SOLUTIONS

PROGRESS TEST-3

GZRS-1902, GZR-1913-1917

GZRK-1905-1906

JEE MAIN PATTERN

Test Date: 25-11-2017

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PHYSICS

1. (B)

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \hat{i}(-1-0) - \hat{j}(-1-0) + \hat{k}(1-0) = -\hat{i} + \hat{j} + \hat{k}$$

$|\vec{A}\times\vec{B}| = \sqrt{3}$, $\quad \hat{n} = \frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} + \hat{k})$

2. (C)

$\vec{A} = 4\hat{i} - 2\hat{j} + 6\hat{k}$, $\quad \vec{B} = \hat{i} - 2\hat{j} - 3\hat{k}$, $\quad \vec{A} - \vec{B} = 3\hat{i} + 9\hat{k}$

The vector along *x*–axis is $\hat{i}$, $\quad (\vec{A} - \vec{B}).\hat{i} = 3$

Also,

$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$ (By definition)

$$\therefore \cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{3}{(1)\,(\sqrt{90})} \text{ or } \cos\theta = \frac{1}{\sqrt{10}} \therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$$

3. (B)

4. (D)

$\vec{a} = 3\hat{i} + 4\hat{j}$. $\quad$ Let $\vec{c} = c_x\hat{i} + c_y\hat{j}$, $\quad \vec{c}$ is perpendicular to $\vec{a}$ $\therefore$ $3c_x + 4c_y = 0$

$c_y = -\frac{3}{4}c_x$ $\quad$ (i)

$|\vec{c}| = 5$, $\quad c_x^{\,2} + c_y^{\,2} = 25$, $\quad c_x^{\,2} + \frac{9c_x^{\,2}}{16} = 25$, $\quad c_x = \pm 4$, $\quad \therefore$

$c_y = \mp 3$

5. (B)

6. (C)

7. (C)

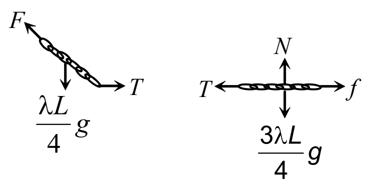
$$\frac{mg}{2} - T = \frac{ma}{2} \quad \ldots(i)$$

$$T\cos 60^\circ = \frac{ma}{\cos 60^\circ} \quad \ldots(ii)$$

Solving (i) and (ii), acceleration of ring = $\frac{2g}{9}$

8. (B)

$$F\cos 37^\circ = \frac{\lambda L}{4} g$$



(where λ is the mass/length of the chain).

$$F\sin 37^\circ = T = f \le \mu N$$

$\Rightarrow \quad \mu \ge \frac{1}{4} \qquad\qquad \Rightarrow \quad \mu_{min} = \frac{1}{4}$

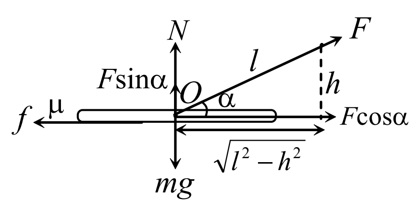
9. (A)

$$N = mg - F\sin\alpha$$

$$F\cos\alpha = f = \mu N$$

$$F\cos\alpha = \mu(mg - F\sin\alpha)$$

$$\mu = \frac{F\cos\alpha}{mg - F\sin\alpha} = \frac{F \times \frac{\sqrt{l^2 - h^2}}{l}}{mg - F \times \frac{h}{l}}$$

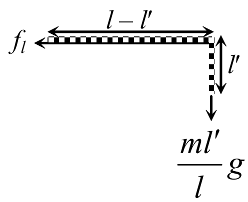


$$\mu = \frac{F\sqrt{l^2 - h^2}}{mgl - Fh}$$

10. (C)

Frictional force will balance the weight of hanging portion of rope.

$$\frac{ml'}{l}g = \mu\frac{m}{l}(l - l')g$$

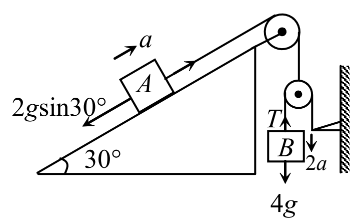


$$l' = \mu l - \mu l'$$

$$l' = \frac{\mu l}{1+\mu}$$

11. (C)

If acceleration of block *A* is *a* upward along the incline, then acceleration of block *B* is 2*a* downward.



For block *B*,

$$4g - T = 8a \qquad \ldots(i)$$

For block *A*,

$$2T - 2g\sin 30° = 2a$$

$$\Rightarrow \quad T - \frac{g}{2} = a \qquad \ldots(ii)$$

From (i) and (ii)

$$9a = \frac{7g}{2}$$

$$a = \frac{70}{18}\ \text{m/s}^2 = \frac{35}{9}\ \text{m/s}^2$$

12. (C)

Friction is static so $a = 0\ \text{m/s}^2$, $f = T\cos 60 = 40\cos 60 = 20\text{N}$

13. (A)

Force on the particle is zero at x_1 and x_2 but potential energy is minimum at x_{-1} so equilibrium will be stable at x_1.

14. (B) **15. (C)** **16. (A)**

17. (C)

p = 1mm, N = 100

Least count, $C = \frac{P}{N} = \frac{1\,\text{mm}}{100} = 0.01\,\text{mm}$

The instrument has a positive zero error $e = +NC = +4 \times 0.01 = +0.04\,\text{mm}$

Main scale reading is 2 × (1 mm) = 2 mm

Circular scale reading is 67 (0.01) = 0.67 mm

$\therefore$ observed reading is $R_0 = 2 + 0.67 = 2.67\,\text{mm}$

So true reading = $R_0 - e$ = 2.63 mm

18. (C)

In case of stretching $R \propto \frac{1}{r^4}$

$\therefore \quad \frac{\Delta R}{R} \times 100 = -4\frac{\Delta r}{r} \times 100$ = 0.4 %

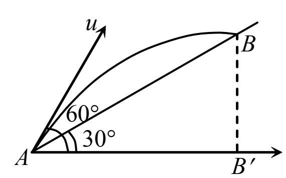
19. (D)

Block B again comes to rest when speed of A = speed of C

$v_A = 6t^2$, $v_c = 3t$, $6t^2 = 3t$, $t = \frac{1}{2}$ s

20. (A)

$AB' = u\cos 60° \times t = \frac{ut}{2}$

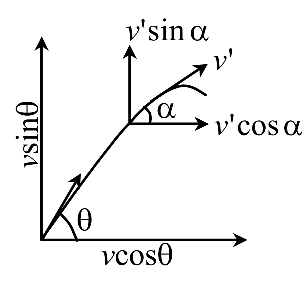


From $\Delta\, ABB'$ $\cos 30° = \frac{AB'}{AB}$

or $AB = \frac{2AB'}{\sqrt{3}} = \frac{2 \times ut}{2 \times \sqrt{3}} = \frac{ut}{\sqrt{3}}$

21. (D)

According to figure



$v'\cos\alpha = v\cos\theta$

$v'\sin\alpha = v\sin\theta - gt$

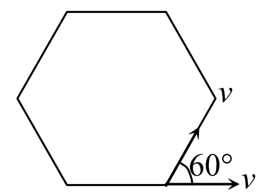
$\therefore \quad \frac{v'\sin\alpha}{v'\cos\alpha} = \frac{v\sin\theta - gt}{v\cos\theta}$

$\therefore \quad \alpha = \tan^{-1}\left(\frac{v\sin\theta - gt}{v\cos\theta}\right)$

22. (A)

$$x_{rel} = u_{x_{rel}} t = v_0\sqrt{\frac{2H}{g}}$$

23. (C)

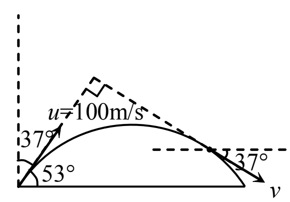


Velocity of approach = $v - \frac{v}{2} = \frac{v}{2}$

$\therefore$ time taken = $\frac{\text{initial separation}}{\text{velocity of approach}} = \frac{2a}{v}$

24. (B)

$u\cos 53° = v\cos 37°$



$\Rightarrow\ 100 \times \frac{3}{5} = v \times \frac{4}{5} \Rightarrow v = 75\,m/s$

$v_y = -v\sin 37° = -45\,m/s$

$u_y = u\sin 53° = 80\,m/s$

$v_y = u_y + gt \ \Rightarrow\ -45 = 80 - 10t$

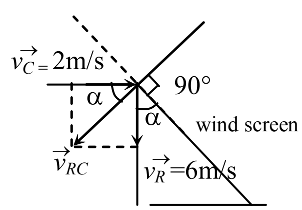
$t = 12.5\,s$

25. (B)

26. (B)

Velocity of rain with respect to car $\vec{v}_{RC} = \vec{v}_R - \vec{v}_C$ should be perpendicular to the wind screen.

From figure,

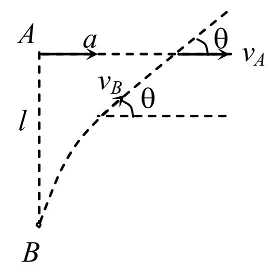


$\tan\alpha = \frac{v_r}{v_c} = \frac{6}{2}$

$\alpha = \tan^{-1}(3)$

27. (A)

Let after time t, the velocity of particle B is directed at an angle θ with the horizontal, then



$$-\frac{ds}{dt} = bt - at\cos\theta$$

$$\Rightarrow \quad -\int_l^0 ds = b\int_0^t t\,dt - a\int_0^t t\cos\theta\,dt$$

and $$\frac{1}{2}at^2 = b\int_0^t t\cos\theta\,dt \qquad \therefore \qquad l = \frac{bt^2}{2} - \frac{a^2t^2}{2b}$$

$$t = \sqrt{\frac{2bl}{b^2 - a^2}}, \qquad S = \frac{1}{2}bt^2 = \frac{1}{2}b\frac{2bl}{b^2 - a^2} = \frac{b^2 l}{b^2 - a^2}$$

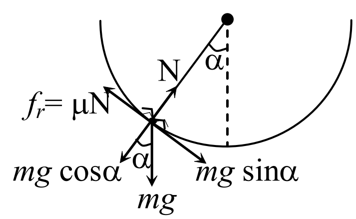
28. (A)

To avoid slipping $f_r = mg\sin\alpha$ at maximum α

$\mu N = mg\sin\alpha$

$\mu mg\cos\alpha = mg\sin\alpha$

$\mu = \tan\alpha$

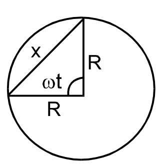


$\therefore \tan\alpha = \frac{1}{3} \qquad \therefore \quad \cot\alpha = 3$

29. (D)

The nature of the motion can be determined only if we know velocity and acceleration as function of time. Here acceleration at an instant is given and not known at other times so D.

30. (D)



$\cos\omega t = \frac{R^2 + R^2 - x^2}{2R^2} \qquad \therefore \ x = 2R\sin\frac{\omega t}{2}$

CHEMISTRY

31. (D)

$I_1 = 24.6\,eV$

$I_2 = I_H \times Z^2$

$= 13.6 \times 4 = 54.4\ eV$

$\therefore$ Required energy = 24.6 + 54.4 = 79 eV

32. (D)

$$\Delta x \Delta \upsilon \geq \frac{h}{4\pi m}$$

$$\Delta x \times \left[\frac{10 \times 0.1}{100}\right] \geq \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 0.2}$$

33. (A)

Angular momentum = $\frac{nh}{2\pi}$

$$r = \frac{n^2}{Z} \times 0.529\ \text{Å}$$

$$n \propto \sqrt{r}$$

$\therefore$ Angular momentum $\propto \sqrt{r}$

34. (D)

$$\lambda = \frac{h}{\sqrt{2Em}}$$

$$\frac{\lambda_m}{\lambda_{2m}} = \frac{h/\sqrt{2Em}}{h/\sqrt{2E(2m)}} = \frac{\sqrt{2}}{1}$$

35. (C)

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{1}{2}mv^2.$$

$$V = \sqrt{\frac{2hc}{m}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)} = \sqrt{\frac{2h}{m}(\lambda_0 - \lambda)\left(\frac{c}{\lambda\lambda_0}\right)}$$

$$K = \frac{c}{\lambda\lambda_0}$$

36. (B)

Spin angular momentum = $\left(\sqrt{s(s+1)} \times \frac{h}{2\pi}\right)$.

where, $s = +\frac{1}{2}$.

$\therefore$ Spin angular momentum = $\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)} \times \frac{h}{2\pi} = \frac{\sqrt{3}}{2} \times \frac{h}{2\pi}$.

37. (B)

$FeSO_4 \longrightarrow Fe^{2+} + SO_4^{2-}$

$Fe_2(SO_4)_3 \longrightarrow 2Fe^{3+} + 3SO_4^{2-}$

Suppose x mol SO_4^{2-} ion are furnished by both $FeSO_4$ and $Fe_2(SO_4)_3$.

Number of moles of $Fe^{2+} = x$

Number of moles of $Fe^{3+} = \frac{2}{3}x$

$Fe^{2+} : Fe^{3+} = x : \frac{2}{3}x$ = 3 : 2

38. (D)

Mass percent of oxygen = $\frac{30 \times 16}{666.43} \times 100$

= 72.

39. (C)

m_S : mo : m_H = 35.79 : 62.92 : 1.13

n_S : no : n_H = $\frac{35.79}{32} : \frac{62.92}{16} : \frac{1.13}{1}$

= 2 : 7 : 2

molecular formula = $(H_2S_2O_7)_n$

40. (C)

The reaction is :

$$2KClO_3(s) \longrightarrow 2KCl(s) + 3O_2(g)$$

$2KClO_3$	$3O_2$
2 mol	3 mol
2×122.5g	3×32g
245g	96g

Mass of O_2 obtained from 10 kg $KClO_3 = \frac{96}{245} \times 10 = 3.92\,kg$

41. (B)

$n_{H_2} = \frac{1}{2} \quad n_{CH_4} = \frac{1}{16}$

$p_{H_2} = x_{H_2} \times P$

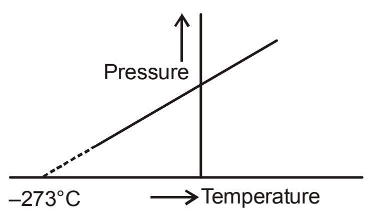
$= \frac{1/2}{1/2 + 1/16} \times P = \frac{\frac{1}{2}P}{9/16} = \frac{8}{9} \times P$

42. (B)

$d = \frac{PM}{RT}$

High pressure and low temperature will give higher density.

43. (D)



$P_T = P_0\left(1 + \frac{T}{273}\right)$

P_T = Pressure at T°C, P_0 = Pressure at 0°C

44. (B)

h × d (glycerine) = h × d (mercury)

5 × 2.72 = h × 13.6

h = 1 m

$P_{gas} = 1760$ mm Hg OR (1000 + 760) mm Hg

PV = nRT

$\frac{1760}{760} \times 10 = n \times 0.082 \times 300$

$n = 0.94$ mol

45. (C)

$$n_{N_2} = \frac{77}{28} = 2.75$$

$$n_{O_2} = \frac{23}{32} = 0.72$$

% by volume of O_2 = % by mol of O_2

$$= \frac{n_{O_2}}{\text{Total moles}} \times 100$$

$$= \frac{0.72}{2.75 + 0.72} \times 100 \qquad = 20.8$$

46. (B)

Ununtrium for atomic number = 113; for Z > 86 pd. no. = 7

113 [Rn]$7s^2$, $5f^{14}$, $6d^{10}$, $7p^1$

p-block

47. (C) **48. (D)** **49. (D)**

50. (C)

$CsBr_3$ exist as $Cs^+\ Br_3^-$, due to lattice energy effect (large cations stabilises by large anion)

51. (B) **52. (D)** **53. (B)** **54. (C)**

55. (A) **56. (C)** **57. (D)** **58. (D)**

59. (A) **60. (A)**

MATHEMATICS

61. (D)

Let S = 1 + 2 + 3 + ... + 100 = 5050

Sum of integers divisible by 3

$$S_3 = \frac{33}{2}[3+99] = \frac{33 \times 102}{2} = 33 \times 51 = 1683$$

Sum of integers divisible by 5

$$S_5 = \frac{20}{2}[5 + 100] = 10 \times 105 = 1050$$

Sum of integers divisible by 3 & 5

$S_{15} = \frac{6}{2}[15 + 90] = 3 \times 105 = 315$

Sum of integers divisible by 3 or 5

$S_{3or5} = S_3 + S_5 - S_{15}$

$= 1683 + 1050 - 315 = 2418$

Sum of integers not divisible by 3 or 5

$= 5050 - 2418 = 2632$

62. (C)

$8 - |x| \geq 4 \quad \Rightarrow |x| - 4 \leq 0 \quad \Rightarrow x \in [-4, 4]$

63. (C)

$\frac{4-4x+2+2x}{(1+x)(1-x)} - 1 < 0 \quad \Rightarrow \frac{6-2x-1+x^2}{(1+x)(1-x)} < 0 \quad \Rightarrow \frac{x^2-2x+5}{(x+1)(x-1)} > 0$

$\Rightarrow \frac{1}{(x+1)(x-1)} > 0$

64. (B)

$|x-2| - 1 = 0 \quad \Rightarrow x = 1, 3 \quad (\text{But } x \neq 1)$

$|x-1| = 1 \quad \Rightarrow x = 2, 0$

65. (C)

Let T_r be the r^{th} term of given series, $T_r = \frac{2r+1}{\frac{r(r+1)(2r+1)}{6}} = \frac{6}{r(r+1)} = 6\left[\frac{1}{r} - \frac{1}{r+1}\right]$

$\sum_{r=1}^{35} T_r = 6\left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + ... + \frac{1}{35} - \frac{1}{36}\right] = 6\left[1 - \frac{1}{36}\right] = \frac{35}{6}$

66. (D)

67. (D)

Let $y = 3^{\log_7 x} \Rightarrow y^2 - 2y + 1 = 0$

$\Rightarrow y = 1 \Rightarrow x = 1$

68. (D)

$\sin A \sec A \sqrt{\operatorname{cosec}^2 A - 1} = \tan A\,|\cot A|$

$= -1 \quad (\because 90^o < A < 180^o)$

69. (B)

We have,

$$\cos\theta \cos 2\theta \cos 2^2\theta \ldots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta} = \frac{\sin(\pi-\theta)}{2^n \sin\theta} \quad [\because 2^n\theta = \pi - \theta]$$

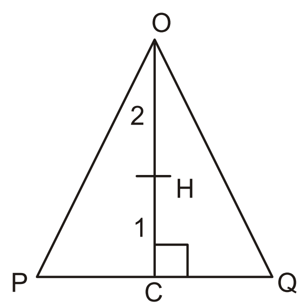
$$= \frac{1}{2^n}$$

70. (A)

In an equilateral triangle the orthocentre and the centroid are the same. OPQ is the equilateral triangle in which $OC \perp PQ$.

Clearly, the point H which divides OC internally in the ratio 2:1 is the orthocentre.

Clearly, $OC = \frac{1}{\sqrt{2}}$. So, $OH = \frac{2}{3} \times \frac{1}{\sqrt{2}}$



$$\therefore \quad H = \left(\frac{2}{3\sqrt{2}}\cos 45°, \frac{2}{3\sqrt{2}}\sin 45°\right)$$

71. (D)

$\frac{3}{4}a + \frac{2}{4}b + c = 0$, thus given line passes through $\left(\frac{3}{4}, \frac{2}{4}\right)$

72. (C)

Let f(t) = $9^t + 9^{1-t}$ where t = $\sin^2 x$, $t \in [0,1]$

Use $A.M. \geq G.M.$

73. (C)

We have,

$(2x - 3y)^2 + (3y - 4z)^2 + (4z - 2x)^2 = 0 \quad \Rightarrow 2x = 3y = 4z$

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in AP $\quad \Rightarrow$ x, y, z are in HP

74. (B)

Let ratio be $\lambda : 1 \Rightarrow \frac{6\lambda - 3}{\lambda + 1} = 0,\ \lambda = \frac{1}{2}$

75. (C)

$$\frac{H_1 + 2}{H_1 - 2} + \frac{H_{20} + 3}{H_{20} - 3} = \frac{\frac{1}{2} + \frac{1}{H_1}}{\frac{1}{2} - \frac{1}{H_1}} + \frac{\frac{1}{3} + \frac{1}{H_{20}}}{\frac{1}{3} - \frac{1}{H_{20}}}$$

$$= \frac{\frac{1}{2} + \frac{1}{2} + d}{\frac{1}{2} - d - \frac{1}{2}} + \frac{\frac{1}{3} + \frac{1}{3} - d}{\frac{1}{3} + d - \frac{1}{3}} = \frac{1 + d}{-d} + \frac{\frac{2}{3} - d}{d} = \frac{\frac{2}{3} - 1}{d} - 2 = 2 \times 21 - 2 = 40$$

76. (C)

$$7\left(\frac{y}{x}\right)^2 + 2c\left(\frac{y}{x}\right) - 1 = 0$$

77. (A)

$$10\tan^4\alpha + 15 = 6(\tan^2\alpha + 1)^2 \Rightarrow \tan^2\alpha = \frac{3}{2} \Rightarrow 9\,\text{cosec}^4\alpha + 8\sec^4\alpha = 75$$

78. (A)

$$3 + \frac{1}{4}(3 + d) + \frac{1}{4^2}(3 + 2d) + ... + \infty = 8$$

$a = 3,\ r = \frac{1}{4}$

Sum of AGP upto ∞

$$S_\infty = \frac{a}{1 - r} + \frac{dr}{(1 - r)^2}$$

$$\Rightarrow 8 = \frac{3}{(3/4)} + \frac{d\left(\frac{1}{4}\right)}{3^2/4^2} \Rightarrow 8 = 4 + \frac{4d}{3^2}$$

$$\Rightarrow 4 = \frac{4d}{3^2} \quad \Rightarrow d = 3^2 \quad \Rightarrow d = 9$$

79. (A)

$\sin(\alpha + \beta) = 1, \Rightarrow \alpha + \beta = \frac{\pi}{2}$ ..... (i)

$\sin(\alpha - \beta) = \frac{1}{2}, \Rightarrow \alpha - \beta = \frac{\pi}{6}$ ..... (ii)

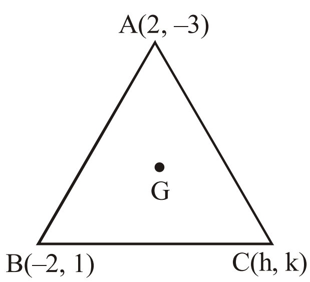
on solving (i) & (ii)

$\alpha = \frac{\pi}{3}, \beta = \frac{\pi}{6}$

$\therefore \tan(\alpha + 2\beta).\tan(2\alpha + \beta) = \tan\left(\frac{2\pi}{3}\right)\tan\left(\frac{5\pi}{6}\right)$

80. (A)

$G\left(\frac{2-2+h}{3}, \frac{-3+1+K}{3}\right) \equiv G\left(\frac{h}{3}, \frac{k-2}{3}\right)$



lies on 2x + 3y = 1,

$\therefore 2\left(\frac{h}{3}\right) + 3\left(\frac{k-2}{3}\right) = 1$

$\Rightarrow 2h + 3K - 6 - 3 = 0 \Rightarrow 2x + 3y - 9 = 0$

81. (C)

cosecA + cotA = 2

$\Rightarrow$ cosecA – cot A = 1/2

$\Rightarrow$ cosecA = 5/4 & cotA = 3/4

$\Rightarrow$ cosA = 3/5

82. (C)

83. **(D)**

Expression reducs to $\sin\theta + \cos\theta$

Which lies between $-\sqrt{2}$ & $\sqrt{2}$

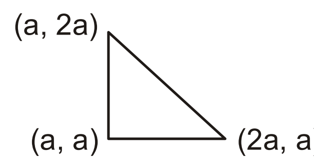
84. **(B)**

Given,

$a_1 = 2$, & $\frac{a_{n+1}}{a_n} = \frac{1}{3} = r$

$$\sum_{r=1}^{20} a_r = \frac{a_1(1-r^{20})}{1-r} = \frac{2\left(1-\left(\frac{1}{3}\right)^{20}\right)}{\frac{2}{3}} = 3\left(1-\frac{1}{3^{20}}\right)$$

85. **(D)**



$\frac{1}{2}a^2 = 72$

$a = \pm 12$

Centroid = (16, 16) or (-16, -16)

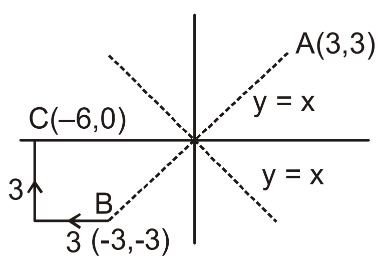
86. **(C)**

$\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

$\sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 1$

$\therefore \cos\theta_1 = 0, \ \cos\theta_2 = 0, \ \cos\theta_3 = 0$

87. **(A)**



88. (A)

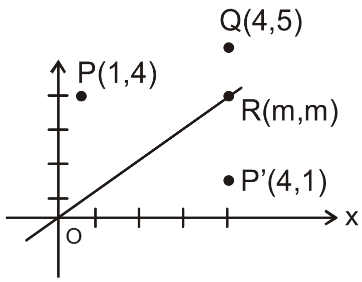


Image of (1, 4) about the line y = x is (4, 1) $\Rightarrow P'(4,1)\,Q(4,5)$ and R(m, m) are collinear.

$\Rightarrow\ m = 4$

89. (D)

Given a_1, a_2, a_3, ... in A.P. with common difference = d & $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24}$ = 225

$\Rightarrow a_1(a_1 + 4d) + (a_1 + 9d) + (a_1 + 14d)$
$+ (a_1 + 19d) + (a_1 + 23d) = 225$

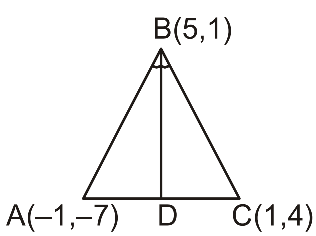
$\Rightarrow 3(2a_1 + 23d) = 225 \ \Rightarrow\ (2a_1 + 23d) = \frac{225}{3}$

$\because$ Sum of terms equidistant fom begining & end is same & is equal to the sum of firat and last term.

$a_1 + a_2 + ... + a_{24} = \frac{24}{2}\,[2a_1 + 23d]$

$= 12 \times \frac{225}{3} = 900$

90. (B)



$$\frac{AD}{CD} = \frac{AB}{BC} = \frac{10}{5} = \frac{2}{1}$$

$$D = \left(\frac{1}{3}, \frac{1}{3}\right) \qquad \therefore BD = x - 7y + 2 = 0$$