

JEE (ADVANCED) 2024 PAPER-1

[PAPER WITH SOLUTION]

HELD ON SUNDAY 26TH MAY 2024

PHYSICS

SECTION 1 (Maximum Marks :12)

- This section contains FOUR (04) questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated <u>according to the following marking scheme</u>:
 - Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered); Negative Marks : -1 In all other cases.

[:Q.1] A dimensionless quantity is constructed in terms of electronic charge *e*, permittivity of free

space ε_0 , Planck's constant *h*, and speed of light *c*. If the dimensionless quantity is written as

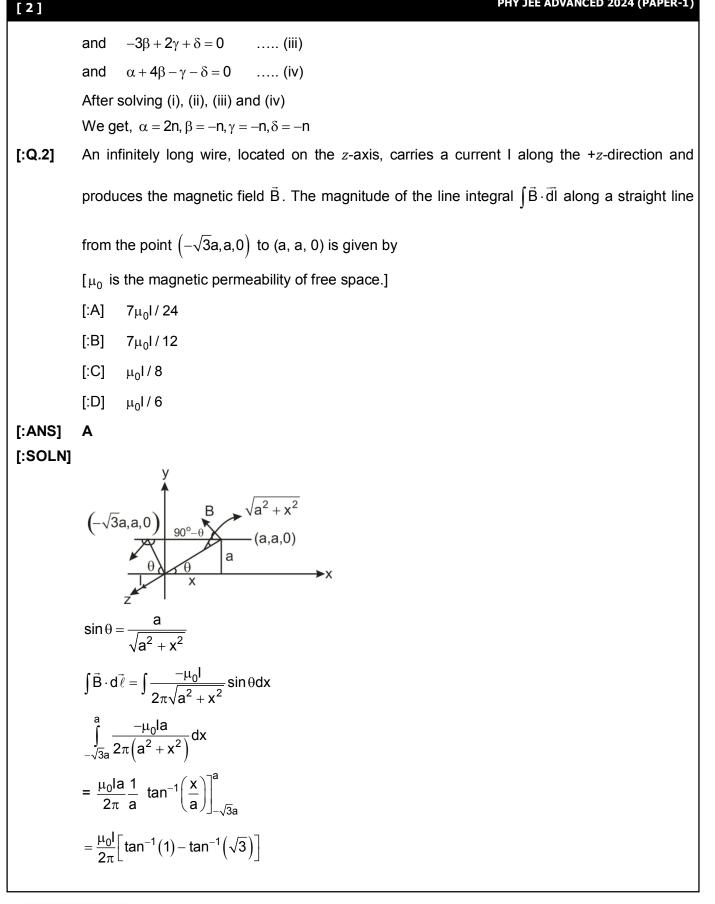
 $e^{\alpha} \varepsilon_0{}^{\beta} h^{\gamma} c^{\delta}$ and n is a non-zero integer, then $(\alpha, \beta, \gamma, \delta)$ is given by

[:A] (2n - n, -n, -n)

- [:B] (n,-n,-2n,-n)
- [:C] (n,-n,-n,-2n)
- [:D] (2n, -n, -2n, -2n)

$$\begin{split} \textbf{[:SOLN]} & \left[e^{\alpha} \epsilon_{o}^{\beta} h^{\gamma} c^{\delta} \right] \\ & M^{0} L^{0} T^{0} A^{0} = \left[A T \right]^{\alpha} \left[M^{-1} L^{-3} T^{4} A^{2} \right]^{\beta} \left[M L^{2} T^{-1} \right]^{\gamma} \left[L T^{-1} \right]^{\delta} \\ & \Rightarrow \quad -\beta + \gamma = 0 \quad \dots (i) \quad \Rightarrow \beta = \gamma \\ & \text{and} \quad \alpha + 2\beta = 0 \quad \dots (ii) \end{split}$$





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$$=\frac{-\mu_0 l}{2\pi}\left(\frac{\pi}{4}+\frac{\pi}{3}\right)=\frac{-7\mu_0 l}{24}, \text{ so the magnutide is } \frac{7\mu_0 l}{24}$$

[:Q.3] Two beads, each with charge q and mass m, are on a horizontal, frictionless, non- conducting, circular hoop of radius R. One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by

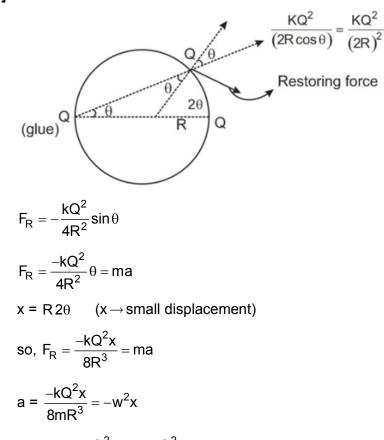
[ϵ_0 is the permittivity of free space.]

[:A] q² /
$$(4\pi\epsilon_0 R^3 m)$$

- [:B] $q^2 / (32\pi\epsilon_0 R^3 m)$
- [:C] $q^2 / (8\pi\epsilon_0 R^3 m)$
- $[:D] \qquad q^2 \, / \left(16 \pi \epsilon_0 R^3 m\right)$



[:SOLN]



so
$$w^2 = \frac{kQ^2}{8mR^3} = \frac{Q^2}{32\pi\epsilon_0 mR^3}$$

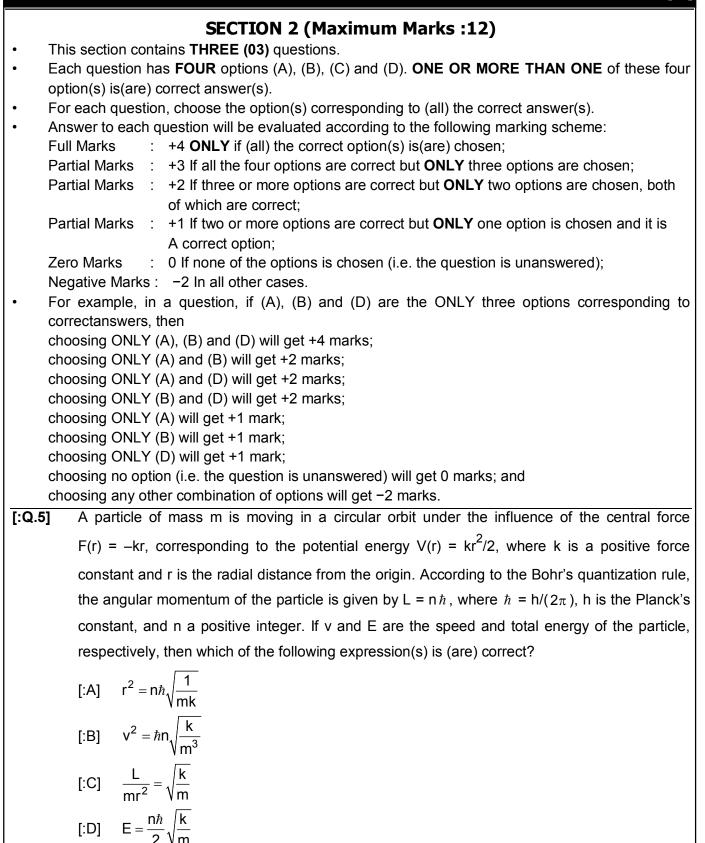
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[:Q.4]						
	the value of x in metre. At time t = 0 s, it is at rest at position $x = 1$ m. The position and					
	momentum of the block at t = $(\pi / 4)$ s are					
	[:A] –0.5 m, 5 kg m/s					
	[:B] 0.5 m, 0 kg m/s					
	[:C] 0.5 m, –5 kg m/s					
	[:D] –1 m, 5 kg m/s					
[:ANS]	C					
[:SOLN]	F = -20x + 10					
	$a = \frac{f}{m} = -4x + 2 \qquad \begin{cases} given \\ u = 0, \text{ at } t = 0 \text{ and } x = 1m \end{cases}$					
	$\frac{vdv}{dx} = -4x + 2$					
	$\int_{0}^{v} v dv = \int_{1}^{x} (-4x+2) dx$					
	$\frac{v^2}{2} = \left[-2x^2 + 2x\right]_1^x = -2x^2 + 2x - (-2 + 2)$					
	$\Rightarrow v^2 = -4x^2 + 4x \Rightarrow v = 2\sqrt{x = x^2}$					
	So, $v = \frac{dx}{dt} = 2\sqrt{x - x^2}$					
	$\int_{1}^{x} \frac{dx}{\sqrt{x - x^{2}}} = \int_{0}^{\pi/4} 2dt$					
	$\Rightarrow \sin^{-1}(2x-1)\Big _{1}^{x} = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$					
	$\Rightarrow \sin^{-1}(2x-1) - \sin^{-1}(1) = \frac{\pi}{2}$					
	$\sin^{-1}(2x-1) = \pi$					
	2x - 1 = 0					
	X = 0.5 m					
	x = 0.5 $+$ $+$ $+$ $+$ $+$ $+$ $+$ $+$ $+$ $+$					
	v will be negative					
	$v = -2\sqrt{0.5 - (0.5)^2} = -1 \text{ m/s}$					
	So/ $P = mv = -5kg m/s$					

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[4]





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[:ANS]	A, B, C				
[:SOLN]	f(r) = -kr				
	$v(r) = \frac{kr^2}{2}$				
	$L=L=\frac{nh}{2\pi}=mvr\qquad \dots \dots (i) \Rightarrow v=\left(\frac{n\hbar}{2\pi mr}\right)=\frac{n\hbar}{mr}$				
	So, $ f(r) = \frac{mv^2}{r}$ for circular motion				
	$\Rightarrow kr = \frac{mv^2}{r}$				
	$v^2 = \frac{kv^2}{m}$ (ii) $\Rightarrow \frac{v}{r} = \sqrt{\frac{k}{m}}$				
	$\left(\frac{n\hbar}{mr}\right)^{2} = \frac{kr^{2}}{m} \Rightarrow r^{4} = \frac{\left(n\hbar\right)^{2}}{mk} \Rightarrow r^{2} = n\hbar\sqrt{\frac{1}{mk}} \qquad \text{option (A)}$				
	$v^2 = \frac{k}{m}r^2 = \frac{k}{m} n\hbar \sqrt{\frac{1}{mk}} = n\hbar \sqrt{\frac{k}{m^3}}$ option (B)				
	$\frac{L}{mr^2} = \frac{mvr}{mr^2} = \frac{v}{r} = \sqrt{\frac{k}{m}}$ option (C)				
	$E = \frac{1}{2}mv^2 + v\left(r\right) = \frac{1}{2}m\left[n\hbar\sqrt{\frac{k}{m^3}}\right] + \frac{kr^2}{2}$				
	$=\frac{n\hbar}{2}\sqrt{\frac{k}{m}}+\frac{k}{2} \qquad n\hbar\sqrt{\frac{1}{mk}}=n\hbar\sqrt{\frac{k}{m}} \qquad \text{option (D)}$				
[:Q.6]	Two uniform strings of mass per unit length μ and $4\mu,$ and length L and 2L, respectively, are				
	joined at point O, and tied at two fixed ends P and Q, as shown in the figure. The strings are				
	under a uniform tension T. If we define the frequency $v_0 = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$, which of the followin				
	statement(s) is (are) correct?				
	$P \xrightarrow{\mu 0 4\mu} Q$				
	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $				

[:A] With a node at O, the minimum frequency of vibration of the composite string is v_0 .

[:B] With an antinode at O, the minimum frequency of vibration of the composite string is $2v_0$.



[6]

- [:C] When the composite string vibrates at the minimum frequency with a node at O, it has 6 nodes, including the end nodes.
- [:D] No vibrational mode with an antinode at O is possible for the composite string.

[:ANS] A, C, D

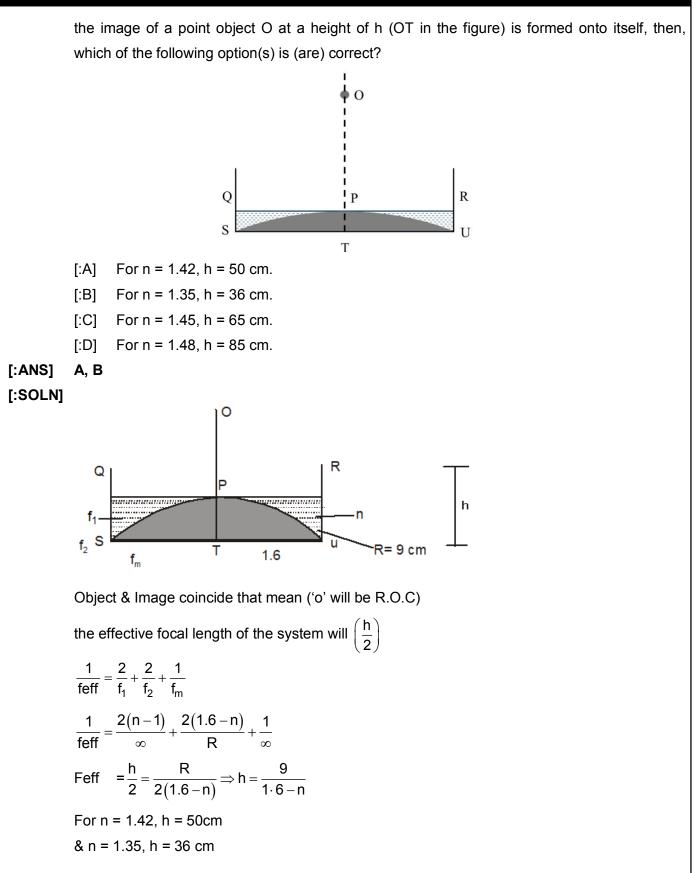
[:SOLN]

Option (A) for min. frequency . for node at O

Suppose 'n' loops in left string and 'm' loops in right string

[:Q.7] A glass beaker has a solid, plano-convex base of refractive index 1.60, as shown in the figure. The radius of curvature of the convex surface (SPU) is 9 cm, while the planar surface (STU) acts as a mirror. This beaker is filled with a liquid of refractive index n up to the level QPR. If







Option (A) and (B).

SECTION 3 (Maximum Marks :24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated <u>according to the following marking scheme</u>: Full Marks : +4 If **ONLY** the correct integer is entered; Zero Marks : 0 In all other cases.

[:Q.8] The specific heat capacity of a substance is temperature dependent and is given by the formula C = kT, where k is a constant of suitable dimensions in SI units, and T is the absolute temperature. If the heat required to raise the temperature of 1 kg of the substance from -73° C to 27° C is nk, the value of n is _____. [Given : 0 K = -273° C]

[:ANS] 25000

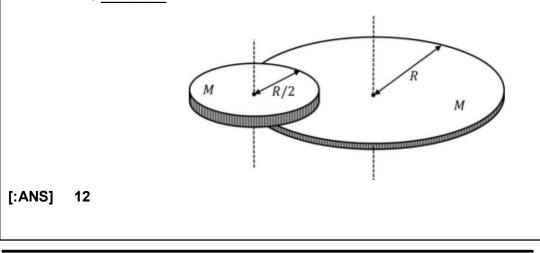
[:SOLN] C = KT = Specific heat \rightarrow m = 1kg

$$\Delta Q = \int msdT = m \int_{200k}^{300k} kTdT \qquad -73^{\circ}C = 200k$$
$$27^{\circ}C = 300k$$

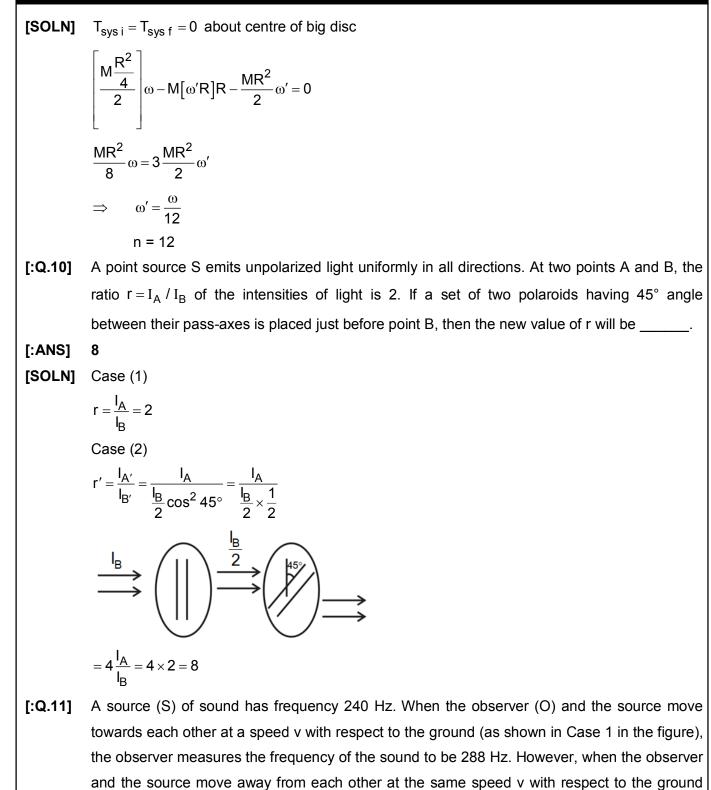
$$\Delta Q = 1 \times \frac{k}{2} \left[T^2 \right]_{200}^{300} = \frac{k}{2} \left[9 - 4 \right] \times 10^4 = 2.5 \times 10^4 \times k$$

∆Q = 2500 k, n = 25000

[:Q.9] A disc of mass M and radius R is free to rotate about its vertical axis as shown in the figure. A battery operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the same mass R and radius R/2 is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed ω . If the angular speed at which the large disc rotates is ω/n , then the value of n is



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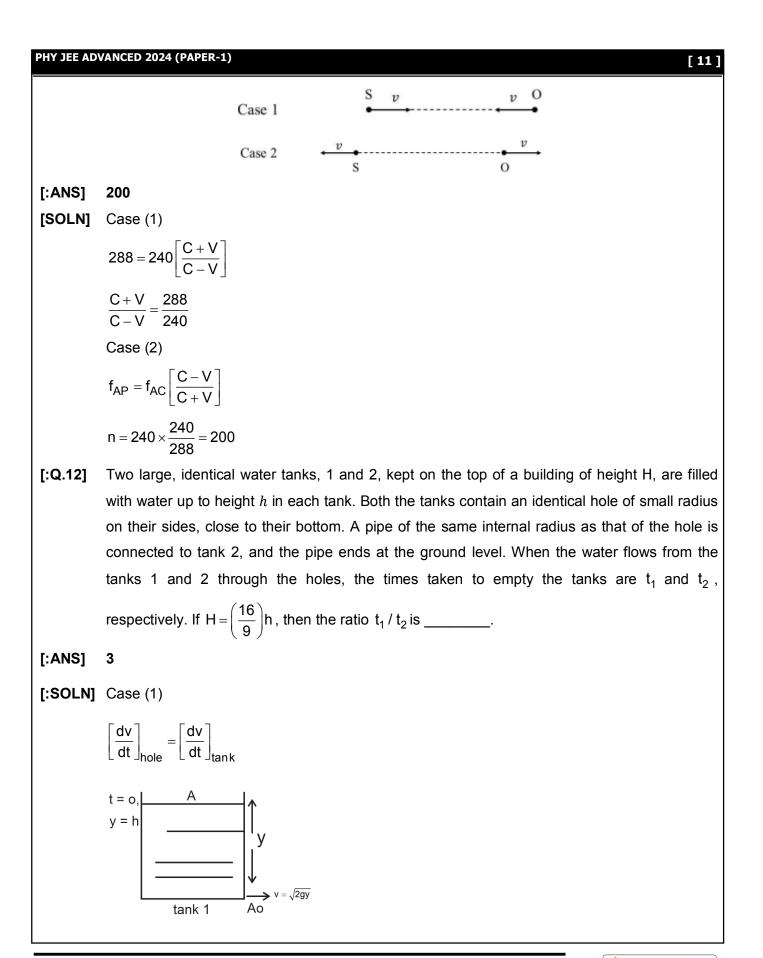


The value of n is _____.

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(as shown in Case 2 in the figure), the observer measures the frequency of sound to be n Hz.

[10]





$$A_{0}\sqrt{2gy} = A \times \left(-\frac{dy}{dt}\right)$$

$$t_{1} = -\frac{A}{A_{0}} \frac{1}{\sqrt{2g}} \int_{y=h}^{y=0} \frac{dy}{\sqrt{y}} = \frac{A}{A_{0}} \sqrt{\frac{2h}{g}}$$
Case (2)
$$\left[\frac{dv}{dt}\right]_{hole} = \left[\frac{dv}{dt}\right]_{tomk}$$

$$A_{0}\sqrt{2g\left(y + \frac{16h}{9}\right)} = A \times \left(-\frac{dy}{dt}\right)$$

$$t_{2} = -\frac{A}{A_{0}\sqrt{2g}} \int_{y=h}^{y=0} \frac{dy}{\sqrt{y + \frac{16h}{9}}}$$

$$= -\frac{A}{A_{0}} \frac{1}{\sqrt{2g}} 2\left[\sqrt{y + \frac{16h}{9}}\right]_{y=h}^{y=0}$$

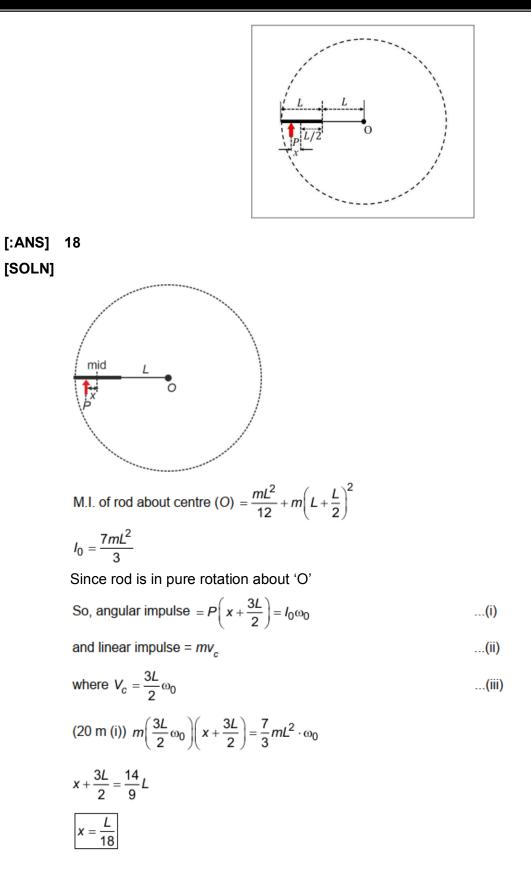
$$= \frac{1}{3} \frac{A}{A_{0}} \sqrt{\frac{2h}{g}}$$
Then $\frac{t_{1}}{t_{2}} = \frac{1}{(1/3)} = 3$

[:Q.13] A thin uniform rod of length L and certain mass is kept on a frictionless horizontal table with a massless string of length L fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point O. If a horizontal impulse P is imparted to the rod at a distance x = L/n from the mid-point of the rod (see figure), then the rod and string revolve together around the point O, with the rod remaining aligned with the string. In such a case, the value of n is ______.

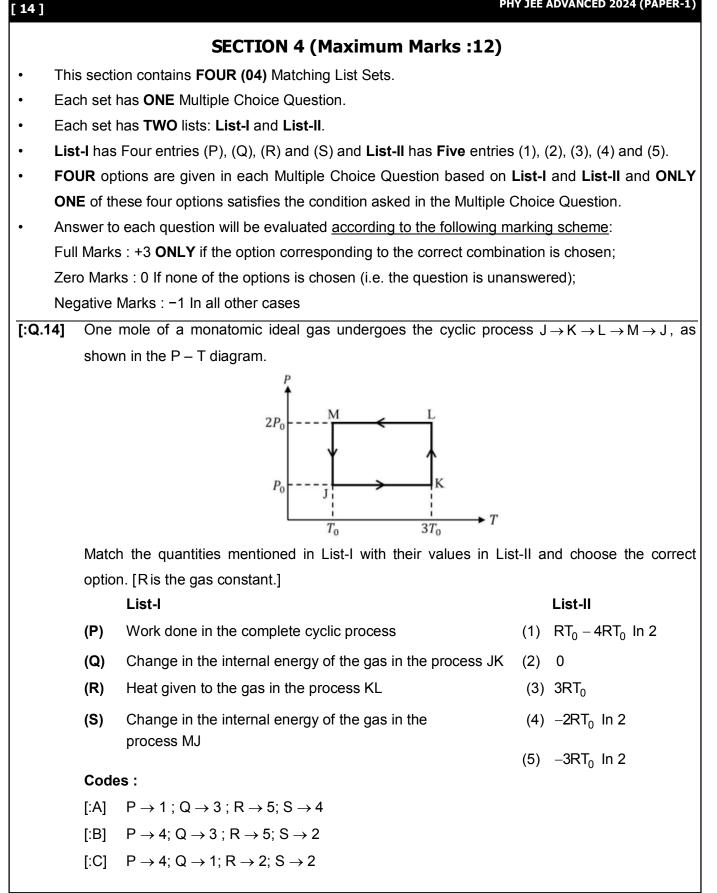


[12]





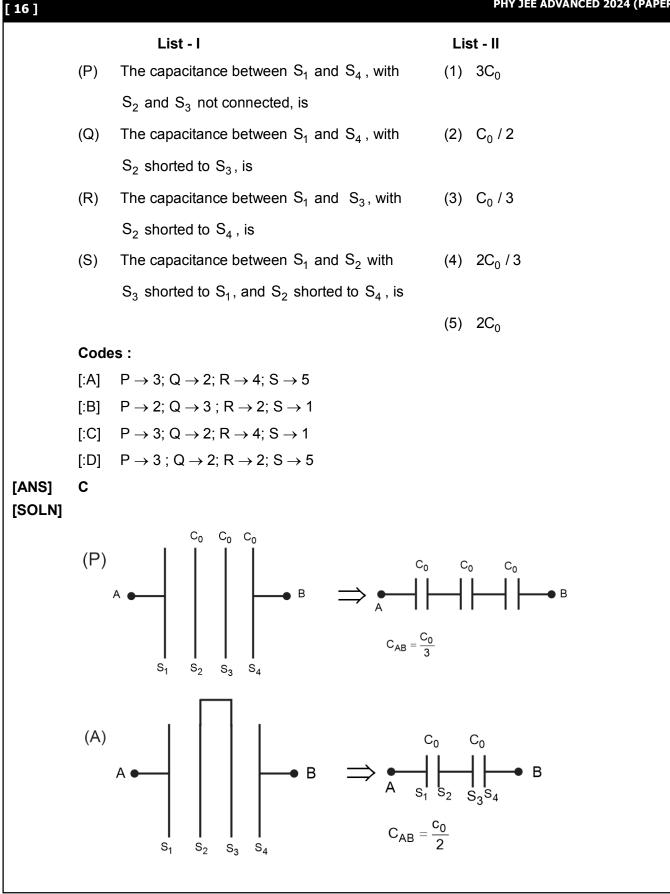




Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

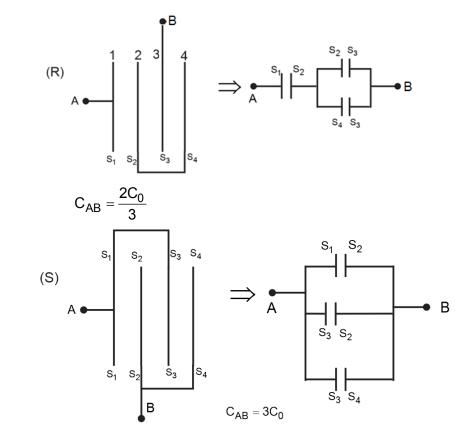


[15]



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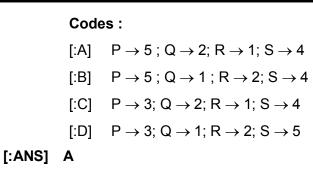
[:Q.16] A light ray is incident on the surface of a sphere of refractive index n at an angle of incidence θ_0 . The ray partially refracts into the sphere with angle of refraction ϕ_0 and then partly reflects from the back surface. The reflected ray then emerges out of the sphere after a partial refraction. The total angle of deviation of the emergent ray with respect to the incident ray is α . Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List - I		List - II
(P)	If n = 2 and $\alpha = 180^{\circ}$, then all the possible values	(1)	30° and 0°
	of θ_0 will be		
(Q)	If $n = \sqrt{3}$ and $\alpha = 180^{\circ}$, then all the possible values of	(2)	60° and 0°
	θ_0 will be		
(R)	If $n = \sqrt{3}$ and $\alpha = 180^{\circ}$, then all the possible	(3)	45° and 0°
	values of φ_0 will be		
(S)	If $n = \sqrt{2}$ and $\theta_0 = 45^\circ$, then all the possible values of	(4)	150°
	α will be		
		(5)	0°

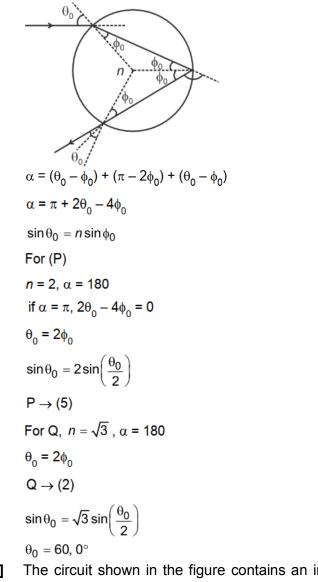
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[17]

[18]



[:SOLN]



[:Q.17] The circuit shown in the figure contains an inductor L, a capacitor C_0 , a resistor R_0 and an ideal battery. The circuit also contains two keys K_1 and K_2 . Initially, both the keys are open and there is no charge on the capacitor. At an instant, key K_1 is closed and immediately after

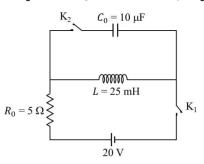
...(i)

...(ii)

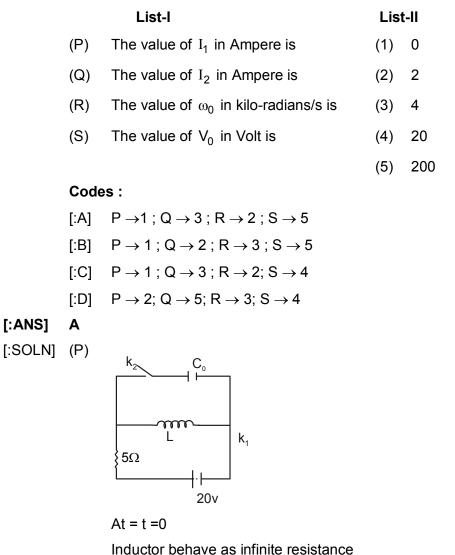


[:ANS]

this the current in R_0 is found to be I_1 . After a long time, the current attains a steady state value I_2 . Thereafter, K_2 is closed and simultaneously K_1 is opened and the voltage across C_0 oscillates with amplitude V_0 and angular frequency ω_0 .



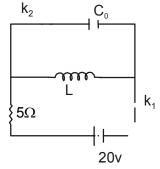
Match the quantities mentioned in List-I with their values in List-II and choose the correct option.





(Q) After long time, inductor behaves as zero resistance,

$$I_{max} = \frac{20}{5} = 4A$$



When K_1 is open and K_2 is closed

The circuit will be LC – oscillation with angular frequency.

$$w_{0} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25 \times 10^{-3} \times 10 \times 10^{-6}}}$$
$$= \frac{1}{\sqrt{25 \times 10^{-8}}} = \frac{10^{4}}{5} \text{ rad/s} = 2000 \text{ rad/s}$$
$$= 2 \text{ kilo-rad/s}$$

(S)
$$\frac{1}{2}LI_0^2 = \frac{1}{2}CV_0^2$$

 $V_0 = \sqrt{\frac{L}{C}}$ $I_0 = \sqrt{\frac{25 \times 10^{-3}}{10 \times 10^{-6}}} \times 4 = 200$ Volt.

