JEE (ADVANCED) 2019 PAPER-2

[PAPER WITH SOLUTION]

HELD ON SUNDAY 27TH MAY, 2019

PHYSICS

SECTION 1 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct option(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4** If only (all) the correct option(s) is(are) chosen.
Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **–1** In all other cases.

- **For example :** In a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answer, then

choosing ONLY (A), (B) and (D) will get +4 marks.
choosing ONLY (A) and (B) will get +2 marks.
choosing ONLY (A) and (D) will get +2 marks.
choosing ONLY (B) and (D) will get +2 marks.
choosing ONLY (A) will get +1 mark.
choosing ONLY (B) will get +1 mark.
choosing ONLY (D) will get +1 mark.
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get –1 mark.

1. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical? [g is the acceleration due to gravity]

(1) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

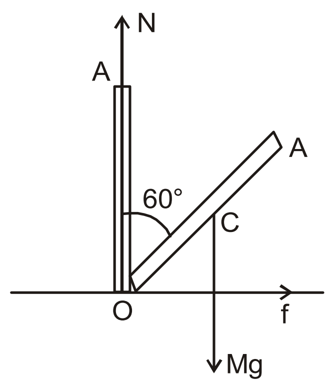
(2) The angular acceleration of the rod will be $\frac{2g}{L}$

(3) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

(4) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

Ans. **(1, 3, 4)**

Sol.



Using $\tau = I\propto$

$$Mg\frac{L}{2}\sin 60^\circ = \frac{ML^2}{3}\propto$$

$$\Rightarrow \propto = \frac{3\sqrt{3}g}{4L}$$

Using COME

$$Mg\frac{L}{4} = \frac{1}{2}\frac{ML^2}{3}\omega^2$$

$$\Rightarrow M\frac{L}{2}\omega^2 = \frac{3Mg}{4} \qquad \Rightarrow \omega = \sqrt{\frac{3g}{2L}}$$

$$(a_{CM})_r = \frac{L}{2}\omega^2 = \frac{3g}{4}$$

$$(a_{CM})_t = \frac{L}{2}\propto = \frac{3\sqrt{3}g}{8}$$

$$Mg - N = M\left[(a_{CM})_r\cos 60^\circ + (a_{CM})_t\cos 30^\circ\right]$$

$$\Rightarrow N = \frac{Mg}{16}$$

2. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0, and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are,

(Given, $2^{1.2} = 2.3;\ 2^{3.2} = 9.2;$ R is gas constant)

(1) The work $|W|$ done during the process is $13RT_0$

(2) The average kinetic energy of the gas mixture after compression is in between $18RT_0$

(3) Adiabatic constant of the gas mixture is 1.6

(4) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$

Ans. (1, 3, 4)

Sol. $F_{mix} = \dfrac{5\times 3 + 1\times 5}{6} = \dfrac{10}{3}$

$$\gamma_{mix} = 1 + \frac{2}{F_{mix}} = 1 + \frac{6}{10} = \frac{8}{5} = 1.6$$

As compression is adiabatic :

$$P_0 V_0^{8/5} = P\left(\frac{V_0}{4}\right)^{8/5}$$

$$P = P_0 \cdot (4)^{8/5} = 9.2\,P_0$$

Now $$|W| = \left|\frac{9.2P_0 \dfrac{V_0}{4} - P_0 V_0}{-0.6}\right|$$

$$= \frac{13}{6} P_0 V_0$$

$$= \frac{13}{6} \times 6RT_0 = 13RT_0$$

Again Using adiabatic condition

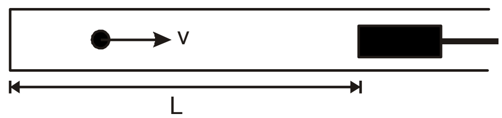
$$T_0 V_0^{0.6} = T\left(\frac{V_0}{4}\right)^{0.6}$$

$$T = T_0 (4)^{0.6} \simeq 2.3T_0$$

$$U = \frac{F}{2} nRT = \frac{10}{3\times 2} \times 6 \times R \times 2.3T_0$$

$$= 23\,RT_0$$

3. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct?



(1) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

(2) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

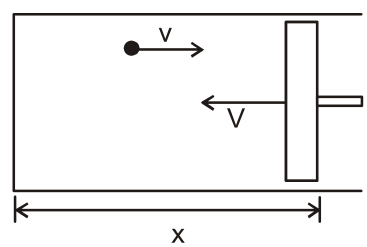
(3) The rate at which the particle strikes the piston is v/L

(4) After each collision with the piston, the particle speed increases by 2V

Ans. (2, 4)

Sol. Let v be the velocity of particle at a certain time just before colliding with piston and v' be it velocity just after collision

As piston is heavy its velocity will not change during collision.



Here e = 1 (As collision is elastic)

$\Rightarrow \quad \frac{v' - V}{v + V} = 1$

$\Rightarrow \quad v' = v + 2V$

Clearly speed of particle increases by 2V during each impact with piston.

Assume that at instant shown in the figure frequency of collision is f.

$$f = \frac{v'}{2x}$$

Now $dv' = (2vf)dt$

Where $dv' =$ change in velocity of particle during a very – very small time 'dt'.

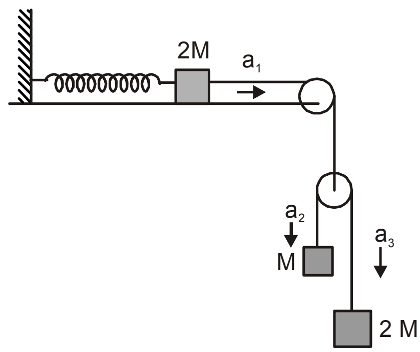
$\therefore \quad dv' = \frac{v'}{x}(-dx)$

$$\int_{v_0}^{v} \frac{dv'}{v'} = -\int_{L}^{x} \frac{dx}{x}$$

$\Rightarrow \quad v' = \frac{v_0 L}{x}$

For $x = \frac{L}{2}$, $v' = 2v_0$ and hence its K.E. will become 4 times.

4. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The acceleration of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct? [g is the acceleration due to gravity. Neglect friction]



(1) $a_2 - a_1 = a_1 - a_3$

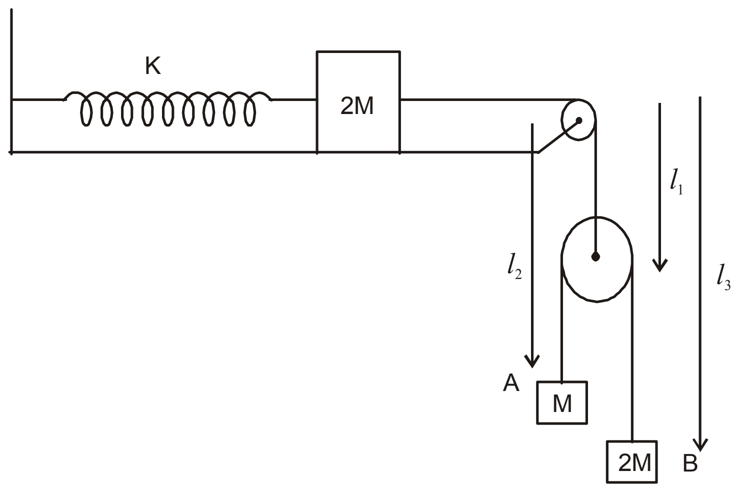
(2) $x_0 = \frac{4Mg}{k}$

(3) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

(4) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

Ans. (1)

Sol.



Consider fixed pulley as origin.

Length of string connecting the blocks A & B can be written as :

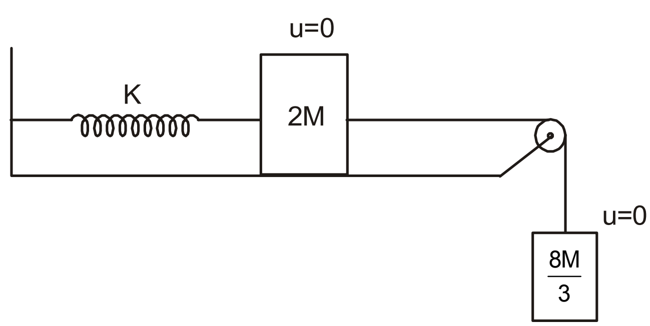
$$\ell = (\ell_2 - \ell_1) + (\ell_3 - \ell_1)$$

$\Rightarrow \quad 0 = \left(\ddot{\ell}_2 - \ddot{\ell}_1\right) + \left(\ddot{\ell}_3 - \ddot{\ell}_1\right)$

$\Rightarrow \quad 0 = (a_2 - a_1) + (a_3 - a_1)$

$\Rightarrow \quad (a_2 - a_1) = (a_1 - a_3)$

To make the problem simpler we can consider it as



Using COME :

$$\frac{8M}{3}gx_0 = \frac{1}{2}Kx_0^2$$

$$x_0 = \frac{16Mg}{3K}$$

At $x = \frac{x_0}{4} = \frac{4Mg}{3K}$, acceleration of 2M connected to spring,

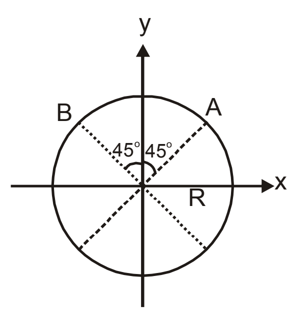
$$a_0 = \frac{4Mg}{3\left(\frac{14M}{3}\right)} = \frac{2g}{7}$$

$x = \frac{x_0}{2}$ is mean position with amplitude

$A = \frac{x_0}{2}$. Velocity of block connected to spring,

$$V = A\omega = \frac{x_0}{x}\sqrt{\frac{K}{\frac{14M}{3}}} = \frac{8Mg}{3K}\sqrt{\frac{3K}{14M}} = 4g\sqrt{\frac{2M}{21K}}$$

5. An electric dipole with dipole moment $\frac{P_o}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are: (ϵ_0 is permittivity of free space. R >> dipole size)



(1) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

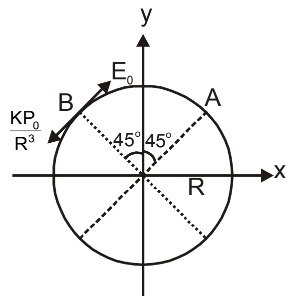
(2) The magnitude of total electric field on any two points of the circle will be same

(3) Total electric field at point B is $\vec{E}_B = 0$

(4) $R = \left(\frac{P_0}{4\pi\epsilon_0 E_0}\right)^{1/3}$

Ans. (3, 4)

Sol. R >> dipole size



Circle is equipotential

So, E_{net} should be perpendicular to surface so $\frac{KP_0}{r^3} = E_0 \Rightarrow r = \left(\frac{KP_0}{E_0}\right)^{1/3}$

At point B net electric field will be zero

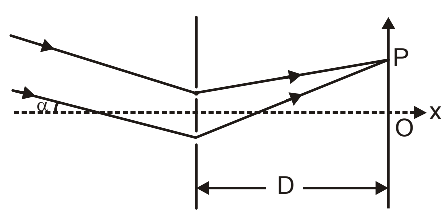
$E_B = 0$

$(E_A)_{Net} = \frac{2KP_0}{r^3} + E_0 = 3E_0$

Electric field at point A $\vec{E}_A = \frac{3}{\sqrt{2}}E_0\left[\hat{i} + \hat{j}\right]$

$(E_B)_{Net} = 0$

6. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1 m. A parallel beam of light of wavelength 600 nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct?



(1) Fringe spacing depends on α

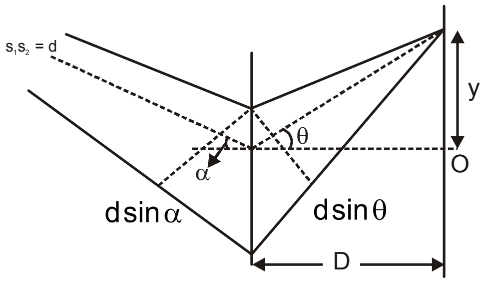
(2) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P.

(3) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O

(4) For $\alpha = 0$, there will be constructive interference at point P.

Ans. (2)

Sol.



$$\Delta x = d\sin\alpha + d\sin\theta$$

If θ & α are very small then $\sin\theta \simeq \tan\theta = \frac{y}{D}$

$$\Delta x = d\alpha + \frac{dy}{D}$$

(1) $\alpha = 0$ $\therefore \Delta x = \frac{dy}{D} = \frac{0.3\times 11}{1000} = 33\times 10^{-4}\,\text{mm}$

Δx in terms of $\lambda, \frac{33\times 10^{-4}}{600\times 10^{-6}} = \frac{11\lambda}{2}$

As $\Delta x = (2n-1)\frac{\lambda}{2}$

There will be destructive interference

(2) $\Delta x = 0.33\,\text{mm}\times\frac{0.36}{\pi}\times\frac{\pi}{180} + \frac{0.3\text{ mm}\times 11\text{ mm}}{1000}$

$= 39\times 10^{-4}\,\text{mm}$

$39\times 10^{-4} = (2n-1)\times\frac{600\times 10^{-9}\times 10^{3}}{2}$

$n = 7$

Destructive interference happened.

(3) $\Delta x = 3\text{mm}\times\frac{0.36}{\pi}\times\frac{\pi}{180} + 0 = 600\,\text{nm}$

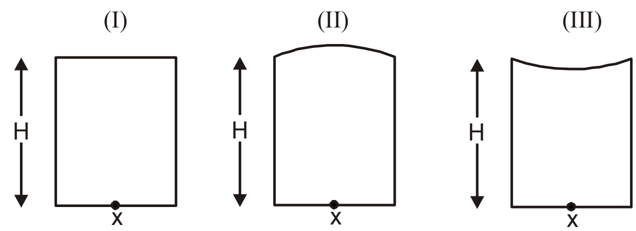
$600\,\text{nm} = n\lambda$

$n = 1$

Constructive interference

(4) Fringe width does not depends on α .

7. Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3m). If H_1, H_2 and H_3 are apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are:



(1) $0.8\text{ cm} < (H_2 - H_1) < 0.9\text{ cm}$

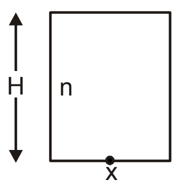
(2) $H_3 > H_1$

(3) $H_2 > H_1$

(4) $H_2 > H_3$

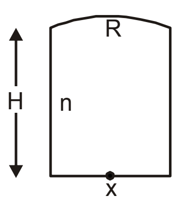
Ans. (3, 4)

Sol. Case I,



$H = 30\,\text{cm}$, $n = \frac{3}{2}$, $H_1 = \frac{H}{n} = \frac{30 \times 2}{3} = 20\,\text{cm}$

Case II,



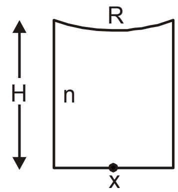
$R = 300\,\text{cm}$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\Rightarrow -\frac{1}{H_2} - \frac{3}{-2 \times 30} = \frac{1 - \frac{3}{2}}{-300}$$

$\Rightarrow\ H_2 = \frac{600}{29} = 20.684\,\text{cm}$

Case III,



$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

$\Rightarrow -\frac{1}{H_3} - \frac{3}{-2\times 30} = \frac{1-\frac{3}{2}}{300}$

$\therefore H_3 = \frac{600}{31} = 19.354\,\text{cm}$

8. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n = 1 to the state n = 4. Immediately after that the electron jumps to n = m state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e, respectively. If $\lambda_a / \lambda_e = \frac{1}{5}$, which of the option(s) is/are correct?

[Use hc = 1242 ev nm; $1\text{nm} = 10^{-9}\text{m}$, h and c are Planck's constant and speed of light, respectively]

(1) The ratio of kinetic energy of the electron in the state n = m to the state n = 1 is $\frac{1}{4}$

(2) m = 2

(3) $\lambda_e = 418\text{nm}$

(4) $\Delta p_a / \Delta p_e = \frac{1}{2}$

Ans. (1, 2)

Sol. $\frac{\lambda_a}{\lambda_e} = \frac{E_4 - E_1}{E_4 - E_m} = \frac{1-\frac{1}{16}}{\frac{1}{m^2} - \frac{1}{16}} = \frac{1}{5}$

On solving m = 2

$$\lambda_e = \frac{12400 \times 4}{13.6} = 3647$$

$$\frac{k_2}{k_1} = \frac{1^2}{2^2} = \frac{1}{4}$$ As kinetic energy proportional to $\frac{1}{n^2}$.

SECTION 2 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value of to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+3** If ONLY the correct numerical value is entered.

  Zero Marks **: 0** In all other cases.

1. Suppose a $^{226}_{88}Ra$ nucleus at rest and in ground state undergoes α-decay to a $^{222}_{86}Rn$ nucleus in its excited state. The kinetic energy of the emitted α-particle is found to be 4.44 MeV. $^{222}_{86}Rn$ nucleus then goes to its ground state by γ-decay. The energy of the emitted γ photon is _________ keV.

[Given: atomic mass of $^{226}_{88}Ra = 226.005u$, atomic mass of $^{222}_{86}Rn = 222.000u$, atomic mass of α particle = 4.000 u, 1 u = 931 MeV/c^2, c is speed of light]

Ans. (135)

Sol. $^{226}_{88}Ra \rightarrow ^{222}_{86}Rn^* + \alpha \rightarrow ^{222}_{86}Rn + \alpha + \gamma$

$$K_\alpha = 4.44 \text{Mev}$$

$$Q = (226.005 - 222.000 - 4.000)c^2 \times \frac{931}{c^2}$$

$$= 0.005 \times 931 \text{ MeV}$$

$$= 4.655 \text{ MeV}$$

Suppose energy of γ-ray = K_γ $\left\{\frac{K_\alpha}{K_{Rn}} = \frac{M_{Rn}}{M_\alpha}\right\}$ $\{K_\alpha + K_{Rn} = Q - K_\gamma\}$

$$K_\alpha = (Q - K_\gamma)\frac{222}{4+222}$$

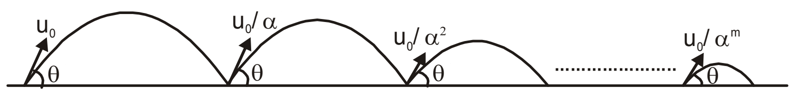
$$4.44 = (4.655 - K_\gamma)\frac{222}{226}$$

$$\frac{4.44 \times 226}{222} = 4.655 - K_\gamma$$

$$K_\gamma = 4.655 - 4.52 = 0.135\,\text{MeV}$$

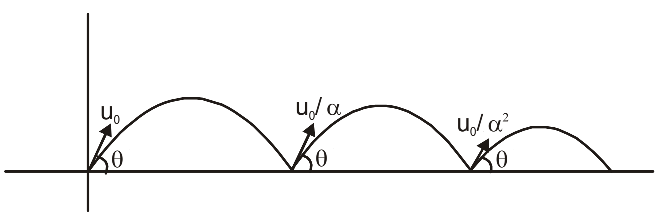
$$K_\gamma = 135\,\text{KeV}$$

2. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is ……… .



Ans. (4)

Sol.



Average velocity first time hitting the ground = $\dfrac{\dfrac{u_0^2 2\sin\theta.\cos\theta}{g}}{\dfrac{2u\sin\theta}{g}} = u\cos\theta$

$u_0\cos\theta = v_1$ ----(i)

Average velocity for entire motion = $\dfrac{\dfrac{2u_0^2\sin\theta\cos\theta}{g} + \dfrac{2u_0^2}{\alpha^2}\dfrac{\sin\theta.\cos\theta}{g} + \dfrac{2u_0^2}{\alpha^4}\dfrac{\sin\theta\cos\theta}{g}}{\dfrac{2u_0\sin\theta}{g} + \dfrac{2u.\sin\theta}{\alpha\, g} + \dfrac{2u\sin\theta}{\alpha^2 g}}$

$$0.8\,V_1 = u_0\cos\theta\frac{\left(1 + \frac{1}{\alpha^2} + \frac{1}{\alpha^4} + ...\infty\right)}{1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + ...\infty}$$

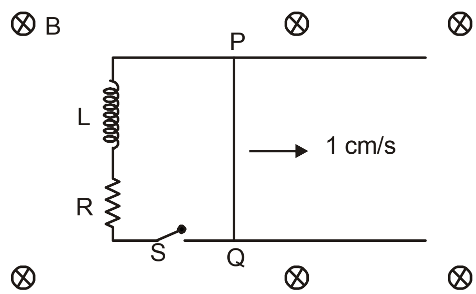
$$0.8V_1 = V_1\left(\frac{1}{1-\frac{1}{\alpha^2}}\right) = \frac{1}{1-\frac{1}{\alpha^2}}\frac{\left(1-\frac{1}{\alpha}\right)}{1}$$

$$0.8 = \frac{1}{\left(1+\frac{1}{\alpha}\right)}$$

$$1+\frac{1}{\alpha} = \frac{5}{4} \Rightarrow \frac{1}{\alpha} = \frac{1}{4} \Rightarrow \alpha = 4$$

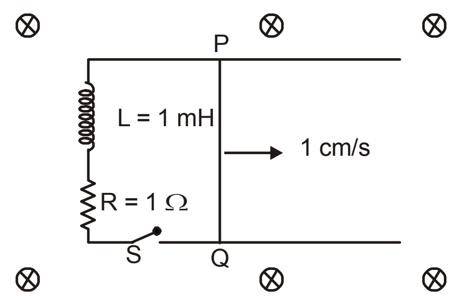
3. A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance $R = 1\Omega$ as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$ A, where the value of x is _______.

[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed. Given: $e^{-1} = 0.37$, where e is base of the natural logarithm]



Ans. (0.63)

Sol.



emf $\xi_{in} = VB\ell$ B = 1 tesla

$= 10^{-2} \times 1 \times 0.1$

$= 10^{-3}V$

$$i = \frac{\xi}{R}\left(1-e^{\frac{-t}{\tau}}\right)$$

$$= \frac{10^{-3}}{1}\left(1-e^{-\frac{10^{-3}}{10^{-3}}}\right) \qquad \tau = \frac{L}{R} = \frac{10^{-3}}{1}$$

$= 10^{-3}(1-e^{-1})$

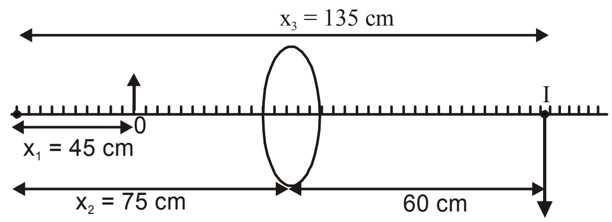
$= 10^{-3}(1-0.37) = 0.63 \times 10^{-3}A$

$x = 0.63A$

4. An optical bench has 1.5 m long scale having four divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is __________.

Ans. (1.38 and 1.39 both)

Sol. (Based on Displacement method)



$$u = x_2 - x_1 = 30\,\text{cm}\ ,\ \Delta u_{max} = \Delta x_2 + \Delta x_1 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\text{cm}$$

$$v = x_3 - x_2 = 60\,\text{cm}\ ,\ \Delta v_{max} = \Delta x_3 + \Delta x_2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\text{cm}$$

As, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, $\quad (v = +60, u = -30)$

$$\Rightarrow \frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f}$$

$$\Rightarrow f = 20\,\text{cm}$$

Again, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

On differentiation of both sides

$$\left|\left(-\frac{1}{v^2}dv\right)\right| + \left|-\frac{1}{u^2}du\right| = \left|-\frac{1}{f^2}\right|df$$

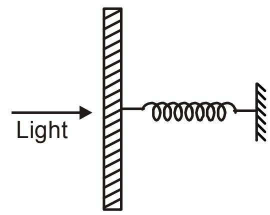
$$\left(\frac{df}{f}\right)_{max} = f\left[\frac{dv}{v^2} + \frac{du}{u^2}\right] = 20\left[\frac{(1/2)}{60^2} + \frac{(1/2)}{30^2}\right]$$

$$\%f = \frac{df}{f} \times 100\% = \frac{50}{36} = 1.3888$$

Ans. 1.38 and 1.39 both

5. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24} m^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by $1\mu m$. If the value of N is $x \times 10^{12}$ then the value of x is ________.

[Consider the spring as massless]



Ans. (1)

Sol. Linear momentum imparted to mirror = $\frac{Nh}{\lambda} + \frac{Nh}{\lambda} = \frac{2Nh}{\lambda}$

At mean position of the mirror $\frac{2Nh}{\lambda} = MV$

At mean position $V = A\omega \quad (A = 1\ \mu m)$

$$\frac{2Nh}{\lambda} = MA\omega \qquad (\lambda = 8\pi \times 10^{-6})$$

$$\Rightarrow N = \frac{M\omega \times 10^{-6} \times \lambda}{2h}$$

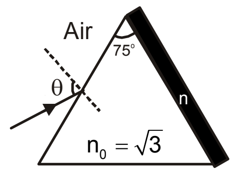
$$N = \frac{4\pi M\omega}{h} \times 10^{-12}$$

$$= 10^{24} \times 10^{-12}$$

$$N = 10^{12} = x \times 10^{12}$$

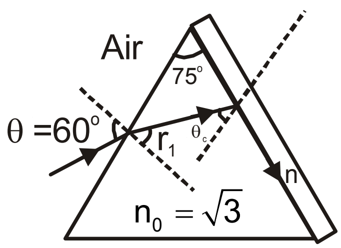
$$\Rightarrow x = 1$$

6. A monochromatic light is incident from air on a refracting surface of a prism of angle 75^o and refractive index $n_0 = \sqrt{3}$. The other refractive surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle $\theta \le 60^o$. The value of n^2 is __________.



Ans. (1.5)

Sol.



At $\theta = 60^o$ ray incident at critical angle at the second interface.

So, $1 \times \sin\theta = \sqrt{3}\sin r_1$ $(\theta = 60^o)$

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin r_1$$

$\Rightarrow r_1 = 30^o$

As $r_1 + r_2 = A$ $(A = 75^o)$

$$\theta_c = 45^o$$

At second interface

$$\sqrt{3}\sin 45^o = n\sin 90^o$$

$$n = \sqrt{\frac{3}{2}} \Rightarrow n^2 = \frac{3}{2} = 1.5$$

SECTION 3 (Maximum Marks : 12)

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions**.**
- Each List-Match set has two lists : **LIST-I** and **LIST-II.**
- **LIST-I** has **Four** entries (I), (II), (III) and (IV) and **LIST-II** has **Six** entries (P), (Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **LIST-I** and **LIST-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the option corresponding to the correct combination is chosen.
  Zero Marks **: 0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : **–1** In all other cases.

Answer the following by appropriately matching the lists based on the information given in the paragraph

A musical instrument is made using four different metal strings. 1, 2 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between in range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

	List-I		List-II
(I)	String-1 (μ)	**(P)**	1
(II)	String-2 (2μ)	**(Q)**	1/2
(III)	String-3 (3μ)	**(R)**	$1/\sqrt{2}$
(IV)	String-4 (4μ)	**(S)**	$1/\sqrt{3}$
		(T)	3/16
		(U)	1/16

1. The length of the string 1, 2, 3 and 4 are kept fixed at $L_0, \frac{3L_0}{2}, \frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. Strings 1, 2, 3 and 4 are vibrated at their 1st, 3rd, 5th and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be,

(1) $I \to T,\ II \to Q,\ III \to R,\ IV \to U$

(2) $I \to P,\ II \to Q,\ III \to R,\ IV \to T$

(3) $I \to P,\ II \to Q,\ III \to T,\ IV \to U$

(4) $I \to P,\ II \to R,\ III \to T,\ IV \to U$

Ans. (3)

Sol. Fundamental frequency $f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

For string 1st : - It is vibrating in 1st harmonic and length of string is L_0.

$\therefore\ f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} = f_0$; $\boxed{\text{Given, } f_1 = f_0}$

$$\frac{1}{2L_0}\sqrt{\frac{T_1}{\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$\Rightarrow \boxed{T_1 = T_0} \qquad \Rightarrow (P)$

For string 2nd :- It is vibrating in 3rd harmonic and length of string is $\frac{3L_0}{2}$

$\therefore f_2 = \frac{3}{2\left(\frac{3L_0}{2}\right)}\sqrt{\frac{T_2}{2\mu}} \qquad [f_0 = f_2]$

Or, $\frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} = \frac{1}{L_0}\sqrt{\frac{T_2}{2\mu}}$ or, $\sqrt{\frac{T_0}{2}} = \sqrt{T_2}$ or, $\boxed{T_2 = \frac{T_0}{2}} \Rightarrow (Q)$

For string 3rd: - It is vibrating in 5th harmonic and its length is $\frac{5L_0}{4}$

$f_3 = \frac{5}{2\left(\frac{5L_0}{4}\right)}\sqrt{\frac{T_3}{3\mu}} \qquad [\because f_0 = f_3]$

Or, $\frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}=\frac{2}{L_0}\sqrt{\frac{T_3}{3\mu}}$ or, $\sqrt{T_0}=4\sqrt{\frac{T_3}{3}}$

Or, $\boxed{T_3=\frac{3T_0}{16}}\Rightarrow(T)$

For string 4^{th} :- It is vibrating in 14^{th} harmonic and its length is $\frac{7L_0}{4}$.

$$f_4=\frac{14}{2\left(\frac{7L_0}{4}\right)}\sqrt{\frac{T_4}{4\mu}} \qquad [\because f_4=f_0]$$

Or, $\frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}=\frac{4}{L_0}\sqrt{\frac{T_4}{4\mu}}$ or, $\frac{1}{2}\sqrt{T_0}=\frac{4}{2}\sqrt{T_4}$

Or, $\boxed{T_4=\frac{T_0}{16}}$ $\Rightarrow(U)$

(i) – P; (ii) – Q; (iii) – (T) ; (IV) – U

Hence, (3) option is correct

2. If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

(1) $I\rightarrow Q,\ II\rightarrow S,\ III\rightarrow R,\ IV\rightarrow P$

(2) $I\rightarrow P,\ II\rightarrow Q,\ III\rightarrow T,\ IV\rightarrow S$

(3) $I\rightarrow Q,\ II\rightarrow P,\ III\rightarrow R,\ IV\rightarrow T$

(4) $I\rightarrow P,\ II\rightarrow R,\ III\rightarrow S,\ IV\rightarrow Q$

Ans. (4)

Sol. For fundamental mode, $f_0=\frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

(i) For string 1^{st} : - $f_1=f_0\Rightarrow(P)$

(ii) For string 2^{nd} : - $f_2=\frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu_0}}=\frac{1}{\sqrt{2}}\times\frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}=\frac{f_0}{\sqrt{2}}\Rightarrow(R)$

(iii) For string 3^{rd} : $f_3=\frac{1}{2L_0}\sqrt{\frac{T_0}{3\mu_0}}=\frac{1}{\sqrt{3}}\times\frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}=\frac{f_0}{\sqrt{3}}\Rightarrow(S)$

(iv) For string 4^{th} : $f_4=\frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu_0}}=\frac{1}{2}\times\frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}=\frac{f_0}{2}\Rightarrow(Q)$

(i) – P; (ii) – (R); (iii) – (S); (iv) – (Q)

Hence, (4) option is correct

Answer the following by appropriately matching the lists based on the information given in the paragraph

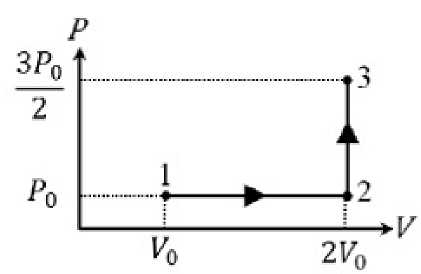
In a thermodynamics process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{R}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$.

Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I		**List-II**	
(I)	**Work done by the system in process** $1 \to 2 \to 3$	**(P)**	$\frac{1}{3}RT_0 \ln 2$
(II)	Change in internal by the system in process $1 \to 2 \to 3$	**(Q)**	$\frac{1}{3}RT_0$
(III)	Heat absorbed by the system in process $1 \to 2 \to 3$	**(R)**	RT_0
(IV)	Heat absorbed by the system in process $1 \to 2$	**(S)**	$\frac{4}{3}RT_0$
		(T)	$\frac{1}{3}RT_0(3 + \ln 2)$
		(U)	$\frac{5}{6}RT_0$

3. If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(1) $I \to S,\ II \to R,\ III \to Q,\ IV \to T$

(2) $I \to Q,\ II \to S,\ III \to R,\ IV \to U$

(3) $I \to Q,\ II \to R,\ III \to S,\ IV \to U$

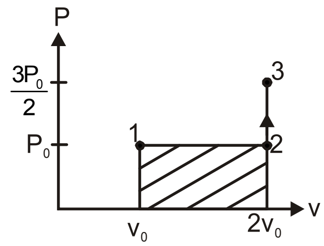
(4) $I \to Q,\ II \to R,\ III \to P,\ IV \to U$

Ans. (3)

Sol.

Method - I :

Given in question, gas is an ideal monatomic gas.



$\left[P_0V_0 = \frac{1}{3}RT_0\right]$

(i) Work done by the system in process $1 \to 2 \to 3$

= Area under P-V graph

$= P_0V_0 = \frac{1}{3}RT_0 \Rightarrow (Q)$

(ii) Change in internal energy in process $1 \to 2 \to 3$

$= \frac{3}{2}R(T_3 - T_1)$ [For 1 mole gas]

$= \frac{3}{2}R\left[\frac{3P_0V_0}{R} - \frac{P_0V_0}{R}\right] = \frac{3}{2} \times 2P_0V_0 = 3P_0V_0 = RT_0 \Rightarrow (R)$

(iii) Heat absorbed by the system in process $1 \to 2 \to 3$

= work done by the system in process $1 \to 2 \to 3$ + change in internal energy of gas in process $1 \to 2 \to 3$

$= \frac{1}{3}RT_0 + RT_0 = \frac{4RT_0}{3} \Rightarrow (S)$

(iv) Heat absorbed by system in process $1 \to 2 = W_{12} + \Delta V_{12}$

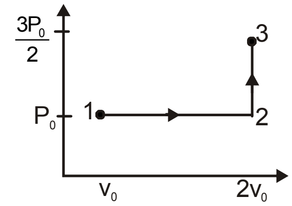
$= \frac{1}{3}RT_0 + \frac{3}{2}R\left[\frac{2P_0V_0}{R} - \frac{P_0V_0}{R}\right] = \frac{RT_0}{3} + \frac{3P_0V_0}{2}$

$= \frac{RT_0}{3} + \frac{RT_0}{2} = \frac{5RT_0}{6} \Rightarrow (U)$

(i) – Q; (ii) – R; (iii) – S; (iv) – (U)

Hence, (3) option is correct

Method - II :



$$\boxed{P_0V_0 = \frac{1}{3}RT_0}$$

$$T_1 = \frac{P_0V_0}{R} = \frac{T_0}{3}$$

$$T_2 = \frac{2P_0V_0}{R} = \frac{2T_0}{3}$$

$$T_3 = \frac{3P_0V_0}{R} = T_0$$

$$x = \frac{3}{2}R\ell n\left(\frac{T}{T_0}\right) + R\ell n\left(\frac{V}{V_0}\right)$$

$$= \frac{3}{2}R\left[\ell nT - \ln T_0\right] + R\left[\ell nV - \ln V_0\right]$$

$$\boxed{dx = \frac{3}{2}R\frac{dT}{T} + \frac{R}{V}dv} \qquad \text{-----(i)}$$

$$\because dQ = Tdx = \frac{3}{2}RdT + \frac{RT}{V}dv$$

Or, $dQ = \frac{3}{2}R\int dT + \int Pdv$ (For monatomic ideal gas)

$\boxed{dQ = \Delta U + W}$ (First law of thermodynamic)

Now, $\int dQ = \frac{3}{2}R\int_{T_1}^{T_3} dT + \int_{V_1}^{V_3=V_2} Pdv$

$$\boxed{Q_{123} = \frac{3}{2}R\left(T_3 - T_1\right) + W_{1\to2\to3}}$$

$$= \frac{3}{2}R\left[T_0 - \frac{T_0}{3}\right] + P_0V_0$$

$$= \frac{3}{2}R \times \frac{2T_0}{3} + P_0V_0 = RT_0 + \frac{RT_0}{3}$$

Or, $\boxed{Q_{123} = \frac{4RT_0}{3}}$ $(iii) \rightarrow (S)$

$$Q_{12} = \frac{3}{2}R[T_2 - T_1] + W_{12}$$

$$= \frac{3}{2}R\left[\frac{2T_0}{3} - \frac{T_0}{3}\right] + P_0V_0 = \frac{3}{2}R \times \frac{T_0}{3} + \frac{RT_0}{3}$$

$$= \frac{RT_0}{2} + \frac{RT_0}{3} = \frac{5RT_0}{6}$$

$(iv) \rightarrow (U)$

$$Q_{23} = \frac{3}{2}R(T_3 - T_2) + W_{2\to3}$$

$$= \frac{3}{2}R\left[T_0 - \frac{2T_0}{3}\right] + 0$$

$$Q_{23} = \frac{RT_0}{2}$$

From 1st law of thermodynamics

$$\Delta U_{23} = Q_{23} - W_{23} = \frac{RT_0}{2} - 0 = \frac{RT_0}{2}$$

$$\Delta U_{12} = Q_{12} - W_{12} = \frac{5RT_0}{6} - P_0V_0 = \frac{5RT_0}{6} - \frac{RT_0}{3}$$

$$= \frac{3RT_0}{6} = \frac{RT_0}{2}$$

$$\therefore \quad \Delta U_{1\to2\to3} = \Delta U_{12} + \Delta U_{23} = \frac{RT_0}{2} + \frac{RT_0}{2} = RT_0$$

$(ii) \rightarrow (R)$

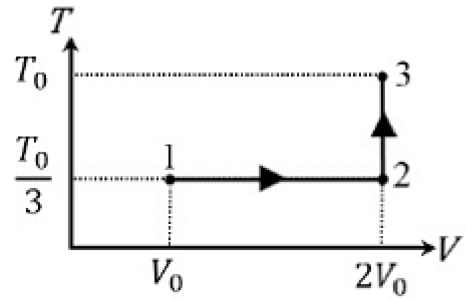
$W_{123} =$ Area under the curve P-V

$= P_0V_0 = \frac{RT_0}{3}$ $\therefore (i) \rightarrow (Q)$

(i) $\rightarrow$(Q); (ii) $\rightarrow$ R; (iii) $\rightarrow$ (S); (iv) $\rightarrow$ (U)

Hence, (3) option is correct

4. If the process on one mole of monatomic ideal gas is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(1) $I \to P,\ II \to T,\ III \to Q,\ IV \to T$

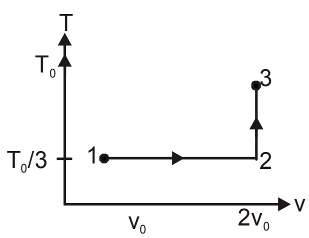
(2) $I \to P,\ II \to R,\ III \to T,\ IV \to S$

(3) $I \to S,\ II \to T,\ III \to Q,\ IV \to U$

(4) $I \to P,\ II \to R,\ III \to T,\ IV \to P$

Ans. (4)

Sol.



(i) Process $1 \to 2$ is isothermal and process $2 \to 3$ is isochoric

Work done by the system in process $1 \to 2 \to 3$

$$= W_{12} + W_{23} = nRT_{12}\ell n\left(\frac{V_2}{V_1}\right) + 0$$

$$= R\frac{T_0}{3}\ell n(2) + 0 = \frac{RT_0}{3}\ell n(2) \Rightarrow (P)$$

(ii) Change in internal energy in process $1 \to 2 \to 3$

$$= \Delta U_{12} + \Delta U_{23} = 0 + \frac{3}{2}nR[T_3 - T_2]$$

$$= 0 + \frac{3}{2} \times 1 \times R\left[T_0 - \frac{T_0}{3}\right] = RT_0 \Rightarrow (R)$$

(iii) Heat absorbed by the system in process $1 \to 2 \to 3$

$= W_{1\to2\to3} + \Delta U_{1\to2\to3} = \frac{RT_0}{3}\ell n(2) + RT_0$

$= \frac{RT_0}{3}(3 + \ell n) \Rightarrow (T)$

(iv) $Q_{1\to2} = W_{1\to2} + \Delta U_{1\to2}$

$= \frac{RT_0}{3}\ell n(2) + 0 = \frac{RT_0}{3}\ell n(2) \Rightarrow (P)$

(i) – P; (ii) – R; (iii) – T; (IV) – (P)

Hence, (4) option is correct