JEE (ADVANCED) 2019 PAPER-1

[PAPER WITH SOLUTION]

HELD ON SUNDAY 27TH MAY, 2019

PHYSICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+3** If ONLY the correct option is chosen.

Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : **–1** In all other cases.

1. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2 (\alpha << 1)$ is made on the shell without affecting the rest of the shell. Which one of the following statements is correct ?

(1) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

(2) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}R$ from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

(3) The magnitude of electric field at a point, located on a line passing through the hole and shell's center, on a distance 2R from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

(4) The potential at the center of the shell is reduced by $2\alpha V_0$

Ans. (2)

Sol. Given that

$$V_0 = \frac{KQ}{R}$$

Pot. at center with hole

$$V_C = \frac{KQ}{R} - \frac{K\alpha Q}{R}$$

$$= \frac{KQ}{R}(1-\alpha)$$

Pot. at $\frac{R}{2}$ from centre

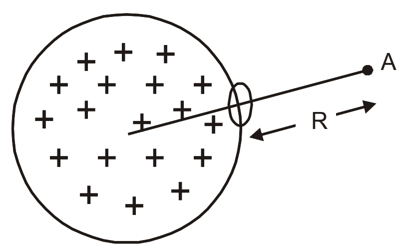
$$V_B = \frac{KQ}{R} - \frac{K\alpha Q}{\frac{R}{2}}$$

$$= \frac{KQ}{R}(1-2\alpha)$$

$$\boxed{\frac{V_C}{V_B} = \frac{1-\alpha}{1-2\alpha}}$$

Electric field at A

$$E_A = \frac{KQ}{4R^2} - \frac{K\alpha Q}{R^2} = \frac{KQ}{4R^2} - \frac{\alpha V_0}{R}$$



So, reduced by $\frac{\alpha V_0}{R}$

Electric field at centre

$$E_C = \frac{K\alpha Q}{R^2} = \frac{\alpha V_0}{R}$$

So, increased by $\frac{\alpha V_0}{R}$

$\therefore$ (2) is correct

2. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is [G is universal gravitational constant]

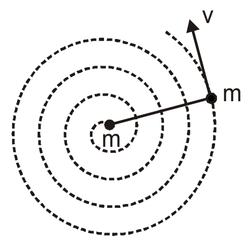
(1) $\frac{K}{6\pi r^2 m^2 G}$ (2) $\frac{K}{2\pi r^2 m^2 G}$

(3) $\frac{K}{\pi r^2 m^2 G}$ (4) $\frac{3K}{\pi r^2 m^2 G}$

Ans. (2)

Sol. $\frac{GMm}{r^2} = \frac{mv^2}{r}$

$\frac{GMm}{r^2} = \frac{2}{r}K$



$M = \frac{2K_r}{Gm}$

$dM = \frac{2K}{Gm}dr$

$\rho\, 4\pi r^2 dr = \frac{2K}{Gm}dr$

$\rho = \frac{2K}{4\pi Gmr^2}$

$\frac{\rho}{m} = \frac{K}{2\pi Gm^2r^2}$

$\therefore$ (2) is correct

3. In a radioactive sample, $^{40}_{19}K$ nuclei either decay into stable $^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable $^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable $^{40}_{20}Ca$ and $^{40}_{19}Ar$ nuclei are produced by the $^{40}_{19}K$ nuclei only. In time $t\times10^9$ years, if the ratio of the sum of stable $^{40}_{20}Ca$ and $^{40}_{18}Ar$ nuclei to the radioactive $^{40}_{19}K$ nuclei is 99, the value of t will be.

[Given : ln 10 = 2.3]

(1) 4.6

(2) 9.2

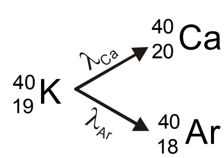
(3) 1.15

(4) 2.3

Ans. (2)

Sol.

$\frac{dN}{dt} = -(\lambda_{Ca} + \lambda_{Ar})N$



$\log_e\left(\frac{N}{N_0}\right) = -(\lambda_{Ca} + \lambda_{Ar})t$

$$2.3 \log_{10}\left(\frac{N_0}{N_0/100}\right) = 5\times 10^{-10} t$$

$$t = 9.2\times 10^{9} \text{ year}$$

$\therefore$ Answer is (2)

4. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as

$$T(t) = T_0(1+\beta t^{\frac{1}{4}})$$

Where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is

(1) $\dfrac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$ (2) $\dfrac{4P(T(t)-T_0)^2}{\beta^4 T_0^2}$

(3) $\dfrac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (4) $\dfrac{4P(T(t)-T_0)}{\beta^4 T_0^2}$

Ans. (1)

Sol. $dQ = CdT$

$$\frac{dQ}{dt} = \frac{CdT}{dt}$$

$$P = CT_0\beta\frac{1}{4}t^{\frac{-3}{4}}$$

$$\frac{4P}{T_0\beta} = t^{-3/4}C \qquad .....(i)$$

From given condition

$$T - T_0 = T_0\beta t^{1/4}$$

$$t^{1/4} = \left(\frac{T-T_0}{T_0\beta}\right)^3 \qquad ......(ii)$$

From equation (i) & (ii)

$$C = \frac{4P\left(T-T_0\right)^3}{T_0^4\beta^4}$$

$\therefore$ Correct answer is (1)

SECTION 2 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct option(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4** If only (all) the correct option(s) is(are) chosen.

Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.

Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen and both of which are correct.

Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : **–1** In all other cases.

- **For example :** In a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answer, then

choosing ONLY (A), (B) and (D) will get +4 marks.

choosing ONLY (A) and (B) will get +2 marks.

choosing ONLY (A) and (D) will get +2 marks.

choosing ONLY (B) and (D) will get +2 marks.

choosing ONLY (A) will get +1 mark.

choosing ONLY (B) will get +1 mark.

choosing ONLY (D) will get +1 mark.

choosing no option (i.e. the question is unanswered) will get 0 marks; and

choosing any other combination of options will get –1 mark.

1. A charged shell of radius R carries a total charge Q. Given Φ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct ? [ε_0 is the permittivity of free space]

(1) If $h > 2R$ and $r = 4R/5$ then $\Phi = Q/5\varepsilon_0$

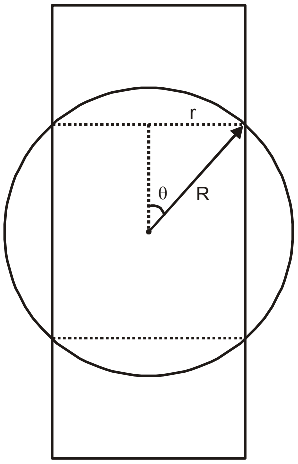
(2) If $h > 2R$ and $r = 3R/5$ then $\Phi = Q/5\varepsilon_0$

(3) If $h > 2R$ and $r > R$ then $\Phi = Q/\varepsilon_0$

(4) If $h < 8R/5$ and $r = 3R/5$ then $\Phi = 0$

Ans. (2,3,4)

Sol.



Option -1

$$\cos\theta = \frac{3}{5}$$

$$\therefore \quad q_{in} = 2\cdot\frac{Q}{2}\left(1-\frac{3}{5}\right) = \frac{2Q}{5}$$

$$\phi = \frac{2Q}{5\,\epsilon_0}$$

Opiton 2

$$\cos\theta = \frac{4}{5}$$

$$\Rightarrow \quad \frac{q_{in}}{\varepsilon_0} = \frac{1}{\varepsilon_0}2\cdot\frac{Q}{2}\left(1-\frac{4}{5}\right) = \frac{Q}{5\varepsilon_0}$$

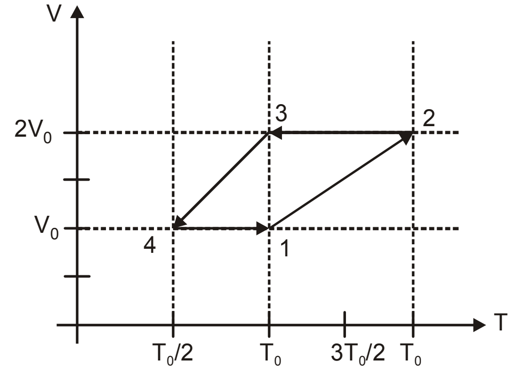
Opition 3

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Opiton 4

$$\phi = \frac{q_{in}}{\varepsilon_0} = 0$$

2. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement (s) is/are : [R is the gas constant]



(1) The above thermodynamics cycle exhibits only isochoric and adiabatic process.

(2) The ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

(3) The ratio of heat transfer during processes $1 \to 2$ and $2 \to 3$ is $\left|\frac{Q_{1\to2}}{Q_{2\to2}}\right| = \frac{5}{3}$

(4) Work done in this thermodynamic cycle $(1 \to 2 \to 3 \to 4 \to 1)$ is $|W| = \frac{1}{2}RT_0$

Ans. (3,4)

Sol.

Opiton 1

Process $1 \to 2$ and $3 \to 4$ is isobaric process as $V \propto T$.

Opiton 2

$$Q_{12} = nC_pT_0,\ Q_{34} = \frac{-nC_pT_0}{2}$$

$$\therefore \left|\frac{Q_{12}}{Q_{34}}\right| = 2$$

Opiton 3

$$Q_{2\to3} = -nC_vT_0$$

$$\therefore \left|\frac{Q_{12}}{Q_{13}}\right| = \frac{C_p}{C_v} = \frac{5}{3}$$

Opiton 4

$$W = W_{12} + W_{23} + W_{34} + W_{41}$$

$$= nRT_0 - nR\frac{T_0}{2}$$

$$= \frac{nRT_0}{2}$$

$$\therefore \quad W = \frac{RT_0}{2} \quad (\because n = 1)$$

3. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement (s) is/are correct ?

(1) The dimension of power is L^{-5}

(2) The dimension of force is L^{-3}

(3) The dimension of linear momentum is L^{-1}

(4) The dimension of energy is L^{-2}

Ans. (2,3,4)

Sol. Let angular momentum is denoted by $'\alpha'$

$$\therefore \quad [\propto] = ML^2\,T^{-1}$$

$$\text{Also, } [T] = \left[\frac{m\ell^2}{\alpha}\right] = \frac{ML^2}{[\alpha]}$$

Opiton 1

$$[P] = ML^2\,T^{-3} = ML^2T^{-1}.T^{-2}$$

$$\therefore \quad [P] \equiv [\alpha]\frac{[\alpha]^2}{M^2L^4} = L^{-4}$$

Opiton 2

$$[F] = ML\,T^{-2} = \frac{ML^2T^{-1}}{L}\cdot T^{-1}$$

$$\therefore \quad [F] \equiv \frac{[\alpha]}{L}\frac{[\alpha]}{ML^2} = L^{-3}$$

Opiton 3

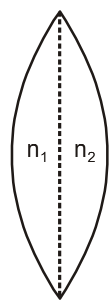
$$[P] = ML\,T^{-1} = \frac{ML^2T^{-1}}{L} \equiv L^{-1}$$

Opiton 4

$$[E] = ML^2T^{-2} = ML^2T^{-1}\cdot T^{-1}$$

$$\therefore \quad [E] \equiv [\alpha].\frac{[\alpha]}{ML^2} = L^{-2}$$

4. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. F is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n-1)$ and $1 < n < 2$, the correct statements (s) is/are



(1) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

(2) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20\,cm$, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place)

(3) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surface of the same radius of curvature.

(4) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

Ans. (1,2,3)

Sol. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{\Delta f}{f_2} = -\frac{\Delta f_2}{f_2^2}$ …(i)

$\frac{1}{f} \approx \frac{2(n-1)}{R}$ …(ii) when $n = \frac{n_1 + n_2}{2}$

$$\frac{1}{f_2} = \frac{(n_2 - 1)}{R} \qquad \ldots\text{(iii)}$$

From (i), (ii), & (iii)

$$\frac{\Delta f}{f} = -\frac{\Delta n}{2(n-1)} \qquad \ldots\text{(iv)}$$

Option 1

$$\frac{\Delta n}{n} < 0 \Rightarrow \frac{\Delta f}{f} > 0$$

Option 2

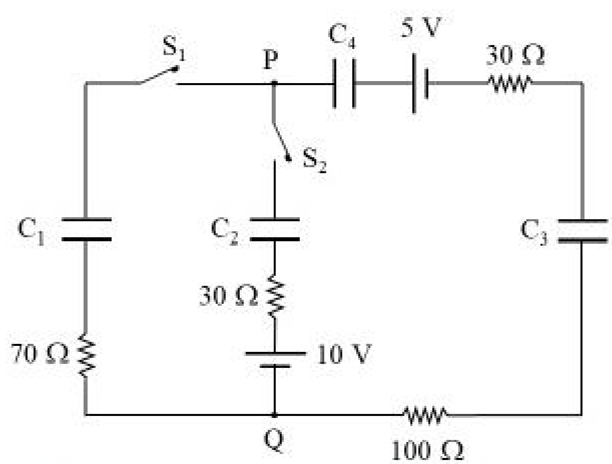
$n = 1.5,\ \Delta n = 10^{-3}\ \&\ f = 20\,\text{cm}$

$\Rightarrow\ |\Delta f| = 2 \times 10^{-3}\ \text{cm} = 0.02\ \text{cm}$

Option 3

In case of concave lens Δf and Δn will have same sign. Hence equation (iv) for concave lens will remain $\frac{\Delta f}{f} = -\frac{\Delta n}{2(n-1)}$. $(\because f < 0 \text{ for concave lens})$

5. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are C_1 = 10 μF, C_2 = 30 μF and $C_3 = C_4$ = 80 μF.



Which of the statement(s) is/are correct ?

(1) If key S_1 is dept closed for long time such that capacitors are fully charged, the voltage across the capacitor C_1 will be 4 V.

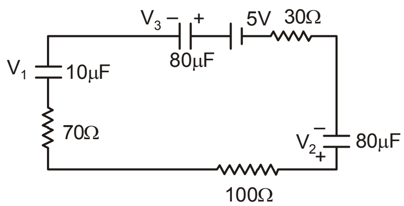
(2) The key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

(3) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

(4) At time t = 0, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA

Ans. (1, 2, 4)

Sol.

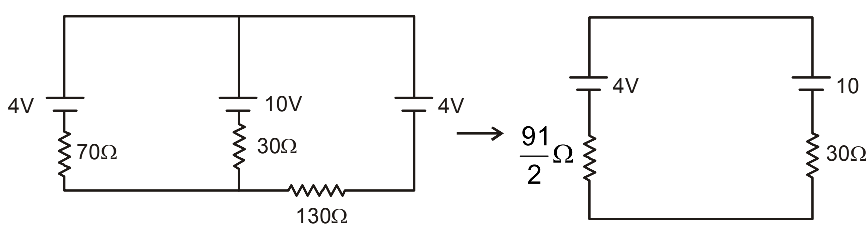


$$V_1 : V_2 : V_3 = 1 : \frac{1}{8} : \frac{1}{8}$$

$$\therefore\ V_1 = \frac{5}{10} \times 8 = 4V$$

$$V_2 = \frac{5}{10} \times 1 = \frac{1}{2} V = V_3$$

When S_2 is made closed (S_1 was already closed) their C_1 acts as a battery of $4V$ & C_3 & C_4 of $\frac{1}{2}V$ each



$$\therefore\ i = \frac{6 \times 2}{151} = 0.079\ A$$

When S_1 in made closed for long time,

$$V_{PQ} = V_{C_1} = 4\ \text{Volt}$$

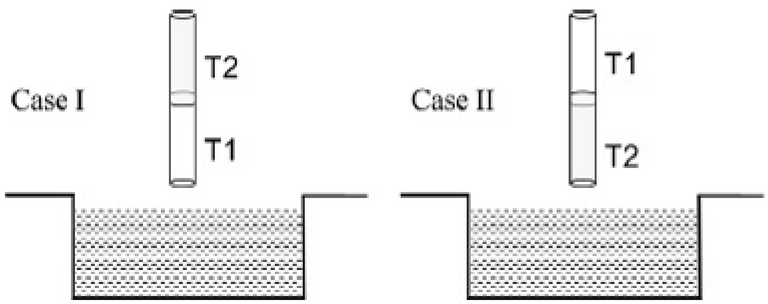
When S_1 is made closed then at this instant,

$$i = \frac{5}{200} = 25\,mA$$

Correct Answer are 1, 2, 4

6. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T1 and T2 of different materials having water contact angles of 0º and 60º, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct ?

[Surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2]



(1) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

(2) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

(3) For case, I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

(4) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm, (Neglect the weight of the water in the meniscus)

Ans. (1, 2, 3)

Sol. For Independent tube

$$h_1 = \frac{25\cos\theta_1}{\rho g r} = \frac{2\times 0.075\times 1}{10^3\times 10\times 2\times 10^{-4}} = 7.5\,\text{cm}$$

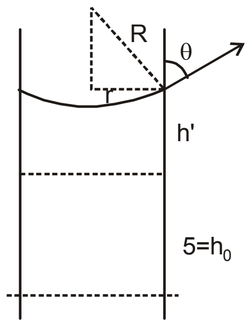
$$h_2 = \frac{2\times 5\cos\theta_1}{\rho g r} = 3.75\,\text{cm}$$

Option (1) is correct

For Case II, $h = 3.75$, when junction of two tube is 5 cm about the water

For Case I, $h = 7.5\,\text{cm}$, when junction of two tube is 8 cm above the water.

For case I, $h = 7.5\,\text{cm}$, but junction is only 5 cm above the water surface so, it may be possible that water rises above the junction also,



$$\frac{25}{R} = \rho g(h_0 + h')$$

$$\frac{25\cos\theta}{\rho g r} = h_0 + h'$$

$$h' = 3.75 - 5 = -1.25\,\text{cm}$$

If means liquid will not cross the function of two tubes. Infact angle of contact will adjust itself to balance the weight

7. Two identical moving coil galvanometers have 10 Ω resistance and full scale deflection at 2 μA current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into an Ammeter of 1 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Omh's law experiment with $R = 1000\ \Omega$ resistor by using an ideal cell. Which of the following statement(s) is/are correct ?

(1) If the ideal cell is replaced by a cell having internal resistacne of $5\ \Omega$ then the measured value of R will be more than 1000 Ω

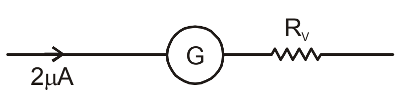
(2) The measured value of R will be $978\ \Omega < R < 982\ \Omega$

(3) The resistance of the Ammeter will be $0.02\ \Omega$ (round off to 2nd decimal place)

(4) The resistance of the voltmeter will be $100\ \text{k}\Omega$

Ans. (2, 3)

Sol.



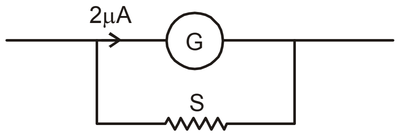
For voltmeter

$$100 \times 10^{-3} = 2 \times 10^{-6}(G + R_V)$$

$$G + R_V = 50000$$

$$R_V = 49990\,\Omega$$

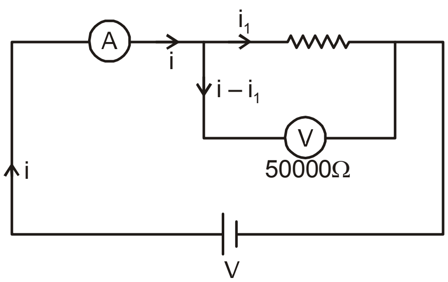
For Ammeter



$(i - i_g)S = G\, i_g$

$S = \frac{G\, i_g}{i} = 0.02\Omega$

$\therefore\ R_A = \frac{GS}{G+S} \approx 0.02\,\Omega$



$i = \frac{V}{\frac{50000}{51}} = \frac{51V}{50000}$ (Neglect R_A)

Also,

$i_1 = \frac{50}{51} i = \frac{50V}{50000}$

$i_1 = \frac{V}{1000}$

Measured value of

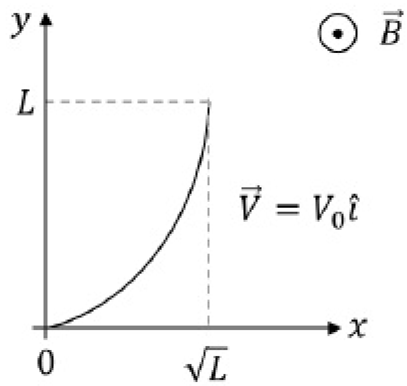
reistance $R = \frac{i_1}{i} \times 1000$

$= \frac{V \times 50000}{1000 \times 51V} \times 1000$

$\approx 980\ \Omega$

Correct option are 2, 3

8. A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnet field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constant and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are :



(1) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$

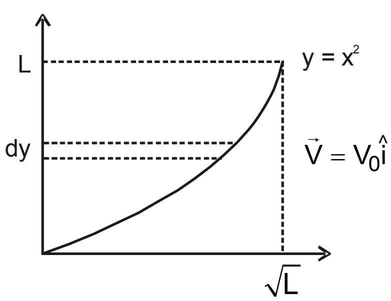
(2) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, y = x initially, of length $\sqrt{2}L$

(3) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.

(4) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$

Ans. (1, 2, 3)

Sol.



$d\Phi = BV_0dy$

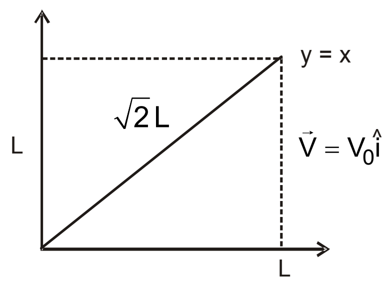
$$\Delta\Phi = \int_0^L B_0\left(1+\frac{y^{\beta}}{L^{\beta}}\right)V_0\,dy$$

$$\Delta\Phi = B_0V_0\left(1+\frac{1}{1+\beta}\right)L$$

$$\therefore\ \Delta\Phi = \frac{4}{3}B_0V_0L \qquad \text{for } \beta = 2$$

$$\Delta\Phi = 2B_0V_0L, \qquad \text{for } \beta = 0$$

Also, $\Delta\Phi \propto L$



$$\Delta\Phi = \int_0^L B_0\left(1+\frac{y^\beta}{L^\beta}\right)V_0\,dy$$

$$\Delta\Phi = B_0V_0\left(1+\frac{1}{1+\beta}\right)L$$

Correct options are 1, 2, 3

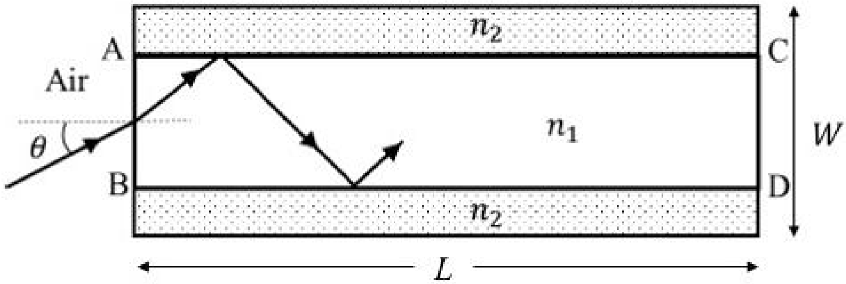
SECTION 3 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value of to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+3** If ONLY the correct numerical value is entered.

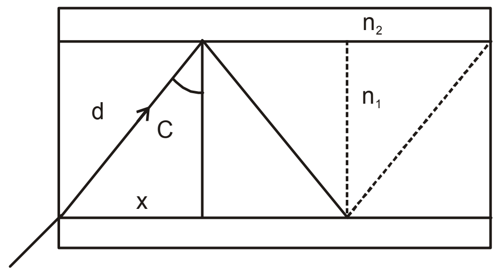
  Zero Marks **: 0** In all other cases.

1. A planar structure of length L and width W is made of two different optical media of refractive indices n_1 = 1.5 and n_2 = 1.44 as shown in figure. If L >> W, a ray entering from and AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6 m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is t × 10^{-9} s, where is _________ .



Ans. (50)

Sol.



$$\sin C = \frac{x}{d}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{x}{d}$$

$$\Rightarrow \frac{1.44}{1.5} = \frac{x}{d}$$

$$\Rightarrow \frac{144}{150} = \frac{x}{d}$$

$$\Rightarrow \frac{72}{75} = \frac{x}{d}$$

$$\Rightarrow \frac{x}{d} = \frac{24}{25}$$

$$d = \frac{25}{24}x$$

If n reflection takes place

$$d_{total} = \frac{25}{24}nx$$

$$d_{total} = \frac{25}{24} \times 9.6\,m$$

$$d_{total} = 10\,m$$

$$VT = 10$$

$$\frac{3 \times 10^8}{1.5}T = 10$$

$T = \frac{10}{3} \times 10^{-8} \times 1.5$

$= 5 \times 10^{-8}$ sec

$= 5 \times 10^{-9}$ sec

$\therefore t = 50$

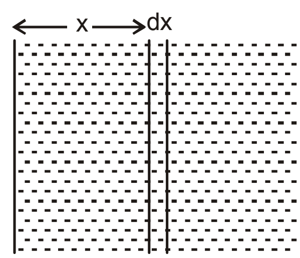
2. A parallel plante capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1 + \frac{m}{N}\right)$. For a very large N (> 10^3), the capacitance C is $\alpha\left(\frac{K\varepsilon_0 A}{d \ln 2}\right)$. The value of α will be _______ .

[ε_0 is the permittivity of free space]

Ans. (1)

Sol.

For m^{th} slab, x is equally divided into m parts



i.e $\frac{x}{m} = \frac{d}{N}$

$\Rightarrow m = \frac{N}{d}x$

$K_m = K\left(1 + \frac{N}{d}\frac{x}{N}\right)$

$= K\left(1 + \frac{x}{d}\right)$

$d\left(\frac{1}{C}\right) = \frac{dx}{\varepsilon_0 AK\left(1 + \frac{x}{d}\right)}$

$\frac{1}{C} = \int_0^d \frac{dx}{\varepsilon_0\, AK\left(1 + \frac{x}{d}\right)}$

$= \frac{1}{\varepsilon_0\, AK}\left[\ell n \frac{1 + \frac{x}{d}}{\frac{1}{d}}\right]_0^d$

$= \dfrac{d}{\varepsilon_0 AK} \ell n2$

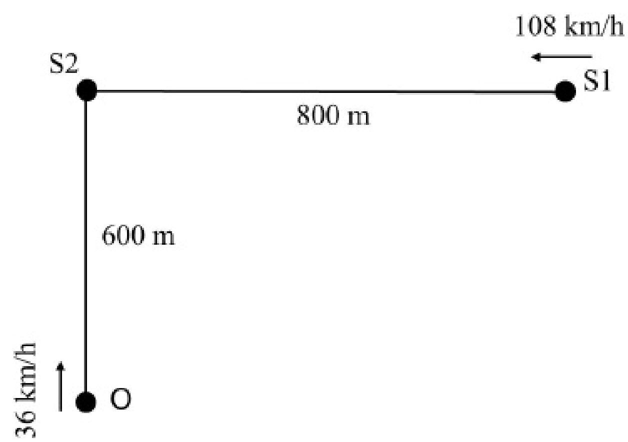
$C = \dfrac{\varepsilon_0 AK}{d \ell n2}$

$\because \quad C = \alpha \dfrac{\varepsilon_0 AK}{d \ell n2}$

$\therefore \alpha = 1$

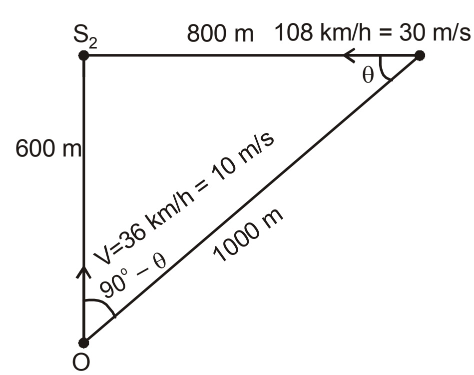
3. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is __________ .

[Speed of the sound = 330 m/s]



Ans. 8.12Hz

Sol.



$\cos\theta = \dfrac{8}{10} = \dfrac{4}{5}$

$\sin\theta = \dfrac{3}{5}$

$f_1 = \dfrac{330 + 10\sin\theta}{330 - 30\cos\theta} \times 120\,\text{Hz}$

$\Rightarrow f_1 = \dfrac{336}{306} \times 120\,\text{Hz}$

Also $f_2 = \frac{330+10}{330} \times 120\,Hz$

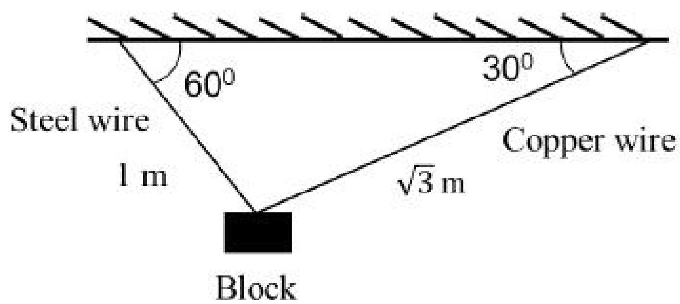
$= \frac{340}{330} \times 120\,Hz$

No. of beats $= \left(\frac{336}{306} - \frac{340}{330}\right) \times 120$

$= 8.12\,Hz$

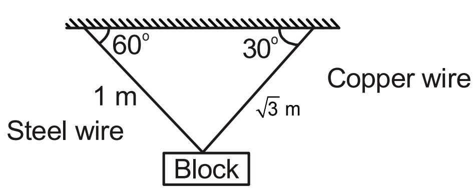
4. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30º and 60º, respectively. If elongation in copper wire is $(\Delta\ell_c)$ and elongation in steel wire is $(\Delta\ell_s)$, then the ratio $\frac{\Delta\ell_c}{\Delta\ell_s}$ is _____ .

[Young's modulus for copper and steel are 1 × 1011 N/m^2 and 2 × 1011 N/m^2, respectively]

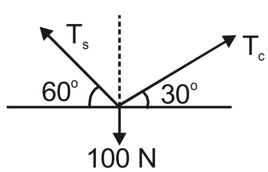


Ans. (2)

Sol.



Weight of block = 100 N



$T_c \cos 30^\circ = T_s \cos 60^\circ$

$\sqrt{3}T_c = T_s$ -------(1)

$T_c \sin 30^\circ + T_s \sin 60^\circ = 100$

$T_c + \sqrt{3}T_s = 200$ --------(2)

From Equation (1) and (2)

$T_c + 3T_c = 200$

$T_c = 50\text{ N}$

And $T_s = 50\sqrt{3}\text{ N}$

Elongation in copper wire $(\Delta\ell_c) = \dfrac{T_c}{AY_c}\ell_c$

Elongation in steel wire $(\Delta\ell_s) = \dfrac{T_s}{AY_s}.\ell_s$

$$\frac{\Delta\ell_c}{\Delta\ell_s} = \frac{T_c\ell_c}{T_s\ell_s}\frac{Y_s}{Y_c} = \frac{50\text{ N}}{50\sqrt{3}}.\frac{\sqrt{3}}{1}.\frac{2\times10^{11}}{1\times10^{11}} = 2$$

5. A liquid at 30ºC is poured very slowly into a Calorimeter that is at temperature of 110ºC. The boiling temperature of the liquid is 80ºC. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50ºC. The ratio of the Latent heat of the liquid to its specific heat will be _________ ºC.
[Neglect the heat exchange with surrounding]

Ans. (270)

Sol.

Heat gain = Heat loss

$\Rightarrow$ $(5\text{gm}).(S)(80-30)+(5\text{gm})L = W(110-80)$ [W = Water equivalent of calorimeter]

$\Rightarrow$ $250S + 5L = 30W$

$\Rightarrow$ $50S + L = 6W$ -----(i)

S : Specific heat of liquid

L : Latent heat of liquid

After pouring another 80 gm of the liquid

$80 \times S \times (50 - 30) = W \times (80 - 50)$

$\Rightarrow$ $80 \times S \times 20 = W \times 30$

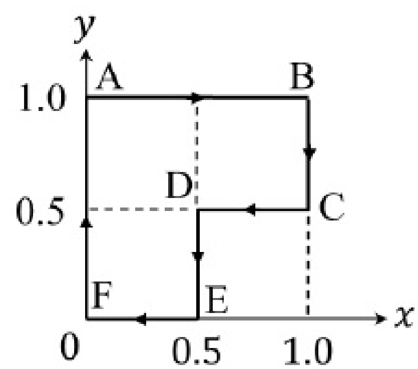
$160S = 3W$ ------(ii)

From Equation (i) and (ii)

$50S + L = 320S$

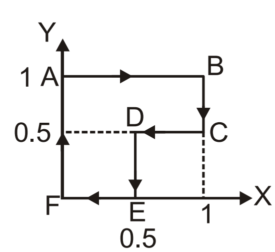
$\boxed{\frac{L}{S} = 270}$ Ans.

6. A particle is moved along a path AB–BC–CD–DE–EF–FA, as shown in figure, in presence of a force $\vec{F} = (ay\hat{i} + 2ax\hat{j})$ N, where x and y are in meter and a = –1 Nm^{-1}. The work done on the particle by this force $\vec{F}$ will be _______ Joule.



Ans. (–1)

Sol.



$\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})N$

Where $\alpha = -1$ N/m

$\vec{F} = -y\hat{i} - 2x\hat{j}(N)$

$-dw = \vec{F}.d\vec{s} = -ydx - 2xdy$

For $A \to B$, $y = 1$ and $dy = 0$

$$W_{AB} = -\int_0^1 y\,dx = -\int_0^1 dx = (-1)\,J$$

For $B \to C$, $x = 1$ and $dx = 0$

$$W_{BC} = -\int_1^{0.5} 2xdy = (-2)\int_1^{0.5} dy = +1\,J$$

For $C \to D$; $y = 0.5$ and $dy = 0$

$$W_{CD} = \int_1^{0.5} -(0.5)dx = +\frac{1}{4}J$$

For $D \to E$; $x = 0.5$ and $dx = 0$

$$W_{DE} = \int_{0.5}^{0} (-2)(0.5)dy = +\frac{1}{2}J$$

For $E \to F$; $y = 0$ and $dy = 0$

$W_{EF} = 0$

For $F \to A$; $x = 0$ and $dx = 0$

$W_{FA} = 0$

So total W. D. on the particle by this force $\vec{F}$ will be

$$= \frac{3}{4}J = 0.75\,J$$