxide of non-metals are acidic in nature.

57. **(A)**

Element is Gd and it is present in 6th period 3 group, as it is a lanthanoid

58. **(B)**

Upon removal of three electrons from valancy shell, noble gas configuration is attained, which has highest IP.

59. **(C)**

EN on pauling scale

C 2.5

N 3

Si 1.8

P 2.1

60. **(D)**

amphoteric oxides are

ZnO, Cr_2O_3, PbO_2

MATHEMATICS

61. **(A)**

62. **(A)**

63. **(C)**

$$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2},\ \left[\tan^{-1} x\right] = -2, -1, 0, 1$$

64. **(C)**

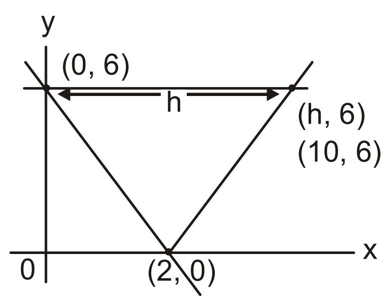
$h^2 = (h-2)^2 + 6^2$

$h^2 = h^2 + 4 - 4h + 6^2$

$4h = 40$

$h = 10$

Centroid $\equiv (4, 4)$



65. **(C)**

Period of $\left(\sin^2 \sqrt{k}x\right)^m = \frac{\pi}{\sqrt{k}} \Rightarrow \sqrt{k} = 1$, k = 1

66. **(B)**

$$y = \frac{x}{2x^2+3} = \frac{1}{2x + \frac{3}{x}}$$

Now, $\frac{2x + \frac{3}{x}}{2} \geq \sqrt{6} \Rightarrow 2x + \frac{3}{x} \geq 2\sqrt{6}$

67. **(C)**

$1 \leq |\sin x| + |\cos x| \leq \sqrt{2}$, $[|\sin x| + |\cos x|] = 1$

68. **(B)**

69. **(A)**

$$\left|\frac{12x - 5y}{13}\right| = \left|\frac{3x + 4y}{5}\right|$$

70. (A)

$\sin^{-1} x = \frac{\pi}{2} \Rightarrow x = 1,\ \sin^{-1} z = \frac{\pi}{2} \Rightarrow z = 1$

$\cos^{-1} y = \pi \Rightarrow y = -1,\ \cos^{-1} w = \pi \Rightarrow w = -1$

$\therefore\ 2x + 3y + 4z + 5w = 2 - 3 + 4 - 5 = -2$

71. (C)

$0 \le x \le \pi,\ 0 \le \frac{x}{2} \le \frac{\pi}{2},\ -1 \le \frac{x}{2} - 1 \le \frac{\pi}{2} - 1$

72. (C)

73. (C)

$\alpha = 3, \beta = -1,$

74. (A)

Homogeneous equation of 2nd degree and perpendicular to each other

75. (C)

$g^{-1}(x) = f(x),\ g(f(x)) = x,\ g'(f(x)) \cdot f'(x) = 1$

$f(x) = -4$ at $x = -2$, $g'(-4) = \frac{1}{f'(-2)} = \frac{1}{2}$

76. (D)

77. (C)

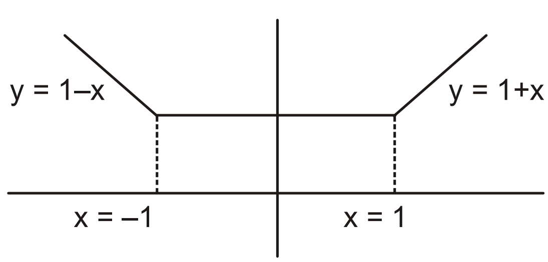
78. (C)

$n(H \cup E) = 1000,\ n(H) = 750,\ n(E) = 400 \Rightarrow n(H \cap E) = 150$

79. (B)

$(A \cup B) \cap B' = A$

80. (D)



81. **(A)**

$\Delta = 0$, $h^2 - ab = 0$

82. **(C)**

$f(x) = x^2 + x + 1, \quad x \neq 1$

83. **(A)**

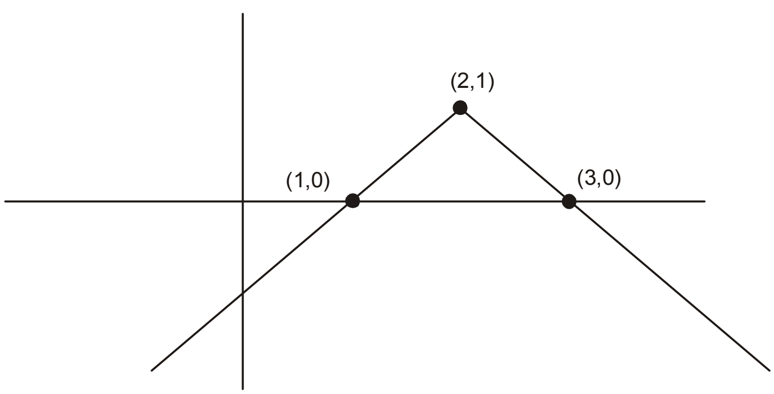
84. **(B)**

$$\sin\left(\cot^{-1}(x+1)\right) = \frac{1}{\sqrt{(x+1)^2 + 1}}$$

$$\cos\left(\tan^{-1} x\right) = \frac{1}{\sqrt{x^2 + 1}}$$

85. **(A)**

y = f(x) is



86. **(A)**

$$\lim_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$$

87. **(A)**

$$y = \sqrt{\sin x^2 + y} \Rightarrow y^2 - y = \sin x^2$$

$$(2y - 1)\frac{dy}{dx} = \cos x^2 \cdot 2x \Rightarrow \frac{dy}{dx} \text{ at } x = \sqrt{\pi}, y = 0 \text{ is } = 2\sqrt{\pi}$$

88. (C)

$$-1 \le \left[\frac{1}{2}\log_2 x\right] \le 1 \Rightarrow -1 \le \frac{1}{2}\log_2 x < 2$$

$$\Rightarrow -2 \le \log_2 x < 4 \Rightarrow \frac{1}{4} \le x < 16$$

89. (D)

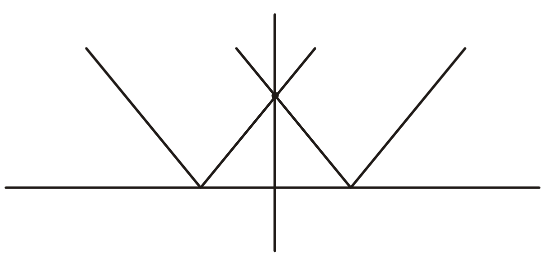
Put $x = -\frac{1}{t}$, $x \to -\infty, t \to 0$

$\therefore$ given limit is = $\lim_{t\to 0} \frac{e^{t^2}-1}{2\tan^{-1}\frac{1}{t^2}-\pi}$

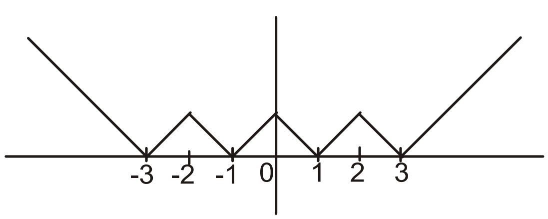
$$\lim_{t\to 0} \frac{e^{t^2}-1}{2\left(\cot^{-1}t^2-\frac{\pi}{2}\right)} = \frac{e^{t^2}-1}{-2\cdot\tan^{-1}t^2} = -\frac{1}{2}$$

90. (D)

graph of y = f(x)



graph of y = f(|x| – 2)



JEE ADVANCED

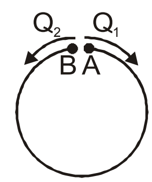
PHYSICS

1. (C)

distance = $\frac{1}{2}\times 2\times 2\times 2+\frac{1}{2}\times 2\times 1+\frac{1}{2}\times 2\times 2\times 2+\frac{1}{2}\times 2\times 1+\frac{1}{2}\times 1\times 1$

= 10.5 m

2. (A)



$\theta_1 = \sqrt{2\pi}\, t - \frac{1}{2}(1)\, t^2$

$\theta_2 = -\frac{1}{2}\sqrt{\frac{\pi}{2}}\,.t$

$\theta_1 = \theta_2$ (at the time of collision)

$\Rightarrow \quad t = 5\sqrt{\frac{\pi}{2}} s$

3. (A)

$\omega = \frac{2\pi}{8T} \quad \therefore\ \theta = \omega\,\Delta t = \frac{2\pi}{8T}(2T - T) = \frac{\pi}{4}$

$\left|\frac{d\vec{r}_2}{dt} - \frac{d\vec{r}_1}{dt}\right| = \left|\vec{v}_2 - \vec{v}_1\right| = \sqrt{v^2 + v^2 - 2vv\cos\frac{\pi}{4}}$

$= v\sqrt{2-\sqrt{2}}$

4. (B)

$|\vec{r}| = \sqrt{16\sin^2 4t + 9\cos^2 4t} = \sqrt{7\sin^2 4t + 9}$

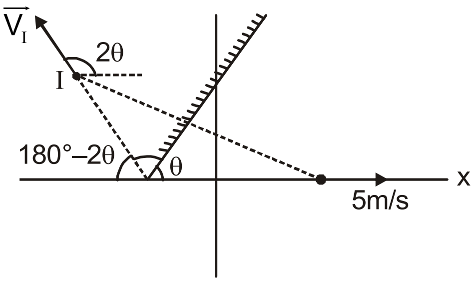
$|\vec{r}|$ is max for sin 4t = 1 $(\Rightarrow \quad \cos 4t = 0)$

$|\vec{r}|_{max} = \sqrt{7+9} = 4m$

$\vec{v} = \frac{d\vec{r}}{dt} = 16\cos 4t\,\hat{i} - 12\sin 4t\,\hat{j}$

$$w = \frac{v}{r} = \frac{\sqrt{16^2 \cos^2 4th + 12 \sin^2 4t}}{4} = 3 \text{ rad/s}$$

5. (A)



$$\left|\vec{V}_I\right| = 5$$

$$\tan\theta = 2$$

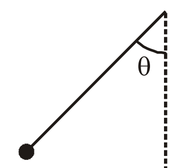
$$\Rightarrow \sin\theta = \frac{2}{\sqrt{5}}$$

$$\cos\theta = \frac{1}{\sqrt{5}}$$

$$\therefore \left|\vec{V}_I\right| = 5\cos 2\theta\,\hat{i} + 5\sin 2\theta\,\hat{j} = -3\hat{i} + 4\hat{j} \text{ m/s}$$

6. (C)

$$T = 2\pi\sqrt{\frac{\ell\cos\theta}{g + a_0}} = \frac{2\pi}{5}$$

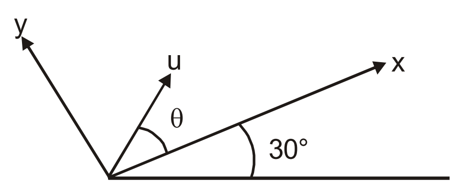


$$\Rightarrow \cos\theta = \frac{6}{25}$$

$$T'\cos\theta = mg + ma_0 \Rightarrow T'\frac{1\times 10 + 1\times 2}{6/25} = 50N$$

7. (B)

Let the angle of initial velocity with the plane = θ



$$v_x = u_x + a_x t$$

$$\Rightarrow 0 = 20\cos\theta - 5\times\frac{2\times 20\sin\theta}{10\times\cos 30^\circ} \quad \Rightarrow \tan\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin\theta = \frac{\sqrt{3}}{\sqrt{7}}$$

$$\cos\theta = \frac{2}{\sqrt{7}}$$

$v_x^2 = u_x^2 + 2a_x.r$

$\Rightarrow \quad \theta = 20^2 \cos^2\theta + 2(-5)\,r$

$\Rightarrow \quad r = \dfrac{20^2 \times \frac{4}{7}}{10} = \dfrac{160}{7}$

$v_y{}^2 = 20^2 \sin^2\theta + 2 \times a_y \times 0$

$v_y = -20\sin\theta$

$|\vec{v}| = v = |v_y| \; (\because v_x = 0)$

$= 20\sin\theta = 20 \times \sqrt{\dfrac{3}{7}}$

$|\vec{r} \times \vec{v}| = rv$

$= \dfrac{160}{7} \times 20\sqrt{\dfrac{3}{7}}$

$= \dfrac{3200\sqrt{3}}{7\sqrt{7}}$

8. (B)

When crossing xy plane, Z-coordinate = 0

$\Rightarrow \quad 2\sin t = 0$

$\Rightarrow \quad \sin t = 0$

$\Rightarrow \quad \cos t = 1$

$\vec{v} = \dfrac{d\vec{r}}{dt} = 2\cos t\,\hat{i} - 2\sin t\,\hat{j} + 2\cos t\,\hat{k}$

$= \quad 2\hat{i} + 2\hat{k}$

$\therefore$ angle with the xy plane = $\tan^{-1}\left(\dfrac{2}{2}\right)$

= 45°

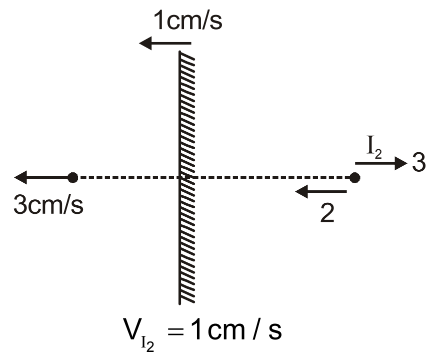
9. (A)

$$\frac{1}{v}-\frac{1}{-15}=\frac{1}{10} \quad \Rightarrow v=+30$$

$$v_{I_1}=\frac{v^2}{u^2}\times v_0=\frac{30^2}{15^2}\times(-1)=-4\text{cm / s (wrt lens)}$$

$$v_{I_1}=-3\text{cm / s wrt ground}$$

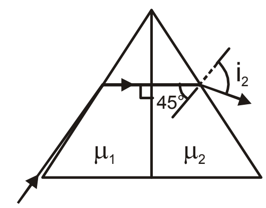
$$v_{I_2}=1\text{cm / s}$$



10. (D)

for right side prism :

$$\sqrt{\frac{3}{2}}\sin 45^\circ = 1\sin i_2$$



$$\Rightarrow \quad i_2 = 60^\circ$$

$$\delta = i_1 + i_2 - A$$

$60° = i_1 + 60° – 90°$

$\Rightarrow$ $i_1 = 90°$

for first prism : 1 sin 90° = μ_1 sin 45°

$$\Rightarrow \quad \mu_1=\sqrt{2}$$

$$\therefore \quad \text{time} = \frac{d}{v_1}+\frac{d}{v_2}$$

$$= \frac{d}{c/\mu_1}+\frac{d}{c/\mu_2}=\frac{2d+\sqrt{3}d}{\sqrt{2}c}$$

11. (A) (C)

$$a_c=\frac{v^2}{r}=2t \quad \Rightarrow v=\sqrt{2rt} \quad \Rightarrow \int_0^{2\pi r} ds=\int_0^t \sqrt{2rt}\,dt \quad \Rightarrow \quad t=3s$$

$\therefore$ (A) is correct

at t = 2s

$a_c = 2t = 2 \times 2 = 4\ m/s^2$

$$v = \sqrt{vt} = \sqrt{\frac{2.6}{\pi^2}t} = \frac{\sqrt{12t}}{\pi}$$

$$\Rightarrow \quad a_T = \frac{dv}{dt} = \sqrt{2r} \times \frac{1}{2\sqrt{t}} = \frac{1}{\pi}\sqrt{\frac{3}{2}}\ m/s^2$$

$$a = \sqrt{a_C^2 + a_T^2} = \sqrt{16 + \frac{3}{2\pi^2}}\ m/s^2$$

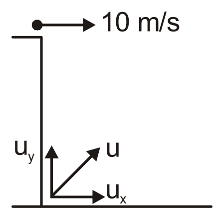
$\therefore$ B is wrong

Speed of the particle increases with time, so second round will take less time than the first round.

$\therefore$ C is correct

12. (A), (B), (C), (D)

for collision to happen,



$$u_x = u_A = 10m/s$$

for A :

A ↓ 5m, $5 = 0 + \frac{1}{2}(10)t^2$

$\Rightarrow \quad t = 1s$ = time of collision $\rightarrow$ (B)

for B

B ↑ 15m, $15 = u_y \times 1 + \frac{1}{2}(-10)1^2$

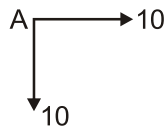
$\Rightarrow u_y = 20$ m/s

$\therefore \quad u = \sqrt{10^2 + 20^2} = 10\sqrt{5}\ m/s \rightarrow (D)$

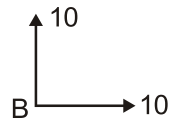
Horizontal disp = 10 × 1 = 10m

$\therefore$ dist from base = $\sqrt{10^2 + 15^2} = 5\sqrt{3}\ m \rightarrow (A)$

at $t = 1s$,



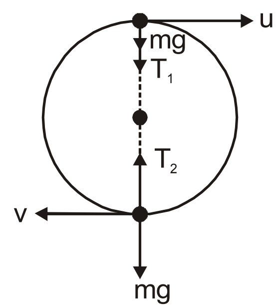
$v_y = 0 + 10 \times 1 = 10$



$v_y = 20 - 10 \times 1 = 10$

$\therefore$ Vel of approach = $10 + 10 = 20\,m/s \rightarrow$ (C)

13. **(A), (D)**



$\frac{1}{2}mu^2 + mg.2\ell = \frac{1}{2}mv^2$ .....(i)

$\frac{T_2}{T_1} = 4$ ...(ii)

$T_2 - mg = \frac{mv^2}{\ell}$ ...(iii)

$T_1 + mg = \frac{mu^2}{\ell}$ ..(iv)

Solving above equeation : $u = \sqrt{3g\ell}$, $v = \sqrt{7g\ell}$

$T_1 = 2mg$, $T_2 = 8mg$

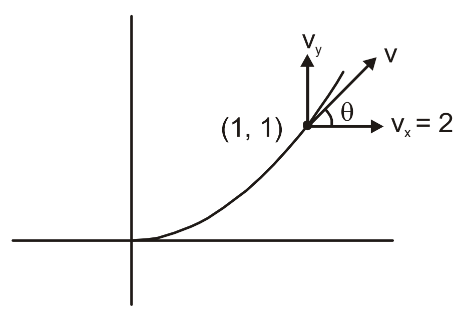
14. **(A), (C)**

The object should lie at the focus or at the centre of curvature of both the mirrors.

So d = 10 + 10 = 20 cm

or d = 20 + 20 = 40 cm

15. **(A, B, C, D)**



$y = kx^2 = x^2$

$\frac{dy}{dx} = 2x = \tan\theta = \frac{v_y}{v_x}$

$\Rightarrow \frac{v_y}{v_x} = 2 \times 1 = 2$

$\Rightarrow v_y = 4m/s$

$v = \sqrt{2^2 + 4^2} = 2\sqrt{5}\ m/s$ .... (D)

$v_x = \text{constt} \quad \Rightarrow a_x = 0$

$\therefore\ a = a_y = \frac{dv_y}{dt} = \frac{d}{dt}(2x \times v_x) = 2v_x \times \frac{dx}{dt}$

$= 2v_x^2 = 8 m/s^2$ ....(A)

$a_T = a\sin\theta = 8 \times \frac{2}{\sqrt{5}} = \frac{16}{\sqrt{5}}$ ...(B)

$a_n = a\cos\theta = 8 \times \frac{1}{\sqrt{5}} = \frac{8}{\sqrt{5}} = \frac{v^2}{r}$

$\Rightarrow\ r = \frac{v^2 \times \sqrt{5}}{8} = \frac{20 \times \sqrt{5}}{8} = \frac{5\sqrt{5}}{2} m.$ ....(C)

16. (4)

For lens + mirror + lens system :

$\frac{1}{v} - \frac{1}{-10} = \frac{1}{10} \Rightarrow v = \infty \Rightarrow$ light rays incident normally on the mirror. So final image is formed at the object itself.

For lenst + water lens + mirror + water lens + lens system :

$\frac{1}{10} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) \Rightarrow R = 10cm$

$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{-10} - \frac{1}{\infty}\right) = -\frac{1}{30}$

$f_2 = -30\ cm$

$f_3 = \infty$

$\therefore\ P_{eq} = 2P_1 + 2P_2 + P_3 = 2 \times \frac{1}{10} + 2 \times \frac{-1}{30} + \frac{1}{\infty} = \frac{4}{30}$

$\therefore\ f_{eq} = -\frac{15}{2} cm$

$$\frac{1}{v}+\frac{1}{-10}=\frac{1}{-15/2} \Rightarrow v=-30\text{cm}$$

$\therefore$ dist between two images = 30 – 10

= 20 cm

= 5 × 4 cm

$\therefore$ n = 4

17. (2)

$$f'=f\frac{(\mu_5-1)\,\mu_\ell}{\mu_5-\mu_\ell}=40\text{ cm}$$

$$\frac{1}{v}-\frac{1}{-60}=\frac{1}{40} \Rightarrow r+120\text{cm}\,(\text{below mirror})$$

image formed by mirror = 120 cm upward.

again for lens :

$$\frac{1}{v}-\frac{1}{-120}=\frac{1}{-40} \Rightarrow v=-30$$

$$v_{I_1}=1\text{cm/s}\times\left(\frac{120}{60}\right)^2=4\text{cm/s}$$

$$v_{I_2}=-4\text{cm/s}\times\left(\frac{30}{120}\right)^2=-\frac{1}{4}\text{cm/s}$$

Velocity of image observed by observer

$$=-\frac{1}{4}\times\frac{1}{4/3}=\frac{-3}{16}\text{cm/s}$$

$$=\frac{3}{16}\text{cm/s}=\frac{3}{8\times n}$$

$\Rightarrow$ n = 2

18. (7)

$$\frac{1}{f_1}=(1.5-1)\left(\frac{1}{20}-\frac{1}{-20}\right)=\frac{1}{20} \Rightarrow f_1=20\text{cm}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = (2-1)\left(\frac{1}{20}\right) + (3-1)\left(\frac{1}{20}\right) = \frac{3}{20}$$

$$\Rightarrow \ f_{eq} = \frac{20}{3}\text{cm}\ (\text{for the lower half})$$

for upper half :

$$\frac{1}{v_1} - \frac{1}{-30} = \frac{1}{20} \ \Rightarrow v_1 = +60$$

$$v_{I_1} = \left(\frac{60}{30}\right)^2 \times 7 = 28\text{cm/s}$$

for the lower half :

$$\frac{1}{v_2} - \frac{1}{-30} = \frac{1}{f_{eq}} = \frac{3}{20} \ \Rightarrow v_2 = \frac{60}{7}$$

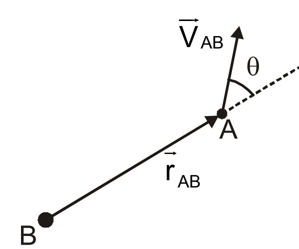
$$\therefore \quad v_{I_2} = \frac{(60/7)^2}{(30)^2} \times 7 = \frac{4}{7}\text{ cm/s}$$

$$\therefore \quad \frac{v_{I_1}}{v_{I_2}} = \frac{28}{4/7} = 7^2 = n^2$$

$$\Rightarrow \quad n = 7$$

19. (4)

At t = 2s



$$\vec{v}_A = \vec{u}_A + \vec{b}_A \times 2 = 6\hat{i} + 7\hat{j} + 9\hat{k}\text{ m/s}$$

$$\vec{r}_A = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \vec{u}t + \frac{1}{2}\vec{a}t^2 = 9\hat{i} + 12\hat{j} + 11\hat{k}\text{ m}$$

$$\vec{v}_B = \vec{u}_B + \vec{a}_B \times 2 = 5\hat{i} + 10\hat{k}\text{ m/s}$$

$$\vec{r}_B = 2\hat{i} + \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2 = 8\hat{i} + 8\hat{j} + 4\hat{k}\text{ m}$$

$$\vec{r}_{AB} = \hat{i} + \vec{u}\,\hat{j} + 7\hat{k}$$

$\vec{v}_{AB} = \overrightarrow{v_A} - \overrightarrow{v_B} = \hat{i} + 7\hat{j} - \hat{k}\ m/s$

$$\omega = \frac{v_{AB}\sin\theta}{|\overrightarrow{r_{AB}}|}$$

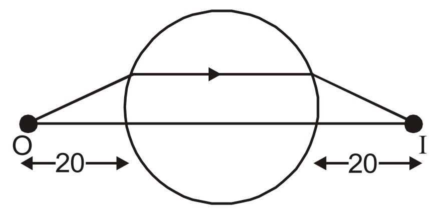
$$= \frac{v\sin\theta}{r}$$

$$= \frac{rv\sin\theta}{r^2} = \frac{|\vec{r}\times\vec{v}|}{r^2}$$

$$= \frac{\sqrt{2882}}{4356} = \frac{\sqrt{2882}}{1089\times n}$$

$\therefore$ n = 4

20. (6)



$$\frac{\mu}{\infty} - \frac{1}{-20} = \frac{\mu-1}{10}$$

$$\Rightarrow \mu = \frac{3}{2}$$

$$= \frac{6}{4} = \frac{n}{4}$$

$\therefore n = 6$

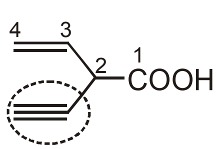
CHEMISTRY

21. (B)

$$N_{mix} = \frac{(250\times3)+(750\times1)}{250+750} = \frac{750+750}{1000} = 1.5N$$

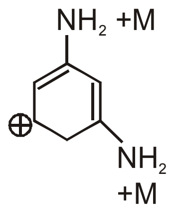
$\therefore$ Volume strength = (1.5)(5.6) = 8.4 V

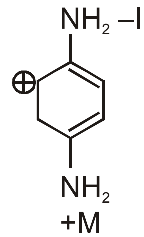
22. (C)

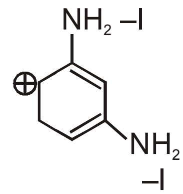


Ethynyl

23. (A)







24. (D)

Li	Be	B	C	N	O	F
1	1.5	2	2.5	3	3.5	4

difference in electronegativity between two element is around 0.5, which is not found in elements of other periods

25. (A)

geq. of Na_3AsO_4 = geq. of $Na_2S_2O_3$

$\Rightarrow \frac{1}{208}\times 2 = (0.2)(V)$

$\Rightarrow V = 0.0481\,L = 48.1\,mL$

26. (B)

geq. of Na_2CO_3 + geq. $NaHCO_3$ = geq. HCl

$\Rightarrow (x \times 2) + (1 - x)(1) = \frac{54.75}{36.5}\times 1 = (1.5)$

$\Rightarrow 1 + x = 1.5 \qquad \Rightarrow x = 0.5$

* After heating moles of Na_2CO_3 = $\left(x + \frac{1-x}{2}\right) = \left(\frac{x+1}{2}\right)$

g.eq. of Na_2CO_3 = g.eq. HCl

$\Rightarrow \left(\frac{x+1}{2}\right)(2) = \frac{w}{36.5}\times 1$

$\Rightarrow$ w = (1.5)(36.5) = 54.75 gm

27. (B)

Mole of HCl = $4\times 10^{-3}\times 2 = 8\times 10^{-3}$ mole

$\underset{8\times10^{-3}\text{ mole}}{2HCl} + \underset{0.58g\text{ in }100\text{ ml}}{Mg(OH)_2} \longrightarrow MgCl_2 + 2H_2O$

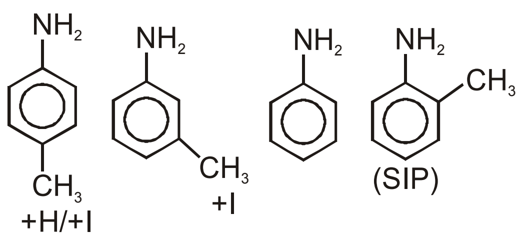
$$\frac{n_{HCl}}{2} = \frac{n_{mg(OH)_2}}{1}$$

$$\frac{8 \times 10^{-3}}{2} = \frac{0.58}{58 \times 100} \times x \text{ (If x ml of solution is required per day)}$$

$$4 \times 10^{-3} = 10^{-4} \times x$$

x = 40 ml

28. (C)



29. (A)

Biggest jump among consecutive IP value is observed after removal of 4th electron (ie IP_4), hence element is carbon.

30. (B)

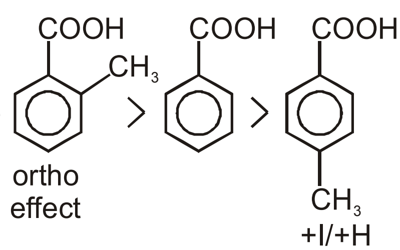
Total number of contributing structure = alpha hydrogen

31. (C,D)

He has highest ionisation potential in periodic table. Electron gain enthalpy of nitrogen is less, because of its stable half filled configuration.

32. (A,C)

(A)



(C) Picric acid > benzoic acid > Acetic acid

Pka- 0.38 4.20 4.76

33. (A,C)

$$2KHCO_3(s) \longrightarrow K_2O(s) + H_2O(g) + 2CO_2(g)$$

$$\frac{\text{Mole } KHCO_3}{2} = \frac{\text{mole} H_2O}{1} = \frac{\text{mole } CO_2}{2}$$

$$\frac{\text{wt KHCO}_3}{100\times 2}=\frac{1.80}{18}=\frac{\text{wt CO}_2}{2\times 44\,\text{g}}$$

$$\text{wt KHCO}_3 = 20\text{g}, \text{wt CO}_2 = 8.8\text{g}$$

$$\text{wt CO}_2\,|_{K_2CO_3} = \text{wt CO}_2\,|\,\text{total} - \text{wt CO}_2\,|_{KHCO_3}$$

$$= 13.20 - 8.8 = 4.4\text{ g}$$

$$K_2CO_3(s) \longrightarrow K_2O + CO_2(g)$$

$$\frac{\text{mole } K_2CO_3}{1}=\frac{\text{mole } CO_2}{1}$$

$$\text{wt } K_2CO_3 = 138\times\frac{4.4}{44}=13.8\,\text{g}$$

$$2KClO_3 \longrightarrow 2KCl + 3O_2(g)$$

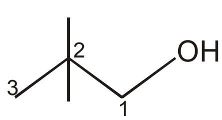
$$\frac{nKClO_3}{2}=\frac{nO_2}{3}$$

$$\frac{\text{wt KClO}_3}{2\times 122.5}=\frac{4}{3\times 32}$$

$$\text{wt KClO}_3 = \frac{245}{24}=10.2\,\text{g}$$

$$\text{wt KCl} = 100-(10.2+13.8+20) = 56\text{ g}$$

34. (A,B,C,D)

(A) 

(B) $\overset{3}{C}H_2{=}\overset{2}{C}H{-}\overset{1}{C}H_2{-}OH$

(C) $\underset{2}{C}H_3{-}\underset{1}{C}HCl_2$

(D) $\overset{1}{C}H_2Cl{-}\overset{2}{C}H_2Cl$

35. (A,B,C)

$$\text{Molarity}=\frac{\text{vol.strength}}{11.2}=\frac{67.2}{11.2}=6$$

1 lit solution has $n_{H_2O_2} = 6,\ \ wt_{H_2O_2} = 6\times 34 = 204\,\text{g}$

$$m_{\text{solution}} = v\times d = 1500\,\text{g}$$

$$m_{\text{solvent}} = 1500 - 204 = 1296\,\text{g}$$

$$\text{molality} = \frac{n_{H_2O_2}}{m_{solvent}} \times 1000 = \frac{6000}{1296} = 4.63$$

$$\% \text{ by } wt_{H_2O_2} = \frac{204}{1500} \times 100 = 13.6\%$$

$$x_{H_2O_2} = \frac{6}{6 + \frac{1296}{18}} = \frac{6}{78} = 0.076$$

36. (8)

$$C_XH_YO_Z \xrightarrow{\text{Combustion}} XCO_2 + \frac{Y}{2}H_2O$$

$$n_{CO_2} = \frac{0.44}{44} = 0.01$$

$$wt_C = 0.01 \times 12 = 0.12\,g$$

$$n_{H_2O} = \frac{0.09}{18} = \frac{0.01}{2}$$

$$wt_H = \frac{0.01}{2} \times 2 = 0.01g$$

$$wt_O = 0.45 - (0.12 + 0.01) = 0.32\,g$$

mole of C : H : O = 1 : 1 : 2 $\Rightarrow$ Empirical formula = CHO_2

$$\underset{0.76\,g}{(Ag_2A)\text{silver salt}} \longrightarrow \underset{0.54\,g}{Ag}$$

By POAC on Ag

$$2 \times \frac{0.76}{\text{M.W Ag} - \text{salt}} = \frac{0.54}{108}$$

mol.wt Ag salt = 304

mol. wt Acid = 304 – 2 × 108 + 2 × 1 = 90

$$\Rightarrow n = \frac{\text{mol.wt}}{\text{Emp.wt}} = \frac{90}{45} = 2$$

Molecular formula = $(CHO_2)_2$ = $C_2H_2O_4$

$$X + Y + Z = 2 + 2 + 4 = 8$$

37. (7)

Aromatic- A, B, D, E, G, H, J

38. (3)

Let valency of metal is X

Metal chloride = MCl_X

$$\underset{0.22g}{MCl_X} + \underset{0.51g}{XAgNO_3} \longrightarrow XAgCl$$

$$\frac{n_{MCl_X}}{1} = \frac{NAgNO_3}{x}$$

$$\frac{0.22}{(\text{At wt M} + 35.5x)} = \frac{0.51}{x(170)} \quad .....(1)$$

from dulong petit rule

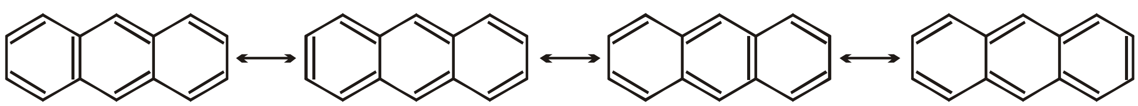
$$\text{At wt}_M = \frac{6.4}{0.057} = 112.3$$

$$(1) \Rightarrow \frac{0.22}{112.3 + 35.5x} = \frac{0.51}{170x}$$

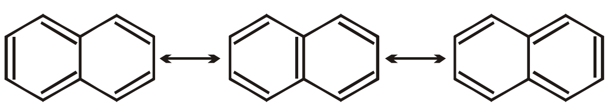
$x \simeq 3$ (Since valency is always integral)

39. (7)

Resonating structures of anthracene



Resonating structures of naphthalene:-



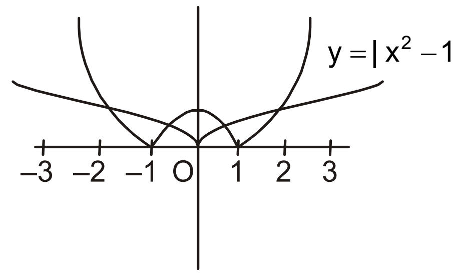
40. (3)

The balanced reaction is

$$2KMnO_4 + 5H_2O_2 + 3H_2SO_4 \rightarrow 2MnSO_4 + K_2SO_4 + 5O_2 + 8H_2O$$

MATHEMATICS

41. (C)



$\sqrt{(x^2+1)^2-4x^2} = |x^2-1|$

$\Rightarrow |\tan^{-1}|x|| = |x^2-1|$

Draw the graph of $y = |\tan^{-1}|x||$ and $y = |x^2-1|$

from the graph it is clear that equation has four roots.

42. (D)

put 2y = y then

$\Rightarrow f(x+y) = f(x) + f(y) + 2xy$

$\Rightarrow$ partially differentiate with respect to x we get

$f'(x+y) = f'(x) + 2y$

put x = 1, y = –1

$f'(0) = f'(1) - 2$

43. (A)

Let $\lim_{x\to\infty} f(x) = A$

then $A + \frac{3A-1}{A^2} = 3$

$\Rightarrow A^3 + 3A - 1 = 3A^2$

$\Rightarrow A^3 - 3A^2 + 3A - 1 = 0$

$\Rightarrow A = 1$

44. (C)

$T_n = \tan^{-1}\left(\frac{2n+2}{1+(n^2+2n)(n^2+2n+1)}\right)$

$= \tan^{-1}((n+1)(n+2)) - (\tan^{-1} n(n+1))$

$$= s_n = \sum_{n=1}^{n} T_n = \tan^{-1}(n+1)(n+2) - \tan^{-1}(2)$$

So, $\lim_{x\to\infty} s_n = \frac{\pi}{2} - \tan^{-1}(2) = \cot^{-1}(2) = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

45. (B)

$f(x) + f(x+\pi) = 2$

$\therefore \frac{1}{2}f(x) + \frac{1}{2}f(x+\pi) = 1$ .... (i)

Also given

$a\,f(x) + b\,f(x+c) = 1$ .... (ii)

After comparing (i) and (ii) we get

$a = b = \frac{1}{2}$ and $c = \pi$

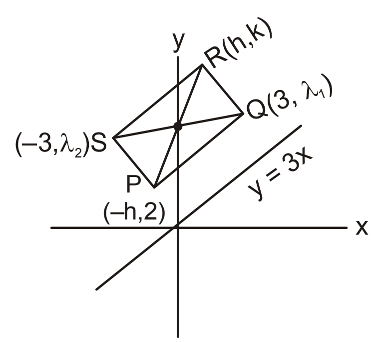
Then value of $\frac{b\cos c}{a} = \frac{\frac{1}{2}}{\frac{1}{2}}\cos(\pi) = -1$

46. (A)

PQ is parallel to y = 3x

Equation of PQ $y = mx + \lambda$ diagonals bisect to each other

$\therefore$ x Co-ordinate of P is -h equation of PQ passing through (–h, 2) then



$\Rightarrow 2 = -mh + \lambda$ where m = 3

$\lambda = 2 + mh$

$y = 3x + 2 + 3h$ .... (i)

Q lies on it

$\lambda_1 = 9 + 2 + 3h$

$Q=(3,11+3h)$

Also slope of PQ × slope of QR = –1

$$\frac{3\left(k-(3\times3+2+3h)\right)}{h-3}=-1$$

$$3(k-11-3h)=-(h-3)$$

$$3k-33-9h=-h+3$$

$$8h-3k+36=0$$

$$h\to x,\quad k\to y$$

$$8x-3y+36=0$$

47. (D)

$$k=\lim_{x\to0^+}\frac{(2e)^x-x-x\ell n2-1}{x^2\left(\frac{\tan x}{x}\right)}$$

$$\lim_{x\to0^+}\frac{(2e)^x-x-x\ell n2-1}{x^2}$$

$$\lim_{x\to0^+}\frac{1+x\ell n(2e)+\frac{x^2}{\lfloor 2}(\ell n2e)^2+\ldots x-x\ell n2-1}{x^2}$$

$$=\frac{(\ell n(2e))^2}{2}$$

$$=\frac{(\ell n2+1)^2}{2}$$

$$=\frac{(\ell n2)^2}{2}+\ell n2+\frac{1}{2}$$

48. (B)

put x = 1 in given equation then

$$f(1+y)=3^y f(1)+2f(y)$$

$y \to x$

$f(1+x) = 3^x f(1) + 2f(x)$ .... (i)

put y = 1 in given eqution

$f(x+1) = 3f(x) + 2^x f(1)$

$f(x+1) = 3f(x) + 2^x$ .... (ii)

from (i) & (ii)

then $3f(x) + 2^x = 3^x + 2f(x)$

$f(x) = 3^x - 2^x$

then f(3) = 27 – 8 = 19

49. (D)

$$f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2\tan^{-1}x, \quad x \ge 1$$

$$g(x) = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) = \pi - 2\tan^{-1}x, \ \ x \ge 0$$

$\therefore f(10) = \pi - 2\tan^{-1}(10)$

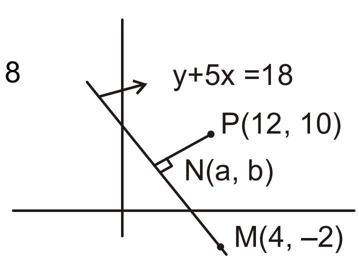
$g(100) = \pi - 2\tan^{-1}(100)$

$f(100) = \pi - 2\tan^{-1}(100)$

$f(10) - g(100) = 2\left(\tan^{-1}(100) - \tan^{-1}(10)\right)$.

50. (B)

(a, b) is the foot of perpendicular of (12, 10) on the line y + 5x = 18



then N we get (2, 8)

$a + b = 10$

51. (A, D)

$$g(x)=\frac{\sin \pi x}{x}+\frac{\sin \pi x}{1-x}$$

$$g(x)=\frac{\sin \pi x}{x(1-x)}=\frac{2\sin\frac{\pi x}{2}\cos\frac{\pi x}{2}}{x(1-x)}$$

$$\therefore\ g(x)=\frac{1}{2}f\left(\frac{x}{2}\right)f\left(\frac{1-x}{2}\right)$$

and $g\left(\frac{1}{2}+x\right)=g\left(\frac{1}{2}-x\right)$

$\therefore$ so symmetrical about line

$2x-1=0$.

52. (A, C)

$$\sin^{-1}\left(\frac{\sqrt{x}}{2}\right)+\sin^{-1}\sqrt{1-\frac{x}{4}}+\tan^{-1}y=\frac{2\pi}{3},\quad x\in[0,4]$$

$$\therefore\ \sin^{-1}\left(\frac{\sqrt{x}}{2}\right)+\cos^{-1}\left(\frac{\sqrt{x}}{2}\right)+\tan^{-1}y=\frac{2\pi}{3}$$

$$\therefore\ \frac{\pi}{2}+\tan^{-1}y=\frac{2\pi}{3}$$

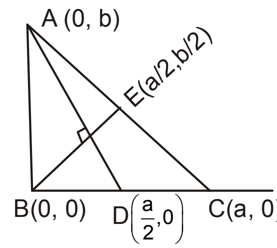
$$\tan^{-1}y=\frac{\pi}{6}$$

$$y=\frac{1}{\sqrt{3}}$$

Then the maximum value of $x^2+y^2=16+\frac{1}{3}=\frac{49}{3}$

and minimum value of $x^2+y^2=0^2+\frac{1}{3}=\frac{1}{3}$

53. (B & D)



Slope of AD × slope of BE = –1

$$\frac{b/2}{a/2}\left(\frac{-b}{a/2}\right)=-1$$

$$\Rightarrow 2b^2=a^2$$

$$a=\pm\sqrt{2}b$$

54. (B, C)

H is ortho centre

and H(0, b')

and also

$$aa'=bb'$$

$$b'=\frac{aa'}{b}$$

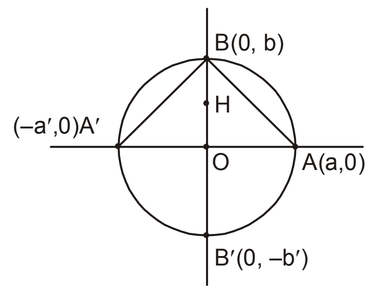


image of orthocentre of tringle always lies on circumfrence of circumcircle

55. (A, B, D)

(A) clearly sin[x] + cos[x] is defined for all 'x'

(B) $\sec^{-1}x$ is always defined if $1+\sin^2 x\geq 1$ is as defined for all 'x'

(C) $\tan(\log x)$ is not defined if $\log x=(2k+1)\frac{\pi}{2}$

(D) $\frac{9}{8}+\cos x+\cos 2x>0\,\forall x\in R$ as D < 0 hence domain is 'R'.

56. (4)

$$\lim_{x\to 1}\frac{x^2\sin f(x^2)\cdot 2x+2x\int_1^{x^2}\sin f(t)\,dt}{2(x-1)}$$

$$\lim_{x\to 1}\frac{2x^3\sin f(x^2)+2x\int_1^{x^2}\sin f(t)\,dt}{2(x-1)}$$

Again applying L' Hospital Rule

then we get $2f'(1) = 4$

57. (9)

$$f(-x) = \frac{1}{e^x + 1}$$

$$f(x) + f(-x) = \frac{1}{e^x + 1} + \frac{1}{1 + e^x} = 1$$

$$f(x) + f(-x) = 1 \qquad \text{........ (i)}$$

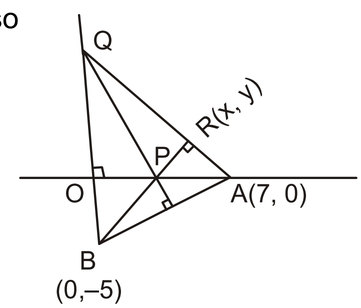
$$S = \left(f(5) + f(-5) + f(4) + f(-4)\right)...\left(f(1) + f(-1)\right) + f(0)$$

$$= 1 + 1 + 1 + 1 + 1 + \frac{1}{1 + e^0} = 5 + \frac{1}{2} = \frac{11}{2}$$

$\therefore$ a = 11, b = 2, a-b = 9

58. (2)

P is the ortho centre of tringle ABQ, so R is foot of 3rd altitude so slope of AQ × slope BR is –1



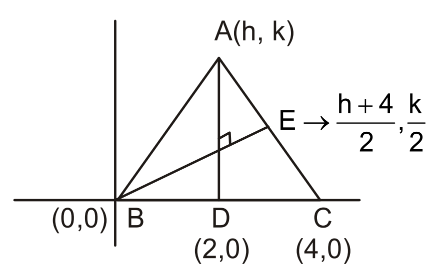
$$\left(\frac{y}{x - 7}\right)\left(\frac{y + 5}{x}\right) = -1 \Rightarrow x^2 + y^2 - 7x + 5y = 0$$

a = 7, b = 5

59. (3)

AD perpendicular BE

$$\left(\frac{k - 0}{h - 2}\right)\left(\frac{k}{2\frac{(h + 4)}{2}}\right) = -1$$



$$k^2 + (h - 2)(h + 4) = 0$$

$$k^2 + h^2 + 2h - 8 = 0 \qquad \text{........ (i)}$$

and AC = 3

$$(h - 4)^2 + k^2 = 9$$

(i) – (ii)

$k^2 + h^2 + 2h - 8 - h^2 - 16 + 8h - k^2 + 9 = 0$

10h = 15

$h = \frac{3}{2}$ then put (i)

$k^2 = 9 - \left(\frac{3}{2} - 4\right)^2$

$k^2 = 9 - \frac{25}{4} = \frac{36 - 25}{4} = \frac{11}{4}$

$k = \sqrt{\frac{11}{4}} = \frac{\sqrt{11}}{2}$

and area of Δ is = 2k

then $2\frac{\sqrt{11}}{2} \Rightarrow 2k = \sqrt{11}$

$[2k] = 3$

60. (3)

We hvae $4\cos^4 x - 2\cos 2x - \frac{1}{2}\cos 4x - x^7$

$= 4\cos^4 x - 2\left(2\cos^2 x - 1\right) - \frac{1}{2}\left(2\cos^2 2x - 1\right) - x^7$

$= 4\cos^4 x - 4\cos^2 x + 2 - (2\cos^2 x - 1)^2 + \frac{1}{2} - x^7$

$= \left(\frac{3}{2} - x^7\right)$

then $f(x) = \left(\frac{3}{2} - x^7\right)^{\frac{1}{7}}$

$ff(x) = \left(\frac{3}{2} - f(x)^7\right)^{\frac{1}{7}} = x$

$ff(12) = 12$