SOLUTIONS

WEEKLY TEST-13

GZRA-1901, GZR-1901(A)

GZRS-1901

(JEE ADVANCED PATTERN)

Test Date: 03-12-2017

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CHEMISTRY

1. (D)

2. (D)

3. (D)

4. (C)

Initially $P_{He}+P_{Ne}=950 - 50 = 900$ torr

$$P_{He} = \frac{2}{3}\times 900 = 600\,\text{torr} \;;\quad P_{Ne} = \frac{1}{3}\times 900 = 300\,\text{torr}$$

In final condition, P_{He} = 1800 torr

$$P_{H_2O}(g) = 100\ \text{torr}$$

$$\frac{n_{He}(g)}{n_{H_2O}(g)} = \frac{1800}{100} = \frac{18}{1}[n\,\alpha\,p\ \text{at constant V and T}]$$

5. (B)

Let volume of balloon is V litre

PV = nRT

$$3V = \frac{n\times 0.0821\times 36}{0.0821}$$

or, $n = \frac{V}{12}$ mole He

$$\text{mass of He} = \frac{V}{12}\times\frac{4}{1000}\text{kg} = \frac{V}{3000}\text{kg}$$

$$\text{Now, } \frac{V}{3000} + 50 + 450 = \frac{V}{1000}$$

or, V = 250 × 3000 litre = 250 × 3m^3 = 750 m^3

6. (B)

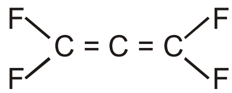
When only attraction force works

$$U(r) = \frac{K\,Q_1Q_2}{r},\ \text{rectangular hyperbola}$$

7. (D)

PF_2Cl_3 – Non-planar molecule

$B_3N_3H_6$ – Planar molecule



– Non-planar molecule

8. (D)

$$\xleftarrow[r]{Sc^{+3},\ Ti^{+3},\ V^{+2},\ Cr^{+3}}$$

- Non-metals having gaint structure have high M.P. i.e. Si.

9. (A), (C), (D)

10. (A,B,C,D)

$$\eta \alpha \rho \bar{u} \lambda_{mean} \text{ or } \eta \alpha \frac{PM}{RT}\sqrt{\frac{8RT}{\pi M}}\ \frac{1}{\sqrt{2}\,\pi\sigma^2 N^*}$$

$$\text{or, } \eta \alpha \frac{PM}{RT}\sqrt{\frac{8RT}{\pi M}}\ \frac{KT}{\sqrt{2}\,\pi\sigma^2 P}$$

$$\text{or, } \eta \alpha \frac{\sqrt{MT}}{\sigma^2}\ ;\ P \alpha T \text{ at const. V}$$

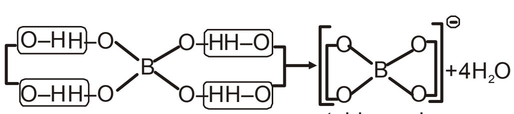
11. (A), (B), (C)

12. (B,D)

(A) $H_3BO_3 + H-OH \rightleftharpoons B(OH)_4^- + H^+$

Weak mono basic lewi's acid

(B) Equalibrium (i) is shifted in forward direction by the addition of syn-diols like-ethylene cly col which form a stable complex with $(O+)_4^-$



stable caplex

(C) It ha a planer sheet like structure due to hydrogen bonding.

(D) H_3BO_3 is a weak electrolyte in water (due to pure water)

13. (A,B,D)

When mixture is passed through hot graphite the following reaction will occur.

$CO_2 + C(s) \rightarrow 2CO(g)$

x ml 2x ml will formed

$\therefore$ Total volume of mixture = 160

100 – x + 2x = 160

x = 60 ml

Volume of CO = 100 – x = 40 ml

14. (C)

For ideal gas, $P_{ideal} = \frac{Mu^2_{rms}}{3V}$

For 1 mole real gas $P_{real} + \frac{a}{V^2} = P_{ideal}$

So, $P_{real} + \frac{a}{V^2} = \frac{Mu^2_{rms}}{3V}$

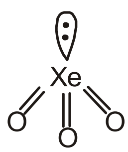
or, $P_{real} = \frac{Mu^2_{rms}}{3V} - \frac{a}{V^2}$ [b is negligible]

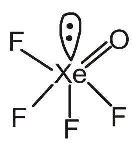
or, $P_{real} = \frac{Mu^2_{rms}V - 3a}{3V^2}$

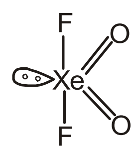
15. (A)

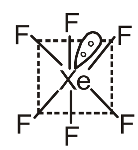
Average kinetic energy energy depends only on temperature.

16. (D)

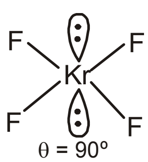
(A) 

(B) 

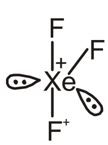
(C) 

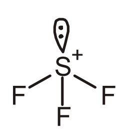
(D) 

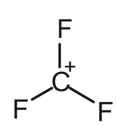
17. (C)



18. (B)

XeF_3^+ = 

SF_3^+ = 

CF_3^+ = 

19. (8)

Let electron jumps to n[th] orbit after absorbing the photon

Then, $E_n - E_1 = 40.8$

or, $\frac{-13.6 \times 4}{n^2} + \frac{13.6 \times 4}{1} = 40.8$

or, $\frac{-13.6}{n^2} + 13.6 = 10.2$

or, $\frac{13.6}{n^2} = 13.6 - 10.2 = 3.4$

or, $n^2 = \frac{13.6}{3.4} = 4$

or, $n = 2$

So, the electron can occupy any of 2s or 2p orbitals (as they have equal energy) with either clockwise or anticlockwise spin.

So, 8 sets of quantum numbers can represent the electron.

20. (6)

As it is s-orbital, probability of electron will be same in every direction at a particular distance.

Hence probability = $m \times 10^{-n} = 1 \times 10^{-5}$.

$\Rightarrow m = 1; n = 5$

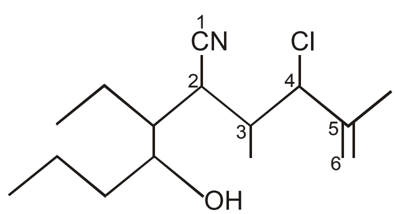
$\therefore m + n = 6$

21. (8)

$$4Zn + 10HNO_3 \longrightarrow 4Zn(NO_3)_2 + 3H_2O + NH_4NO_3$$

$a = 4;\ b = 3; c = 1 \Rightarrow a + b + c = 4 + 3 + 1 = 8$

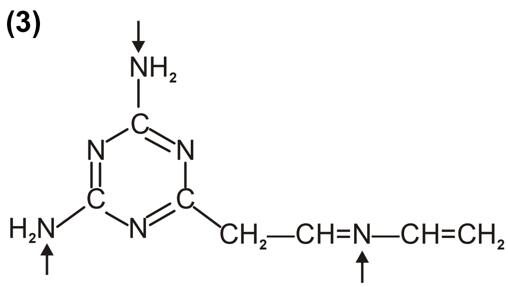
22. (6)



23. (7)

24. (6)

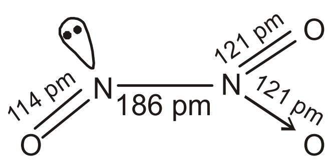
25. (3)



26. (3)

	MP (ºC)	BP (ºC)	density (g/cm^3)
B	2076	3927	2.35
Al	660	2467	2.69
Ga	29.8	2237	5.9
In	157	2080	7.3
Tl	304	1457	11.8

27. (3)



28. (3)

$$3Cu + 8H\overset{+5}{N}O_3 \longrightarrow 3Cu(\overset{+5}{N}O_3)_2 + 2\overset{+2}{N}O + 4H_2O$$

n.f. = 5 – 2 = 3

MATHEMATICS

29. (B)

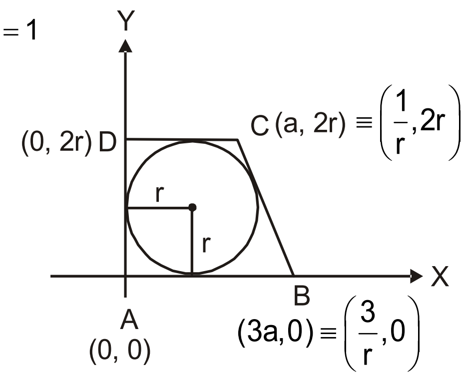
Area of trapezium ABCD $= \frac{1}{2}(a+3a)(2r) = 4 \Rightarrow ar = 1$

Equation of line BC is $y = -r^2\left(x - \frac{3}{r}\right)$

or, $y + r^2x - 3r = 0$

$\because$ BC is the tangent to the circle

$$\Rightarrow \frac{|r + r^3 - 3r|}{\sqrt{1+r^4}} = r \Rightarrow r^4 + 4 - 4r^2 = 1 + r^4 \Rightarrow r = \frac{\sqrt{3}}{2}$$



30. (C)

Given, $|4x + 3| + |3x - 4| = 12$

When $x \le \frac{-3}{4}$

$-(4x + 3) - (3x - 4) = 12$ ...(i)

$-7x = 11 \Rightarrow x = -\frac{11}{7}$ (Accepted)

When $\frac{-3}{4} < x \le \frac{4}{3}$

$4x + 3 - (3x - 4) = 12$; $x = 5$ (Rejected) ...(ii) when $x > \frac{4}{3}$

$4x + 3 + (3x - 4) = 12$; ...(iii)

$7x = 13 \Rightarrow x = \frac{13}{7}$ (Accepted)

From (i) $x = -\frac{11}{7}$ From (ii) $x = 5$ (reject)

From (iii) $x = \frac{13}{7}$

31. (D)

$$= \frac{2^{\log_2(a^4)} - 3^{\log_3(a^2+1)} - 2a}{7^{\log_7(a^2)} - a - 1} = \frac{a^4 - (a^2+1) - 2a}{a^2 - a - 1}$$

$$= \frac{(a^2)^2 - (a+1)^2}{(a^2 - a - 1)} = a^2 + a + 1$$

32. (A)

a, b, c in A.P. $\Rightarrow 2b = a + c$ ...(i)

p, q, r, in H.P. $\Rightarrow q = \frac{2pr}{p+r}$ ...(ii)

ap, bq, cr in G.P. $\Rightarrow b^2q^2 = acpr$ ...(iii)

From (ii) & (iii), we get

$$\Rightarrow \frac{b^2.4(pr)^2}{(p+r)^2} = ac\,pr \Rightarrow \frac{(a+c)^2 pr}{(p+r)^2} = ac \text{ (from (i))}$$

$$\Rightarrow \frac{(p+r)^2}{pr} = \frac{(a+c)^2}{ac} \Rightarrow \frac{p^2+r^2}{pr} + 2 = \frac{a^2+c^2}{ac} + 2$$

$$\Rightarrow \frac{p}{r} + \frac{r}{p} = \frac{a}{c} + \frac{c}{a}$$

33. (C)

A rational number of the desired category is of the form $2015.x_1x_2...x_K$

$(1 \le K \le 9 \text{ and } 9 \ge x_1 > x_2 > ... > x_K \ge 1)$

total $= {}^9C_1 + {}^9C_2 + ... + {}^9C_9 = 2^9 - 1$

34. (A)

$x_i > 0, i = 1, 2, ..., 50 \ \& \ x_1 + x_2 + x_3 + ... + x_{50} = 50$

or $\sum_{1}^{50} x_i = 50 \Rightarrow \frac{\Sigma x_i}{50} = 1$

$\because$ A.M. $\ge$ H.M.

$$\frac{\left(\sum_{1}^{50} x_i\right)}{50} \ge \frac{50}{\left(\sum_{1}^{50} \frac{1}{x_i}\right)} \Rightarrow 1 \ge \frac{50}{\left(\sum_{1}^{50} \frac{1}{x_i}\right)}$$

$\Rightarrow \sum_{1}^{50} \frac{1}{x_i} \ge 50$, Minimum value of $\Sigma \frac{1}{x_i} = 50$

35. (B)

x-cordinate

$$S_\infty = 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + ... \infty$$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-1/4} = \frac{4}{3}$$

y cordinate $\rightarrow$

$$\frac{1}{2} - \frac{1}{8} + \frac{1}{32} - \frac{1}{1028} .... \infty$$

$$S_\infty = \frac{a}{1-r} = \frac{1/2}{1+1/4} = \frac{1/2}{5/4} = \frac{2}{5}$$

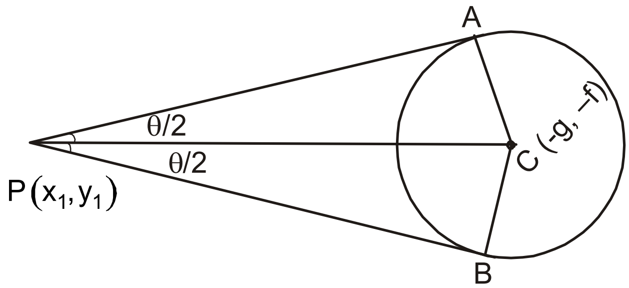
$$(x, y) = \left(\frac{4}{3}, \frac{2}{5}\right)$$

36. (C)

$$\left.\begin{matrix} \_\,\_\,20 \\ \_\,\_\,40 \\ \_\,\_\,60 \\ \_\,\_\,04 \end{matrix}\right\} \underline{|3} \times 4 = 24 \qquad \left.\begin{matrix} \_\,\_\,12 \\ \_\,\_\,16 \\ \_\,\_\,24 \\ \_\,\_\,64 \end{matrix}\right\} 2 \times 2 \times 4 = 16$$

Total number of numbers = 24+16= 40

37. (B, C, D)



Let PA & PB be the tangent from $P(x_1, y_1)$ to the given circle with centre C (–g, –f) such that

$$\angle APB = \theta \text{ then } \angle APC = \angle CPB = \frac{\theta}{2}$$

from right angle triangle PAC

$$\tan\frac{\theta}{2} = \frac{CA}{PA} = \frac{\sqrt{g^2 + f^2 - c}}{\sqrt{S_1}} \quad \Rightarrow \theta = 2\tan^{-1}\sqrt{\frac{g^2 + f^2 - c}{S_1}} \;\; \& \cot\frac{\theta}{2} = \frac{\sqrt{S_1}}{\sqrt{g^2 + f^2 - c}}$$

$$\text{Also } \cos\theta = \frac{1 - \tan^2\theta/2}{1 + \tan^2\theta/2} = \frac{S_1 - \left(g^2 + f^2 - c\right)}{S_1 + \left(g^2 + f^2 - c\right)} = \frac{S_1 + c - g^2 - f^2}{S_1 - c + g^2 + f^2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1 - \tan^2\frac{\theta}{2}}{2\tan\frac{\theta}{2}} = \frac{s_1 + c - g^2 - f^2}{2\sqrt{s_1\left(g^2 + f^2 - c\right)}}$$

38. (A,C)

Let A = cos 2α + cos 2β + 2cos (α + β)

= 2 cos (α + β) cos (α – β) + 2 cos (α + β)

= 2 cos (α + β) [cos (α – β) + 1]

& B = sin 2α + sin 2β + 2 sin(α + β)

$= 2\sin(\alpha+\beta)\cos(\alpha-\beta) + 2\sin(\alpha+\beta)$

$= 2\sin(\alpha+\beta)[\cos(\alpha-\beta) + 1]$

In a right angled triangle,

Hypotneuse = $\sqrt{A^2 + B^2}$

$= \sqrt{4\cos^2(\alpha+\beta)[\cos(\alpha-\beta)+1]^2 + 4\sin^2(\alpha+\beta)[\cos(\alpha-\beta)+1]^2}$

$= 2[\cos(\alpha-\beta) + 1]\sqrt{\sin^2(\alpha+\beta)+\cos^2(\alpha+\beta)}$

$= 2[\cos(\alpha-\beta) + 1]$

or $2 \cdot 2\cos^2\left(\frac{\alpha-\beta}{2}\right) = 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$

39. (B,C)

Given that,

$\tan x = \frac{2b}{a-c}$, $a \neq c$

& $y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$ ....(i)

& $z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$ ....(ii)

By adding (i) & (ii), we get

$(y+z) = a + c$ (option B correct)

Subtracting (i) & (ii)

$(y-z) = (a-c)\cos^2 x - (a-c)\sin^2 x + 4b\sin x\cos x$

$\Rightarrow (y-z) = (a-c)(\cos^2 x - \sin^2 x) + 2b\sin 2x$

$\Rightarrow y - z = (a-c)[1 - 2\sin^2 x + \left(\frac{2b}{a-c}\right)2\sin x\cos x]$

$\Rightarrow y - z = (a-c)[1 - 2\sin^2 x + \tan x\, 2\sin x\cos x]$

$\Rightarrow y - z = (a-c)[1 - 2\sin^2 x + 2\sin^2 x]$

$\Rightarrow y - z = a - c$ (option C correct)

40. (A,C)

$$\sum_{r=1}^{n}\frac{1}{\sqrt{a+rx}+\sqrt{a+(r-1)x}} = \sum_{1}^{n}\frac{\sqrt{a+rx}-\sqrt{a+(r-1)x}}{x}$$

$$= \frac{1}{x}\sum_{1}^{n}(\sqrt{a+rx}-\sqrt{a+(r-1)x})$$

$$= \frac{1}{x}\,[\sqrt{a+nx} - \sqrt{a}\,]$$

Upon rationalizing

$$= \frac{1}{x}\;\frac{a+nx-a}{\sqrt{a+nx}+\sqrt{a}} = \frac{n}{\sqrt{a+nx}+\sqrt{a}}$$

41. (A,B)

$$BD = \sqrt{1^2+7^2} = 5\sqrt{2}$$

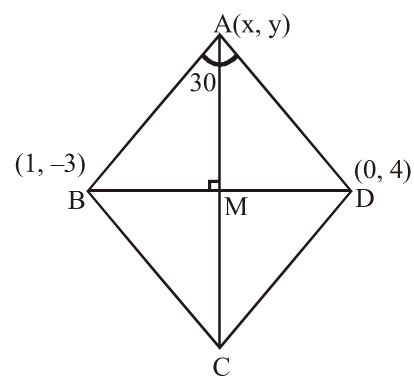
$$BM = \frac{5}{\sqrt{2}}$$

$$\frac{AM}{BM} = \tan 60^o$$

$$M\left(\frac{1}{2},\frac{1}{2}\right)$$

for point A

$$\frac{x-\frac{1}{2}}{\cos\theta} = \frac{y-\frac{1}{2}}{\sin\theta} = \pm\frac{5\sqrt{6}}{2}$$



$$m_{BD} = \frac{7}{-1}$$

$$m_{AC} = \frac{1}{7} = \tan\theta$$

$$\sin\theta = \frac{1}{\sqrt{50}} = \frac{1}{5\sqrt{2}}$$

$$\cos\theta = \frac{7}{5\sqrt{2}}$$

42. (C)

Given that,

sin A sin B sin C = p & cos A cos B cos C=q

$\Rightarrow$ tan A tan B tan C= $\frac{p}{q}$

$\Rightarrow$ Σ tan A = Π tan A = $\frac{p}{q}$

43. (B)

In a triangle, A + B + C = π

L.H.S. = tan C (tan A + tan B) + tan A tan B

= tan C tan (A + B) (1 – tan A tan B) + tan A tan B

= – $\tan^2$ C + tan A tan B ($\tan^2$ C + 1)

= 1 – $\sec^2$ C + tan A tan B $\sec^2$ C

= 1 – $\sec^2$ C (1 – tan A tan B)

= 1 – $\sec^2$ C $\frac{\cos(A+B)}{\cos A \cos B}$

(since, A + B = π – C)

= 1 + $\sec^2$ C cos C sec A sec B

= 1 + sec A sec B sec C

44. (C)

$\tan^3A + \tan^3B + \tan^3C$

$\tan^3A + \tan^3B + \tan^3C = 3 \tan A \tan B \tan C$

$+ (\tan A + \tan B + \tan C)(\Sigma\tan^2A - \Sigma\tan A\tan B)$

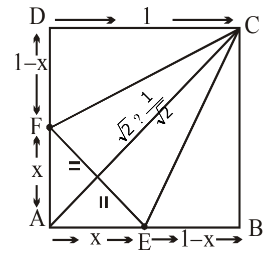
$\Sigma\tan^3A = 3(\Pi \tan A) + (\Sigma\tan A)(\Sigma\tan^2A - \Sigma\tan A\tan B)$

$= 3(\Pi\tan A) + (\Sigma\tan A)[(\Sigma\tan A)^2 - 3(\Sigma\tan A\tan B)]$

$$= \frac{3p}{q} + \frac{p}{q}\left[\frac{p^2}{q^2} - 3\frac{(1+q)}{q}\right] = \frac{3p}{q} + \frac{p^3}{q^3} - \frac{3p(1+q)}{q^2}$$

$$= \frac{3pq^2 + p^3 - 3pq - 3pq^2}{q^3} = \frac{p^3 - 3pq}{q^3}$$

45. (C)



Area of Quadrilateral CDFE =

Area ΔCDF + area FCE

$$= \frac{1}{2}(1-x)\times 1 + \frac{1}{2}\left(\sqrt{2} - \frac{1}{\sqrt{2}}x\right)\sqrt{2}\,x$$

$$= \frac{1}{2}(1 - x + (2 - x)x)$$

$$= \frac{1}{2}(1 - x + 2x - x^2)$$

$$A = \frac{1}{2}(-x^2 + x + 1)$$

for max value of A

$$\frac{dA}{dx} = 0$$

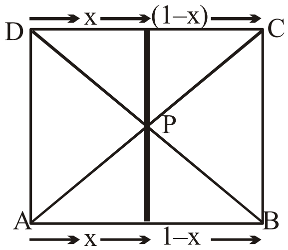
$$\frac{1}{2}(-2x + 1) = 0$$

$$-2x = 1$$

$$x = \frac{1}{2}$$

$$A = \frac{1}{2}\left(-\frac{1}{4} + \frac{1}{2} + 1\right) \qquad = \frac{5}{8}$$

46. (D)



(i) $PA^2 - x^2 = PB^2 - (1-x)^2$

$PA^2 - PB^2 = x^2 - (x^2 - 2x + 1)$

$PA^2 - PB^2 = 2x - 1$

(ii) $PC^2 - (1-x)^2 = (PD)^2 - x^2$

$PD^2 - (PC)^2 = x^2 - (1-x)^2$

$PD^2 - (PC)^2 = 2x - 1$

$PA^2 - (PB)^2 = (PD)^2 - (PC)^2$

$(PA)^2 - (PB)^2 + (PC)^2 - (PD)^2 = 0$

47. (8)

Let $x = \cos\theta, y = \sin\theta$

$-\sqrt{2} \le x + y \le \sqrt{2} \qquad (\because x + y > 0)$

48. (6)

Let $\frac{y}{x}$ = m'. So on putting y = m'x in the given

circle, we get

$(x - 3)^2 + (m'x - 3)^2 = 6$

$\Rightarrow x^2(1 + m'^2) - 6x(1 + m') + 12 = 0$

Putting discriminant = 0

$\therefore$ m' = $3 \pm \sqrt{8}$

So m = $3 - \sqrt{8}$ and M = $3 + \sqrt{8}$

Hence (M + m) = 6

49. **(7)**

Equation of circle circumscribing ΔPAB is

$(x - 1)(x - 3) + (y - 8)(y - 2) = 0$

$\Rightarrow \quad x^2 + y^2 - 4x - 10y + 19 = 0 \quad ...(i)$

Equation of circle passing through points of intersection of circles $x^2 + y^2 - 2x - 6y + 6 = 0$ and $x^2 + y^2 + 2x - 6y + 6 = 0$ is given by $(x^2 + y^2 - 2x - 6y + 6) + \lambda(x^2 + y^2 + 2x - 6y + 6) = 0$

$\Rightarrow x^2 + y^2 - 2\frac{(1-\lambda)}{(1+\lambda)}x - 6y + 6 = 0 \ ...(ii)$ As circle (ii) is orthogonal to circle (i), so

$$-2\frac{(1-\lambda)}{(1+\lambda)} - 5(-6) = 19 + 6$$

$\Rightarrow \quad 2 = 5 \Rightarrow 4\lambda - 4 = 5\lambda + 5 \Rightarrow \lambda = -9$

Hence required equation of circle is

$$x^2 + y^2 + \frac{5}{2}x - 6y + 6 = 0$$

$$\text{Radius} = \frac{\sqrt{73}}{4}$$

$\therefore$ p = 73, q = 4

Hence minimum value of (p + q) = 73 + 4 = 77

50. **(8)**

$$\because \quad \frac{1}{\sqrt{n+\sqrt{n^2-1}}} = \frac{1}{\sqrt{\left(\sqrt{\frac{n+1}{2}}+\sqrt{\frac{n-1}{2}}\right)^2}} = \frac{1}{\sqrt{\frac{n+1}{2}}+\sqrt{\frac{n-1}{2}}} = \frac{\sqrt{\frac{n+1}{2}}-\sqrt{\frac{n-1}{2}}}{\frac{n+1}{2}-\frac{n-1}{2}}$$

$$= \sqrt{\frac{n+1}{2}} - \sqrt{\frac{n-1}{2}}$$

Hence $a + b\sqrt{2} = \sum_{n=1}^{49}\left(\sqrt{\frac{n+1}{2}} - \sqrt{\frac{n-1}{2}}\right)$

$$\Rightarrow\ a+b\sqrt{2}=\left(\sqrt{\frac{2}{2}}-0\right)+\left(\sqrt{\frac{3}{2}}-\sqrt{\frac{1}{2}}\right)+\left(\sqrt{\frac{4}{2}}-\sqrt{\frac{2}{2}}\right)+\left(\sqrt{\frac{5}{2}}-\sqrt{\frac{3}{2}}\right)+......+\left(\sqrt{\frac{49+1}{2}}-\sqrt{\frac{49-1}{2}}\right)$$

$$=\sqrt{\frac{49+1}{2}}+\sqrt{\frac{48+1}{2}}-\frac{1}{\sqrt{2}}-0=5+3\sqrt{2}$$

$\Rightarrow\ a=5,\ b=3$ and $a+b=8.$

51. **(1)**

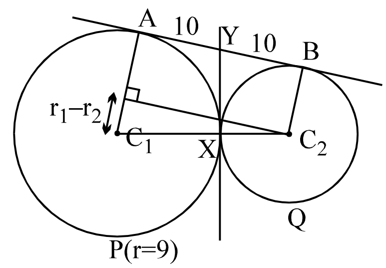
AY = BY ($\because$ Y lies on radical axis)

$(AB)^2=(r_1+r_2)^2-(r_1-r_2)^2=4r_1r_2$

$AB=2\sqrt{r_1r_2}$

$AB=20=2\sqrt{9r_2}$

$r_2=\dfrac{100}{9}$



52. **(1)**

$$a=\frac{n!}{(n-r+1)!}k,\ b=\frac{n!}{(n-r)!}k\ ;\ c=\frac{n!}{(n-r-1)!}k$$

$$\frac{b^2}{a(b+c)}=\frac{\left[\frac{n!}{(n-r)!}\right]^2}{\frac{n!}{(n-r+1)!}\left[\frac{n!}{(n-r)!}+\frac{n!}{(n-r-1)!}\right]}=1$$

53. **(1)**

Circle $(x-2)^2+(y-3)^2+\lambda(x+y-5)=0$

it passes through (1, 2)

$1+1+\lambda(1+2-5)=0\ \Rightarrow \lambda=1$

$x^2-4x+4+y^2-6y+9+x+y-5=0$

$(x-3/2)^2+(y-5/2)^2=\dfrac{1}{2}$

54. **(6)**

Let $f(t)=9^t+9^{1-t}$ where $t=\sin^2x,\ t\in[0,1]$

Use $A.M.\ge G.M.$ (for the numbers 9^t and 9^{1-t})

55. (6)

$$p(x) = x^6 - x^5 - x^3 - x^2 - x + x^4 - x^4 = x^2(x^4 - x^3 - x^2) + (x^4 - x^3 - x^2) - x$$

$$= (x^2+1)(x^4 - x^3 - x^2 - 1) + x^2 - x + 1 = x^2 - x + 1$$

$$\therefore \quad p(a) + p(b) + p(c) + p(d) = \sum a^2 - \left(\sum a\right) + 4$$

$$= (a+b+c+d)^2 - 2(ab+bc+cd+ad+ac+bd) - \sum a + 4$$

$= 1 - 2(-1) - 1 + 4 = 6.$

56. (1)

$$t_3 = t_1 + t_2 ; t_7 = 1000 ; t_1 = 1$$

$$\because \quad t_7 = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$$

$$\Rightarrow \quad 1000 = 2(t_1 + t_2 + t_3 + t_4 + t_5) = 8(t_1 + t_2 + t_3)$$

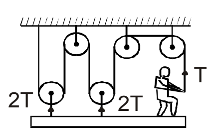
$$\Rightarrow \quad 1000 = 16(t_1 + t_2) \Rightarrow t_1 + t_2 = \frac{1000}{16} \Rightarrow t_2 = \frac{123}{2}$$

PHYSICS

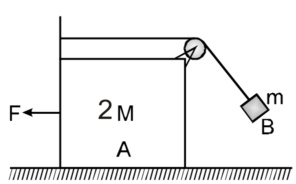
57. (C)

5T = 75 g

T = 15g = 150 N.

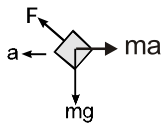


58. (A)



Applying Newton's law on the system in horizontal direction F = (2M + m) a.

Now consider the equilibrium of block B w.r.t. block 2M



$$F^2 = (mg)^2 + (ma)^2 = (mg)^2 + \left(m\frac{F}{m+2M}\right)^2$$

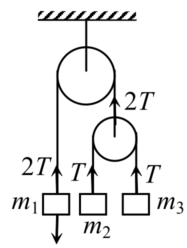
$$\therefore \quad F^2 = \frac{m^2g^2}{1-\frac{m^2}{(m+2M)^2}} \; ; \quad F = \frac{mg}{\sqrt{1-\left(\frac{m}{m+2M}\right)^2}}$$

59. From constraint relation $\nu_C = \frac{3\nu}{5}$

$\therefore$ **(B)**

60. **(B)**

If m_1 remains at rest



$2T = m_1 g$ …(i)

$T = \frac{2m_2m_3g}{m_2+m_3}$ …(ii)

From (i) and (ii)

$$\frac{4m_2m_3g}{m_3+m_2} = m_1 g$$

$$\frac{1}{m_1} = \frac{m_3+m_2}{4m_2m_3}, \; \frac{4}{m_1} = \frac{1}{m_2}+\frac{1}{m_3}$$

$$\Rightarrow \quad \frac{4}{m_1} = \frac{1}{2m}+\frac{1}{3m}$$

$$\frac{4}{m_1} = \frac{3+2}{6m}$$

$$\Rightarrow \quad \boxed{m_1 = \frac{24}{5}m}$$

61. **(A)**

$$\frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin 2\theta}{g}\times\frac{1}{2} \quad \Rightarrow \sin^2\theta = 2\sin\theta\cos\theta$$

$$\Rightarrow \quad \tan\theta = 2$$

62. (D)

Gravitational potential energy of the block at the position 1;

$U_1 = mgh_1$

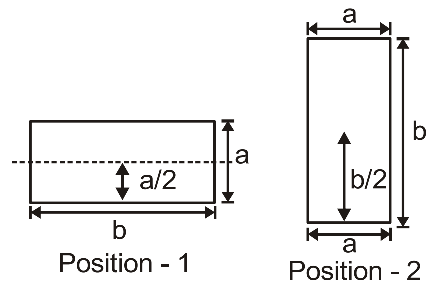
Gravitational potential energy of the block at the position 2,

$U_2 = mgh_2$

Thus, workdone by the external agent

$W = \Delta U = U_2 - U_1 = mg(h_2 - h_1)$

$\therefore \quad W = mg\left(\frac{b}{2} - \frac{a}{2}\right) = \frac{mg(b-a)}{2}$



63. (B)

Horizontal component of velocity of *A* is 10 cos 37° or 8 m/s which is equal to the velocity of *B* in horizontal direction. They will collide at *C* if time of flight of both the particles are equal i.e.

$t_A = t_B$

$$\frac{2u\sin\theta}{g} = \sqrt{\frac{2h}{g}}\ ; \text{ As } \left(h = \frac{1}{2}gt_B^2\right)$$

or $h = \dfrac{2u^2\sin^2\theta}{g}$

$$= \frac{2(10)^2\left(\frac{3}{5}\right)^2}{10} = \frac{2\times10\times9}{5\times5} = \frac{36}{5} = 7.2\text{m}$$

64. (A)

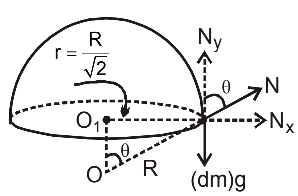
$(dm)g = \dfrac{W}{2\pi r}\times r(2\alpha) = \dfrac{W\alpha}{\pi}$

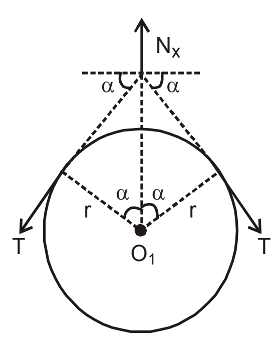
$N_x = 2T\sin\alpha$

$N_x = 2T(\alpha), \quad \because (\alpha <<)$

$N_y = (dm)g$

$\Rightarrow \tan\theta = \dfrac{N_x}{N_y} = \dfrac{2T\alpha}{(dm)g}$





$\Rightarrow \tan 45° = \frac{2T\alpha}{(dm)g} \quad \Rightarrow 1 = \frac{2T\alpha}{\frac{W\alpha}{\pi}}$

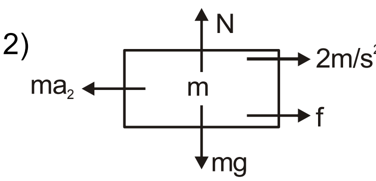
$1 = \frac{T(2\pi)}{W}$

$\therefore T = \left(\frac{W}{2\pi}\right)$

65. (A, D)

for observer (2),

the displacement of the block with respect to trolley (observer 2)



$d_{12} = \frac{1}{2} a_{12} t^2 = \frac{1}{2} \times 2 \times 10^2 = 100\text{ m}$

Hence workdone by pseudo force on block as shown by observer (2),

$W_2 = -(ma_2)d_{12} = -5 \times 1 \times 100 = -500\text{J}$

The observer (3) will observe a pseudo force $ma_3(\leftarrow)$.

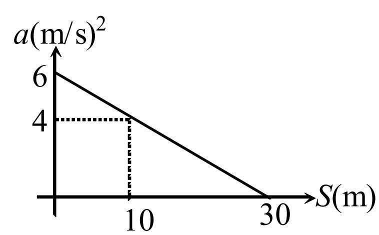
Now, $S_{13} = \frac{1}{2} a_{13} t^2$

Here $a_{13} = a_1 - a_3 = (a_{12} + a_2) - a_3 = 2 + 1 - 3 = 0$

$\therefore \quad S_{13} = 0$

$\therefore \quad W_3 = 0$ and also $W_{pseudo} = 0$.

66. (B, C)



Area $= \frac{1}{2} \times 10 \times (6+4) = \frac{v^2}{2}$

v = 10 m/s

Area upto 30 m $= \frac{1}{2} \times 30 \times 6 = \frac{v^2}{2}$

$v^2 = 180$

$v_{max} = \sqrt{80} < 14$

67. (A, B, C, D)

$\vec{v}_A = 4\hat{i} + 4\hat{k}$ $\vec{a}_A = -g\hat{k}$

$\vec{v}_B = 3\hat{j} + 4\hat{k}$ $\vec{a}_B = -g\hat{k}$

$\vec{v}_A - \vec{v}_B = 4\hat{i} - 3\hat{j}$ $\vec{a}_{AB} = \vec{O}$

$|\vec{v}_{AB}| =$ 5 m/s

Time of flight $t_A = \frac{2\times 4}{g} = \frac{8}{g}$, $t_B = \frac{2\times 4}{g} = \frac{8}{g}$

Separation when they hit the ground $= 5\times\frac{8}{g} =$ 4m

68. $h_1 = \frac{u^2}{2g}\sin^2\alpha$ and $h_2 = \frac{u^2}{2g}\cos^2\alpha$

$R = \frac{u^2}{g}2\sin\alpha\cos\alpha$

$R = \frac{u^2}{g}\times 2\frac{\sqrt{2gh_1}}{4}\frac{\sqrt{2gh_2}}{4}$, $R = 4\sqrt{h_1h_2}$

$\frac{t_1}{t_2} = \frac{2u\sin\alpha/g}{2u\sin(90-\alpha)/g} = \tan\alpha$

69. u_x= 3 m/s

a_x = – 1.0 m/s^2

$\therefore\ v_x^2 = u_x^2 + 2a_x.x$

or $0 = (3)^2 + 2(-1)(x)$ or x = 4.5 m

Also $v_x = u_x + a_xt$

$0 = 3 - (1.0)t$ or t = 3s

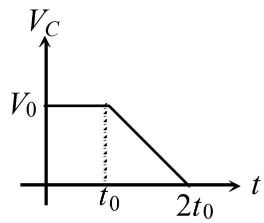
$y = u_yt + \frac{1}{2}a_yt^2 = 0 + \frac{1}{2}(-0.5)(3)^2 =$ -2.25 m

and $v_y = a_yt = (-0.5)(3) = -$ 1.5 m/s

$\therefore \quad \vec{v} = v_x\hat{i} + v_y\hat{j} = 0 - 1.5\hat{j} = \left(-1.5\hat{j}\right)$ m/s

and $\vec{r} = x\hat{i} + y\hat{j} = \left(4.5\hat{i} - 2.25\hat{j}\right)$m

70. The velocity-time graph of cat is shown in figure. For first t_o seconds V_0 is constant and then V_0 velocity is decreasing linearly for t_0 seconds and it becomes zero.



71. From graph, $v_0t_0 + \frac{1}{2}v_0t_0 = 2d$

$\frac{3}{2}v_0t_0 = 2d$ … (i)

and from motion inside the groove $\frac{2\pi R}{2v_0} = t_0 \Rightarrow v_0t_0 = \pi R$ … (ii)

From equation (i) and (ii) $\frac{3}{2}\pi R = 2d \Rightarrow d = \frac{3\pi R}{4}$

The distance travelled by the cat in $t_0 = v_0t_0 = \frac{4d}{3}$ (from equation (i))

72. Average speed of rat in t_0 second $= \frac{d}{t_0} = \frac{3}{4}\frac{v_0t_0}{t_0} = \frac{3}{4}v_0$

73. **(A)**

74. **(C)**

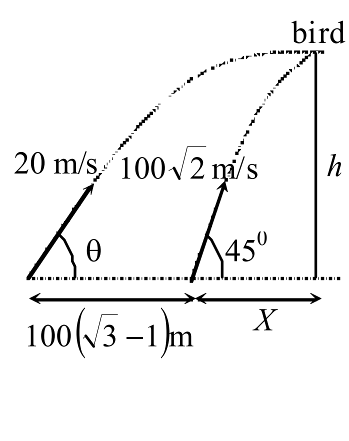
$h = 200\sin\theta\, t - \frac{1}{2}gt^2$ … (i)

$h = 100t - \frac{1}{2}gt^2$ … (ii)

$\sin\theta = \frac{1}{2} \Rightarrow$ θ = π/6

$200\cos\theta t = 100\left(\sqrt{3} - 1\right) + X$ … (iii)

$100t = X$ … (iv)



$$200 \times \frac{\sqrt{3}}{2} \times t = 100\left(\sqrt{3}-1\right) + 100t$$

$$100\sqrt{3}\,t - 100\,t = 100\left(\sqrt{3}-1\right)$$

t = 1 s

$$h = 100 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 95\,\text{m}$$

x = 100 m

75. (5)

In the frame of the lift first pulley will be stationary so velocity of second pulley will be zero & Hence in the frame of ground it moves with 5 m/s

76. (8)

$$T = \frac{4m_1m_2\,m_3\,g}{4m_1m_2 + m_2m_3 + m_1m_3}$$

77. (2)

From constraint relation, $a_B = 4a_A = 2\text{ms}^{-2}$

78. (4)

Force of friction between the two will be maximum i.e., μmg.

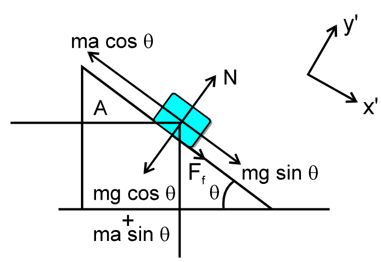
Retardation of A is $a_A = \frac{\mu mg}{m} = \mu g$

and acceleration of B is $a_B = \frac{\mu mg}{3m} = \frac{\mu g}{3}$

Required acceleration = 4

79. (3)

FBD of block B w.r.t. wedge A , for maximum 'a' :



Perpendicular to wedge :

$\Sigma f_{y'}$ = (mg cos θ + m a sin θ – N) = 0.

and $\Sigma f_{x'}$ = mg sin θ + μN – ma cos θ = 0 (for maximum a)

⇒ mg sin θ + μ(mg cos θ + ma sin θ) – ma cos θ = 0

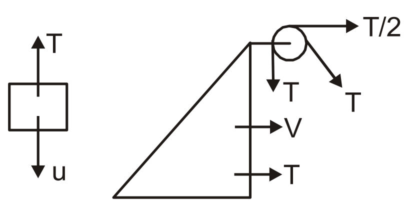
$$\Rightarrow a = \frac{(g\sin\theta + \mu g\cos\theta)}{\cos\theta - \mu\sin\theta}$$

for θ = 45°

$$a = g\left(\frac{\tan 45° + \mu}{\cot 45° - \mu}\right) \quad ; \quad a = g\left(\frac{1+\mu}{1-\mu}\right) = 20$$

hence, $\frac{60}{a} = 3$

80. (4)



$$\left(\frac{3T}{2} \times v\right) + (-Tu) = 0$$

$\therefore \; N = 4$

81. (6)

82. (6)

$$S = (8 \times 3) - \frac{1}{2} \times (0.4 \times 10) \times 3^2 = 24 - 18 = 6 \text{ m/s}$$

83. (4)

Since mg sin 37° > μmg cos37°, the block has a tendency to slip downwards.

Let F be the minimum force applied on it, so that it does not slip. Then,

N = F + mg cos37°

$\therefore \quad mg\sin 37^\circ = \mu N = \mu(F + mg\cos 37^\circ)$

or $F = \dfrac{mg\sin 37^\circ}{\mu} - mg\cos 37^\circ$

$$= \frac{(1)(10)(3/5)}{0.5} - (1)(10)\left(\frac{4}{5}\right) = 4\,N$$

84. (2)

$$y = u\sin\theta t - \frac{1}{2}gt^2$$

$$15 = 52 \times \frac{5}{13}t - \frac{1}{2} \times 10t^2$$

$$5t^2 - 20t + 15 = 0$$

$$t^2 - 4t + 3 = 0$$

$$t^2 - 3t - t + 3 = 0$$

$t_1 = 1s$ and $t_2 = 3\,s$

Then, $t_2 - t_1 = 3 - 1 = 2\,s$