SOLUTIONS

PHASE TEST-1

RB-1813-1815, RBK-1806

RBS-1803-1804

JEE ADVANCED PATTERN

Test Date: 25-11-2017

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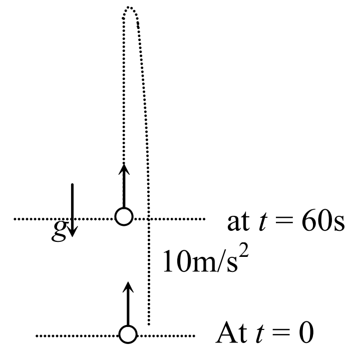
PHYSICS

1. (B) and (D)

Total height $= \frac{1}{2}\times 10\times (60)^2 + \frac{(10\times 60)^2}{2\times 10}$ = 36000 m = 36 km



Total time $= 60 + 60 + \sqrt{\frac{2\times 36000}{10}}$

$= 60s + 60s + 60\sqrt{2}s$

$= \left(120 + 60\sqrt{2}\right)$s

2. (A) and (C)

Due to symmetry net force on M is zero. Hence its acceleration is also zero and acceleration of B is $\frac{mg}{2m} = \frac{g}{2}$.

3. (A), (C) and (D)

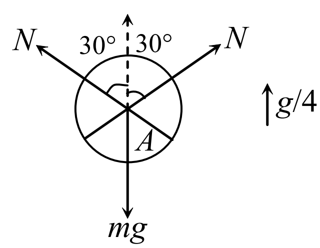
$F - T - \mu_2 m_2 g = m_2 a$, $T - \mu_2 m_2 g = m_1 a$

for just equilibrium a = 0 , $F = 2\mu_2 m_2 g = 4\,\text{N}$

If F = 6N, a = 1 m/s² $\Rightarrow$ T = 3N

4. (B) and (D)

Net upward force on three spheres applied by bottom



= $3mg + \frac{3}{4}mg = \frac{15mg}{4}$

For sphere A, $N\sqrt{3} = mg + \frac{mg}{4}$, $N = \frac{5mg}{4\sqrt{3}}$

5. (A) and (C)

$-\frac{1}{F} = P = 2P_{l_1} + 2P_{l_2} + P_m$ …(1)

$$P_{l_1} = \frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$P_{l_1} = [(1.5 - 1]\left[-\frac{1}{10} - \frac{1}{15}\right] = -\frac{1}{12} \quad \ldots(2)$$

$$P_{l_2} = \frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$P_{l_2} = \left(\frac{4}{3} - 1\right)\left[\frac{2}{15}\right] = \frac{2}{45} \quad \ldots(3)$$

$$P_m = -\frac{1}{f} = +\frac{2}{15} \quad \ldots(4)$$

$$-\frac{1}{F} = P = 2\left[-\frac{1}{12} + \frac{2}{45}\right] + \frac{2}{15} = -\frac{1}{6} + \frac{4}{45} + \frac{2}{15} = \frac{1}{18}$$

$F = -18\,\text{cm}$. Focus is negative means system will behave as concave mirror.

6. (A), (C) and (D)

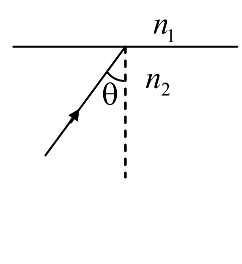
$$\theta > \theta_{C_1} = \sin^{-1}\frac{n_1}{n_2}$$

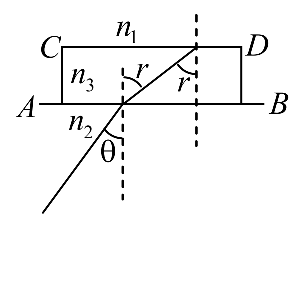
when slab is placed,

$\theta_{C_2} = \sin^{-1}\frac{n_3}{n_2}$ if $n_3 < n_1$, $\quad \theta_{C_2} < \theta_{C1} < \theta$

Hence ray will totally reflect at surface *AB*.

$$\sin\theta_{C_1} = \frac{n_1}{n_2} \quad \theta > \theta_{C_1}$$





at surface *AB*

$$\frac{\sin r}{\sin\theta} = \frac{n_2}{n_3}$$

$$\sin r = \sin\theta\frac{n_2}{n_3}$$

$$\sin r > \sin\theta_{C_1}\frac{n_2}{n_3}$$

$$\sin r > \frac{n_1}{n_2}\times\frac{n_2}{n_3}$$

$$\sin r > \frac{n_1}{n_3}$$

r is incident angle at surface *CD* which is greater the critical angle at that surface so any value of n_3 say will reflect at surface.

7. **(A, B, C)**

In equilibrium position, if x_0 is stretch of spring then $kx_0 = qE$ or $x_0 = \frac{qE}{k}$

If x_m is maximum stretch of spring , $\frac{1}{2}kx_m^2 = qEx_m$ or $x_m = \frac{2qE}{k}$

Amplitude will be $x_m - x_0 = \frac{2qE}{k} - \frac{qE}{k} = \frac{qE}{k}$

8. **(A) and (C)**

$E_A = i + 2\hat{j} + 3\hat{k}$, $E_B = \hat{i} + \hat{j} - \hat{k}$, $\vec{E}_A . \vec{E}_B = 0 \Rightarrow \vec{E}_A \perp \vec{E}_B$

$E_B = \frac{kq}{3}$, $E_C = \frac{kq}{12}$, $|E_B| = 4|E_C|$

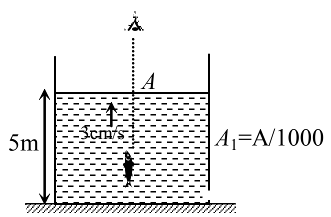
9. **(A)**

Velocity of efflux $v = \sqrt{2g\times 5} = 10$ m/s

Velocity of surface $= \frac{10\times A/1000}{A}$ = 1 cm/s

10. **(A)**

Velocity of fish w.r.t surface in air $= \frac{3+1}{\mu} = 3$ cm/s



Velocity as viewed directly by the observer $= 3 - 1 =$ 2 cm/s

11. (B)

$$\frac{1}{V+f}+\frac{1}{U+f}=\frac{1}{f} \Rightarrow (U+f+V+f)f=(V+f)(U+f) \Rightarrow VU = f^2$$

12. (A)

13. (A)

$at+bt^2=ct^2+dt^3$ $\Rightarrow$ t = 2s

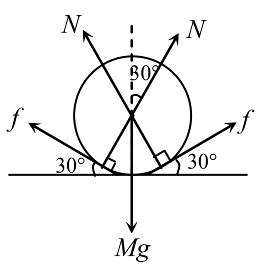
14. (B)

$\frac{d}{dt}(x_B-x_A)=0$ $\Rightarrow$ $a+2bt=2ct+3dt^2$ $\Rightarrow$ $t=\frac{2}{\sqrt{3}}$ s

15. (D)

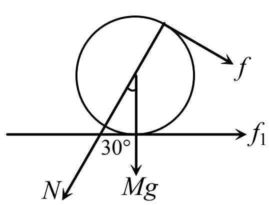
16. (C)

$f_1=f$ for no rotation of bottom cylinder ...(i)



FBD of top cylinder :

$2N\cos 30+2f\sin 30=Mg$ ...(ii)



FBD of bottom cylinder (left one)

$N\sin 30=f_1+f\cos 30$ ...(iii)

Solving (i), (ii) and (iii)

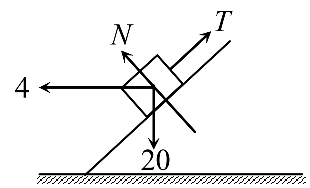
$N=\frac{Mg}{2}$ $f=\frac{Mg}{2(2+\sqrt{3})}$

17. (A)

Acceleration of 2kg relative to wedge = 2 m/s^2

Acceleration of 2 kg relative to ground

= $\sqrt{2^2+2^2+2\times2\times\cos120^\circ}$ = 2 m/s^2



$20\sin60^\circ+4\cos60^\circ-T=4$

$T=10\sqrt{3}-2$

$N=20\cos60^\circ-4\sin60^\circ=10-2\sqrt{3}$ N

Net force = 2 × 2 = 4 N

18. (C)

For (I) and (II)

As μ_1 > tan 37°, tension in the string will be zero.

So, for 2 kg block,

$2g\sin37^\circ-0.6\times2g\cos37^\circ=2a$

$\Rightarrow$ a = 1.2 m/s^2

For (III) and (IV)

If the tension in the string is T, then

$4g\sin37^\circ-0.6\times4g\cos37^\circ-T=4a$ …(i)

$2g\sin37^\circ-0.8\times2g\cos37^\circ+T=2a$ …(ii)

By solving above equations

a = 0.67 m/s^2, T = 2.13 N

19. (D)

$\mu\sin90^0=\mu_0\sin C$ and $\mu_0\sin(90-C)=1\sin i$

If $i=45^0$ and $\mu_0=\sqrt{2}$

$\mu=\sqrt{3/2}$ and $\sin C=\frac{\sqrt{3}}{2}$

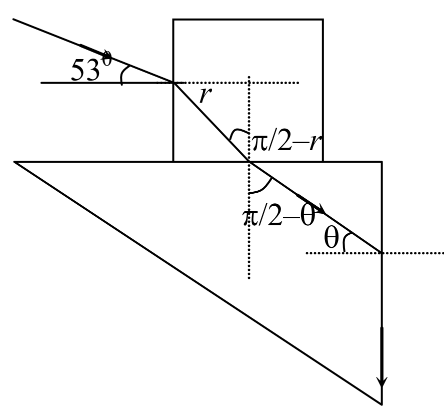
If ray just fail to energy from the prism , then i = 90^0

μ = 1

$\mu_0\sin\theta=1$

θ = 45^0 and $\sin\left(\frac{\pi}{2}-r\right)=\mu_0\sin\left(\frac{\pi}{2}-\theta\right)$

$\mu\cos r$ = 1



$1\sin 53^0 = \mu \sin r$

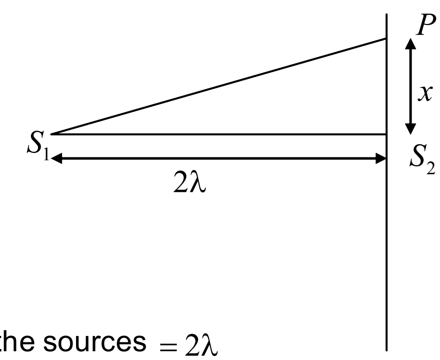
on solving $\mu = \frac{\sqrt{41}}{5}$

20. **(A)**

Let the person finds minimum intensity at point *P* at distance *x* from S_2

Path difference

$$\sqrt{(2\lambda)^2 + x^2} - x = (2n-1)\frac{\lambda}{2}$$



To have minimum intensity at smallest distance from S_2

$$\sqrt{(2\lambda)^2 + x^2} - x = 3\frac{\lambda}{2} \qquad (2)$$

$\Rightarrow \quad x = \frac{7\lambda}{12}$

Maximum possible path difference = distance between the sources $= 2\lambda$

Let intensity of light from each source is *I*, then $I = I + I + 2\sqrt{II}\cos\phi$

$\Rightarrow \quad \cos\phi = -\frac{1}{2} \Rightarrow \qquad \phi = \frac{2\pi}{3} \quad \Rightarrow$ path diff. $= \frac{\lambda}{3}$

CHEMISTRY

21. **(A,B,C,D)**

Molarity = 2M →
- n_{solute} = 2 mole
- $V_{solution}$ = 1 L = 1000 mL

$d_{solution}$ = 1.20 g/mL

$w_{solution}$ = 1200 g

w_{solute} = 2 × 60 g

$w_{solvent}$ = 1080 g

Molality = $\frac{2}{1080} \times 1000$m

Xsolute = $\frac{2}{2 + \frac{1080}{18}}$

$$\%\frac{w}{w}=\frac{120}{1200}\times 100=10\%$$

$$\%\frac{w}{v}=\frac{120}{1000}\times 100=12\%$$

22. (A,D)

23. (B,C,D)

$$M_R=\frac{\frac{11.2}{11.2}\times 100+\frac{22.4}{11.2}\times 200+\frac{33.6}{11.2}\times 200}{1000}=\frac{1100}{1000}$$

= 1.1 M = 1.1 mol/L = 1.1 × 34 g/L

V.S = 1.1 × 11.2 = 12.32 V

24. (A,B,D)

25. (A,B,D)

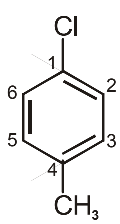
Product formed is tropyllium ion. Which is aromatic in nature. So, high resonance energy.

So, Option (C) is wrong.

26. (B,D)

(A) & (C) are anti-aromatic.

27. (A,B)



28. (A,B,C)

$$B_2Cl_4+6NaOH\longrightarrow 2NaBO_2+2H_2O+H_2+4NaCl$$

$$n_{B_2Cl_4}=\frac{1362}{22400}=0.0608\text{ mole}$$

$$w_{B_2Cl_4}=0.06\times 164=9.97\,g$$

29. (C)

30. (D)

31. (B)

$2SO_2 + O_2 \longrightarrow 2SO_3$

10 mole 4 mole

Limiting reagent

32. (B)

Mole of SO_2 consume = 8

Moles of SO_2 left = 10 – 8 = 2.

33. (B)

Hyperconjugation is the properties of α_{C-H} bond.

34. (A)

Stabality of free radical α no. of α_H

35. (B)

36. (B)

37. (A)

(A) 100 ml of 0.2 M $AlCl_3$ = 20 milimole

400 ml of 0.1 M HCl = 40 milimole

On mixing $[AlCl_3] = \frac{20}{500} = \frac{1}{25} = 0.04$

$[HCl] = \frac{40}{500} = 0.08M$

[Cation] = 0.04 + 0.08 = 0.12 M

$[Cl^-] = 3 \times 0.04 + 0.08 = 0.2M.$

(B) $[KCl] = \frac{50 \times 0.4}{100} = 0.2$

$Cl^- = 0.2M.$

(C) 30 ml of .2 M K_2SO_4 + 70 ml H_2O

$[K_2SO_4] = \frac{30 \times 0.2}{30 + 70} = \frac{6}{100} = 0.06$

$[SO_4^{2-}] = 0.06\ M$

$[Cation] = 0.12\ M.$

(D) $[H_2SO_4] = \frac{245}{98} = 2.5M$

$[SO_4^-] = 2.5$ M.

38. (C)

(A) $Zn + 2HCl \longrightarrow ZnCl_2 + H_2$

2 2

(LR)

1 mole O 1 mole 1 mole = 22.4 L

(B) $AgNO_3$ + HCl $\longrightarrow$ AgCl + HNO_3

170 g 18.25

1 mole 0.5 mole

0.5 mole Limiting reagent 0.5 mole 0.5 mole

(C) $CaCO_3 \longrightarrow CaO + CO_2$

100 g

1 mole 1 mole 1 mole = 22.4 L

(D) $2KClO_3 \longrightarrow 2KCl + 3O_2$

$\frac{2}{3}$mole $\frac{2}{3}$mole $\frac{2}{3} \times \frac{3}{2} = 1 \text{ mole} = 22.4 \text{ L}.$

39. (D)

40. (A)

MATHEMATICS

41. (B, D)

Range of f (sgn(x)) is $\{\sin 1, -\sin(5/3), -\sin(1/3)\}$ range of sgn(f(x)) is {– 1, 1, 0}

f(x) is one-one on $\left(\frac{3(2n-1)\pi+2}{8}, \frac{3(2n+1)\pi+2}{8}\right) \forall \ n \in I$

42. (A, B)

Only h(x) is periodic function because

domain of f = [–1, 1]

domain of g = $(-\infty, -1] \cup [1, \infty)$

domain of h = R

43. (A, B, C)

$\cos x + \cos y = a$

$\cos^2 x + \cos^2 y + 2\cos x \cos y = a^2$ .....(1)

$\cos 2x + \cos 2y = b$

$\therefore \quad \cos^2 x + \cos^2 y = \frac{b+2}{2}$ .....(2)

$\therefore$ From (1) and (2), $\cos x \cos y = \frac{a^2}{2} - \frac{b+2}{4}$

Now, $\cos 3x + \cos 3y = c$

$\therefore \quad 4\cos^3 x - 3\cos x + 4\cos^3 y - 3\cos y = c$

$4(\cos x + \cos y)(\cos^2 x + \cos^2 y - \cos x \cos y) - 3(\cos x + \cos y) = c$

$\Rightarrow \quad 2a^3 + c = 3a(1 + b).$

44. (A, B)

$1 \le |\sin x| + |\cos x| \sqrt{2}$

45. (A, C)

The bases of all triangles PAB (A, B fixed P moving indeed A, B, P lie on a circle) will be fixed

$\Rightarrow$ area = $\frac{1}{2}$ × base × height

Thus if area is maximum then height must be maximum which will be true if P lies on perpendicular to bisector of AB.

Thus (A) and (C) are connect choices.

46. (A, B , C, D)

Equation of line through A (4, 3) is

$\frac{x-4}{\cos\theta} = \frac{y-3}{\sin\theta} = r$ .....(i)

$A \equiv (4 + r\cos\theta, 3 + r\sin\theta)$.

$4 + r\cos\theta = 8 \Rightarrow r = 4\sec\theta$.

$\therefore$ AB = 4 $\sec\theta$.

Similarly AC = 3 cosec θ

$$\frac{16}{AB^2} + \frac{9}{AC^2} = \cos^2\theta + \sin^2\theta = 1$$

So $AB + AC = \frac{4}{\cos\theta} + \frac{3}{\sin\theta} = \frac{2(4\sin\theta + 3\cos\theta)}{\sin 2\theta}$

47. (A, B)

$\alpha x - \beta y = 8$ divides the area of the region enclosed by the curve $x^2 + y^2 - 4x + 2y - 5 = 0$.

$\Rightarrow 2\alpha + \beta = 8$

Also, $\frac{2\alpha+\beta}{2} \ge \sqrt{2\alpha\beta}$

$4 \ge \sqrt{2\alpha\beta} \Rightarrow \alpha\beta \le 8$

48. (A, B)

If $\angle B + \angle D > 180°$

$\Rightarrow$ B and D lie inside the circle through other three points.

Let $\angle B + \angle D < 180°$, then B and D lie outside the circle through other three points.

49. (A)

$f(x) = (x-1)^2 - 2, a = 1, b = -2$

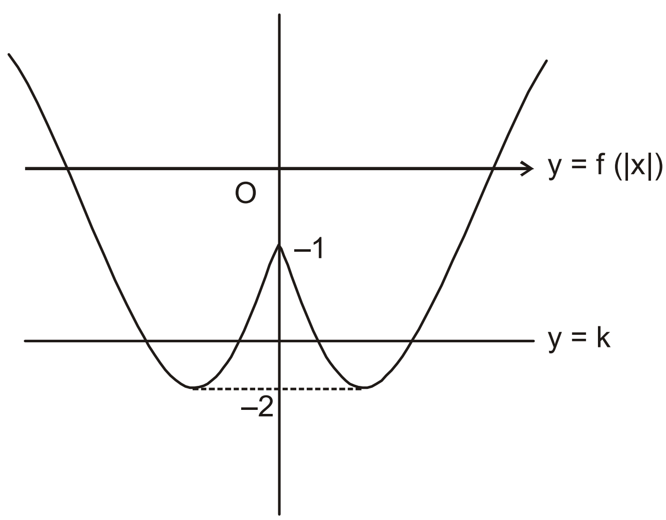
$f : [1, \infty) \to [-2, \infty)$

then $f^{-1} : [-2, \infty) \to [1, \infty)$, $f(x) = y \Rightarrow x^2 - 2x - (1+y) = 0$

$\therefore x = \frac{2 \pm \sqrt{4 + 4(1+y)}}{2}$, $x = 1 \pm \sqrt{2+y}$

$\because f^{-1}(y) = 1 + \sqrt{2+y} \Rightarrow f^{-1}(x) = 1 + \sqrt{2+x}$

50. (A)



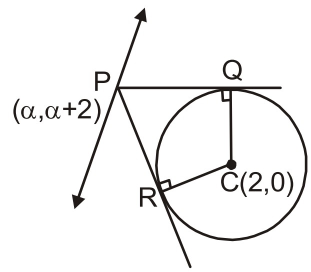
For f (|x|) = k to be four distinct solutions, $k \in (-2, -1)$

51. (A)

Circumcircle of ΔPQR passes through the centre C of the circle with PC as a diameter.

Let S(h, k) be the circumcentre, then

S is mid point of PC



$\Rightarrow \quad (h,k) = \left(\frac{\alpha+2}{2}, \frac{\alpha+2}{2}\right)$

$\Rightarrow \quad h = k$

$\therefore$ Locus is y = x.

52. (B)

Orthocentre of ΔPQR is the image of C about QR. Let $P \equiv (\alpha, \alpha+2)$ and H(h, k) be the orthocentre.

Equation of chord of contact QR is

$$x\alpha + y(\alpha+2) - \frac{4(x+\alpha)}{2} = 0$$

i.e., $(\alpha - 2)x + (\alpha + 2)y = 2\alpha$

$$\therefore \quad \frac{h-2}{\alpha-2} = \frac{k-0}{\alpha+2} = \frac{-2(2(\alpha-2)-2\alpha)}{(\alpha-2)^2+(\alpha+2)^2}$$

$$\Rightarrow h-2 = \frac{8(\alpha-2)}{(\alpha-2)^2+(\alpha+2)^2},\ k = \frac{8(\alpha+2)}{(\alpha-2)^2+(\alpha+2)^2}$$

$$\Rightarrow (h-2)^2 + k^2 = \frac{64}{(\alpha-2)^2+(\alpha+2)^2} \text{ and } h-k-2 = \frac{-32}{(\alpha-2)^2+(\alpha+2)^2}$$

$$\Rightarrow (h-2)^2 + k^2 = -2(h-k-2)$$

$\Rightarrow h^2 + k^2 - 2h - 2k = 0$

$\therefore$ Locus is $x^2 + y^2 - 2x - 2y = 0$

53 (A)

$x - 1 = 3\cos\theta$

$y - 2 = 4\sin\theta$

$x + y = 3 + 3\cos\theta + 4\sin\theta$

Maximum value = 3 + 5 = 8

54. (B)

$$\sin x \cos\frac{\pi}{6} + \cos x.\sin\frac{\pi}{6} + \cos x = \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x + \cos x = \frac{\sqrt{3}}{2}\sin x + \frac{3}{2}\cos x$$

maximum value $\sqrt{\frac{3}{4}+\frac{9}{4}}=\sqrt{3}$

55. (D)

f(0) = –3, f(1) = 1, f(2) = –1 and f(3) = 3.

So f(x) = 0 has one root in each of the intervals (0, 1), (1, 2) and (2, 3) and hence no root in (3, 4).

56. (C)

Let $\alpha \le \beta \le \gamma$, then

$\alpha \in (0, 1), \beta \in (1, 2), \gamma \in (2, 3)$

$\therefore$ $[\alpha] = 0, [\beta] = 1$ and $[\gamma] = 2$

$\therefore$ $\{\alpha\} + \{\beta\} + \{\gamma\} = (\alpha + \beta + \gamma) - ([\alpha] + [\beta] + [\gamma]) = \frac{9}{2} - (0+1+2) = \frac{3}{2}$.

57. (B)

(P) $x^3 + x^2 + 4x = -2\sin x$

In $[0, \pi]$,

LHS ≥ 0 and RHS ≤ 0

$\therefore$ x = 0 is the only solutions.

In $[\pi, 2\pi]$,

LHS > $\pi^3 + \pi^2 + 4\pi$, RHS ≤ 2

$\therefore$ No solution.

(Q) $\frac{1}{2}\sin 2e^x = \frac{1}{4}(2^x + 2^{-x})$

LHS $\le \frac{1}{2}$, RHS $\ge \frac{1}{2}$ $\therefore$ LHS = RHS = $\frac{1}{2}$ which has no solution.

(R) $\sin 2x = 1$

$\cos 4x = 1$

(S) $y = |\sin x|$ ... (1)

$y = \frac{x}{30}$ ... (2)

Drawing graphs of (1) and (2), we get four solutions in $[0, 2\pi]$

58. (B)

P. Let $x = 4\cos\theta, y = 3\sin\theta$, than x + y = $4\cos\theta + 3\sin\theta \le 5$

$\therefore$ $\log_5(x+y) \le 1$

Q. $\because \quad 2^{\sqrt{\log_2 3}} = \left(2^{\log_2 3}\right)^{\frac{1}{\sqrt{\log_2 3}}} = 3^{\sqrt{\log_3 2}}$ and $3^{\log_3 2} - 2^{\log_2 3} = 2 - 3 = -1$

$\therefore \quad \alpha + \beta = 3 \text{ and} \alpha\beta = 2$

$\Rightarrow \quad \alpha^2 + \alpha\beta + \beta^2 = (\alpha + \beta)^2 - \alpha\beta = 7$

R. $3\log_{18} 96 \log_{12} 3 + \log_{18} 96 + 3\log_{12} 3$

$= \left(\log_{18} 96 + 1\right)\left(3\log_{12} 3 + 1\right) - 1$

$= \log_{18}(96 \times 18).\ \log_{12}(27 \times 12) - 1$

$= \left(3\log_{18} 12\right)\left(2\log_{12} 18\right) - 1 = 6 - 1 = 5$

S. $2^{\log_x 3} = y^{\log_5 y} \Rightarrow \log_x 3\ \ln 2 = \log_5 y \ln y$

$\Rightarrow (\ln x)\ (\ln y)^2 = \ln 2 \ln 3 \ln 5$

similarly, $3^{\log_y 5} = x^{\log_2 x} \Rightarrow (\ln x)^2 (\ln y) = \ln 2 \ln 3 \ln 5$

$\therefore \quad (\ln x)(\ln y)^2 = (\ln x)^2(\ln y) \Rightarrow \ln y = \ln x \Rightarrow x = y$

$$\therefore \quad \frac{\left(x^{\log_y x} + y^{\log_x y}\right)^2}{\left(x^{\log_x y}\right)^2 + \left(y^{\log_y x}\right)^2} = \frac{(x+x)^2}{x^2 + x^2} = 2$$

59. (A)

(P) $\frac{x-4}{5} = \frac{y+13}{1} = \frac{-2(20-13+6)}{26}$

$x = -1, y = -14$

(Q) $|a| \in (1, \sqrt{2})$

(R) Equation of circle passes though origin and touching the line y = x is $x^2 + y^2 + \lambda(y - x) = 0$ therefore according to quesiton equation of common chord will be $(6x + 8y - 7) + \lambda(x - y) = 0$ and this common chord always passes though the point (1/2, 1/2)

(S) The point at shortest distance form the line and lying on the circle is (2, 1)

60. (C)

(P) $f(x) = \sin^{-1} x$

$$\lim_{x \to \frac{1}{2}^+} f(3x - 4x^3) = \ell - 3\left(\lim_{x \to \frac{1}{2}} f(x)\right)$$

$\Rightarrow \ \ell = \pi \ \ [\ell] = 3$

(Q) $\sin\left(\frac{1}{2}\left(\tan^{-1} x\left(\frac{2x}{1-x^2}\right)\right) - \tan^{-1} x\right)$ $\qquad \tan^{-1} x = \theta;\ x = \tan\theta$

$\sin\left(\frac{1}{2}\left(\tan^{-1}(\tan 2\theta)\right) - \theta\right)$ $\qquad \sin(\theta - \pi/2 - \theta) = \sin \pi/2 = -1$

(R) Domain of given question is x = –1 and 1 and x = 1 satisfy the equation

(S) $\tan^{-1} x + \tan^{-1}\frac{1}{x} = -\frac{\pi}{2}$ when $x < 0$