Mentors Eduserv

INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN ASTRONOMY (NSEA)2024

(QUESTION PAPER CODE 42)

Date : 23.11.2024 **Time : 120 Minute** **Maximum Marks: 216**

Writethequestionpapercode(mentionedabove)onYOUR OMRAnswerSheet (inthespaceprovided), otherwise yourAnswerSheetwillNOTbeevaluated,NotethatthesameQuestionpapercodeappears oneachpageofthequestionpaper.

INSTRUCTIONS

1. Use of mobile phone, smart watches, and iPad during examination is STRICTLY **PROHIBITED.**

2. In addition to this question paper, you are given OMR Answer Sheet along with candidate's copy.

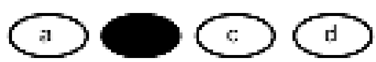
3. On the OMR sheet, make all the entries carefully in the space provided ONLY in BLOCK CAPITALS as well as by properly darkening the appropriate bubbles.
Incomplete/ incorrect/ carelessly filled information may disqualify your candidature.

4. On the OMR Answer sheet, use only **BLUE or BLACK BALL POINT PEN** for making entries and filling bubbles.

5. Your **Ten-digit roll number and date of birth** entered in the OMR Answer sheet shall remain your login credentials (means login id and password respectively) for accessing your performance/result in National Standard Examination in Astronomy – 2023.

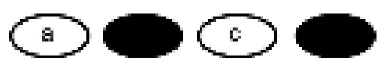
6. Question paper has two parts. In part A1 (**Q. No.1 to 48**) each question has four alternatives, out of which only one is correct. Choose the correct alternative (s) and fill the appropriate bubbles(s), as shown.

Q.No.22



In part A2 (**Q. No. 49 to 60**) each question has four alternative out of which any number of alternative (s) (1, 2, 3, or 4) may be correct. You have to choose all correct alternative(s) and fill the appropriate bubbles(s), as shown

Q.No.54



7. For **Part A1,** each correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer. In **Part A2,** you get 6 marks. If all the correct alternative are marked. No negative marks in this part.

8. Rough work should be done only in the space provided. There are __printed pages in this paper.

9. Use of **non-programmable scientific** calculator is allowed

10. No candidate should leave the examination hall before the completion of the examination.

11. After submitting answer paper, take away the question paper & candidate's copy of OMR for your reference
Please DO NOT make any mark other than filling the appropriate bubbles properly in the space provided on the OMR answer sheet.
OMR answer sheets are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED, Scratching or overwriting may result in wrong score.
DO NOT WRITE ON THE BACK SIDE OF THE OMR ANSWER SHEET.

Name of Student :...

Batch : ....

Enrolment No. | | | | | | | | | | | | | | | | |

Mentors Eduserv : Plot No. : 136/137, ParusLok Complex, Boring Road Crossing, Patna-1, Ph. No. : 0612-3223681/2 | 7544015993/6/7 | 7070999604/5

INDIAN ASSOCIATION OF PHYSICS TEACHERS

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PAPER CODE -42

Date of Examination – 23.11.2024

SOLUTIONS

Attempt All Sixty Questions

(NSEA) PART : A–1

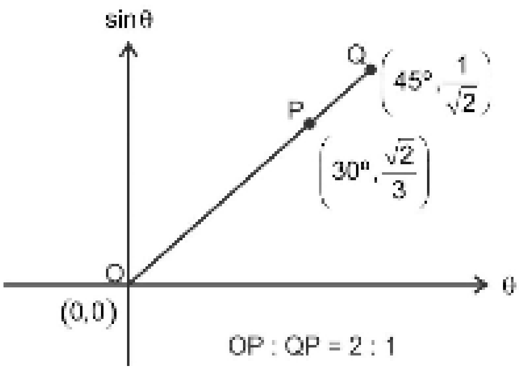
ONLY ONE OUT OF FOUR OPTIONS IS CORRECT, BUBLE THE CORRECT OPTION.

[:Q.1] The percentage error in determining sin 30º by assuming that between 0º and 45º the sine function may be approximated by a straight line is

[:A] 9.3% [:B] 7.5% [:C] 5.7% [:D] 3.6%

[:ANS] **C**

[:SOLN]



$$\% \text{ error} = \frac{\left(\frac{1}{2} - \frac{\sqrt{2}}{3}\right)}{\left(\frac{1}{2}\right)} \times 100\%$$

$\approx 5.7\%$

C is correct

[:Q.2] The relation R on N, the set of natural numbers, defined $(x, y) \in R$ if and only if xy is a perfect square is

[:A] reflexive, symmetric but not transitive [:B] reflexive, symmetric and transitive

[:C] symmetric but neither reflexive nor transitive [:D] symmetric, transitive but not reflexive

[:ANS] **B**

[:SOLN] x.x is perfect square $\forall x \in N$

$\therefore$ R is reflexive

xy is perfect square

$\Rightarrow$ yx is perfect square $\forall x, y \in N$

$\therefore$ R is symmetric

xy and yz are perfect squares

$\Rightarrow$ xy .yz is perfect square

$\Rightarrow$ xz is perfect square

$\therefore$ R is transitive.

[:Q.3] If A is a 10 × 10 matrix with determinant 2, what is the determinant of 2A ?

[:A] 4 [:B] 512 [:C] 1024 [:D] 2048

[:ANS] **D**

[:SOLN] $|2A| = 2^{10}|A| = 2^{10} \times 2 = 2048$

[:Q.4] Consider the following system of linear equations:

$kx + y + z = 2023$

$k^2x + ky + z = 2024$

$k^3x + k^2y + z = 2025$

Which of the following is true ?

[:A] The system has a unique solution for k = 1

[:B] The system has a unique solution for any value of k

[:C] The system is always inconsistent for any value of k

[:D] The system has infinitely many solutions for k = 1

[:ANS] **C**

[:SOLN] $\Delta = \begin{vmatrix} k & 1 & 1 \\ k^2 & k & 1 \\ k^3 & k^2 & 1 \end{vmatrix} = k\begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ k^2 & k^2 & 1 \end{vmatrix} = 0$

$\Delta_1 = \begin{vmatrix} 2023 & 1 & 1 \\ 2024 & k & 1 \\ 2025 & k^2 & 1 \end{vmatrix}$

$= \begin{vmatrix} 2023 & 1 & 1 \\ 1 & k-1 & 0 \\ 1 & k^2-k & 0 \end{vmatrix}$ $(R_2 \to R_2 - R_1, R_3 \to R_3 - R_2)$

$= k^2 - k - k + 1 = (k-1)^2$

$\Delta_1 = 0 \Rightarrow k = 1$

but for k = 1, system is inconsistent.

so it is always inconsistent.

[:Q.5] The number of real values of x in the closed interval $[0, 2024\pi]$ that satisfy the equation $\sin^2 x + \sin x - 2 = 0$ is

[:A] 0 [:B] 1 [:C] 1012 [:D] 2024

[:ANS] **C**

[:SOLN] $\sin^2 x + \sin x - 2 = 0$

$\Rightarrow$ $(\sin x + 2)(\sin x - 1) = 0$

$\Rightarrow$ $\sin x = 1 \Rightarrow x = 2n\pi + \dfrac{\pi}{2}$

But $x \in [0, 2024\pi]$

$\therefore$ solution set is $\left\{\dfrac{\pi}{2}, 2\pi + \dfrac{\pi}{2}, \ldots\ldots., 2022\pi + \dfrac{\pi}{2}\right\}$

so total 1012 solutions

[:Q.6] What is the centre and radius of the circle represented by $\left|\dfrac{z+1}{z-2}\right| = 2$?

[:A] Centre (–1, 2), Radius 2
[:B] Centre (1, –2), Radius 2
[:C] Centre (0, 3), Radius 2
[:D] Centre (3, 0), Radius 2

[:ANS] **D**

[:SOLN] $\left|\dfrac{z+1}{z-1}\right| = 2$

Taking $z = x + iy$

The equation reduces to $x^2 + y^2 - 6x + 5 = 0$.

[:Q.7] In how many ways can 10 identical balls be distributed among three children ?

[:A] 30
[:B] 66
[:C] 120
[:D] 156

[:ANS] **B**

[:SOLN] Number of non-negative integral solutions of $x + y + z = 10$ is 66.

[:Q.8] Consider the function $f(x) = \begin{cases} 1 + 2x + 3x^2, & x \geq 2024 \\ 3 + 2x + x^2, & x < 2024 \end{cases}$ Which of the following is **not** correct ?

[:A] f is discontinuous at x = 2024
[:B] f has removable discontinuity at x = 2024
[:C] f is continuous at c where c < 2024
[:D] f is differentiable at c where c > 2024

[:ANS] **B**

[:SOLN] $\lim\limits_{x \to 2024} f(x) = 4100627$

$\lim\limits_{x \to 2024^+} f(x) = 12293777$

$f(2024) = 12293777$

[:Q.9] What is the value of the following limit ? $\lim_{n\to\infty}\left(\frac{1}{n^3+1}+\frac{4}{n^3+8}+\frac{9}{n^3+27}+......+\frac{1}{2n}\right)$

[:A] 0 [:B] $\ell n2$ [:C] $\ell n\, 2^3$ [:D] $\ell n\, 2^{\frac{1}{3}}$

[:ANS] **D**

[:SOLN] $L = \lim_{n\to\infty}\left(\frac{1}{n^3+1}+\frac{4}{n^3+8}+\frac{9}{n^3+27}+....+\frac{1}{2n}\right)$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{r^2}{n^3+r^3}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{\left(\frac{r}{n}\right)^2}{1+\left(\frac{r}{n}\right)^3}\right)\cdot\frac{1}{n}$$

$$=\int_0^1\frac{x^2}{1+x^3}dx=\frac{1}{3}\left[\ell n\left(1+x^3\right)\right]_0^1$$

$$=\frac{1}{3}\ell n2=\ell n\left(2^{\frac{1}{3}}\right)$$

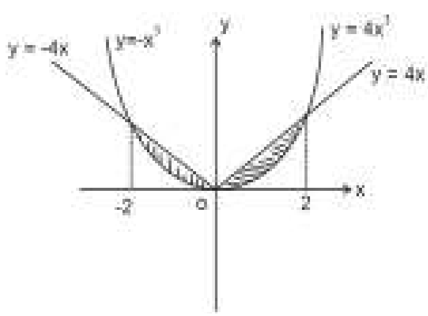
[:Q.10] The area (arbitrary units) bounded by the curves $y=4|x|$ and $y=x^2|x|$ is

[:A] 4 [:B] 8 [:C] 12 [:D] 16

[:ANS] **B**

[SOLN] $y=4|x|=\begin{cases}4x: & x\ge 0\\ 4x: & x<0\end{cases}$

$$y=x^2|x|=\begin{cases}x^3: & x\ge 0\\ -x^3: & x<0\end{cases}$$



$$\left.\begin{aligned}&x^2=4x\\&x=0,\pm 2\end{aligned}\right.$$

Area = $2\int_0^2\left(4x-x^3\right)dx=2\left(2x^2-\frac{x^4}{4}\right)_0^2$

$$=2(8-4)=8\text{ sq. units.}$$

[:Q.11] Consider the set S of all functions from $A=\{2021,2022,2023,2024\}$ to itself. What is the probability that a randomly picked function from S is a one-one function?

[:A] $\frac{3}{32}$ [:B] $\frac{3}{64}$ [:C] $\frac{3}{128}$ [:D] $\frac{1}{64}$

[:ANS] **A**

[SOLN] $A=\{2021,2022,2023,2024\}$

$\int : A \to A$

$P(E) =$ Prob. That a randomly Picked from S is a one-one Function

$$=\frac{4!}{44}=\frac{4\times3\times2}{4\times4\times4\times4}=\left(\frac{3}{32}\right)$$

[:Q.12] Auroral Kilometric Radiation occurs due to emission form charged particles moving in helical paths around planetary magnetic field lines at or around the electron gyro frequency and/or its lower harmonics. Considering the emission from relativistic particles, the emission peaks around a frequency $\sim\gamma^2\upsilon_c$ where γ, the Lorentz factor is $\gamma=\frac{1}{\sqrt{\left(1-\left(\frac{v}{c}\right)^2\right)}}$ and the cyclotron frequency is $\upsilon_c=\frac{eB}{2\pi m}$. Determine the frequency of peak emission by electrons moving at one-third the speed of light near the Earth's poles. The average strength of the Earth's magnetic field (which is approximately a dipole) at its surface is 50μ Tesla. It varies from about the surface.

[:A] 1.78×10^{12}Hz [:B] 1.89 MHz [:C] 2×10^{12}Hz [:D] 2 MHZ

[:ANS] **D**

[:SOLN] Lorentz factor $=\frac{1}{1-\frac{v^2}{c^2}}=\frac{9}{8}$

$$f=\frac{eB}{2\pi m}$$

$$\therefore\ f_{peak}=\frac{9}{8}f=2\text{ MHz}$$

[:Q.13] Thermal diffusivity of a material is related to the speed with which thermal equilibrium is reached. Thermal diffusivity $a=\frac{k}{\rho c}$ where k is the thermal conductivity, ρ is the density and c is the specific heat capacity of the material. Vessel of thick walls and bottom, made of which of the following metals/alloys aluminium (k = 385 W/m.K; ρ = 2.7 g/cc; c = 0.9 J/g. °C all at room temperature) or copper (k = 205 W/m.k; ρ = 8.96 W/m.K; c = 0.385 J/g. °C) or steel (k = 50.2 W/m.k; ρ = 7.75 W/m.K; c = 0.42 J/g.°C) is more likely to develop cracks if cold water is suddenly poured in after the vessel has been heated to 400 kelvin. The other conditions like size, shape and thickness of the walls and the bottom of the vessels remaining the same.

[:A] Aluminium [:B] Steel

[:C] Copper [:D] All the three equally likely

[:ANS] **B**

[:SOLN] $a = \dfrac{k}{\rho c}$

For Al: $\dfrac{385}{2.7 \times 0.9} = 158.4$

For Cu: $\dfrac{205}{8.96 \times 0.385} = 59.4$

For Steel: $\dfrac{50.2}{7.75 \times 0.42} = 15.4$ of thermal diffusivity is lower, the material is more likely to break

$\Rightarrow$ Steel is more likely to develop cracks

[:Q.14] Light from a sodium vapour lamp is diffracted by a plane transmission diffraction grating with N = 4000 lines per cm. The light is incident normally on the grating placed in air. The maximum angular separation (in minute of arc) achieved between the sodium doublets?

[:A] 0.014’ [:B] 0.029’ [:C] 0.155’ [:D] 9.9’

[:ANS] **D**

[:SOLN] Sodium doublet wavelength $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm

Grating element spacing = $a = \dfrac{1\text{ cm}}{1000} = 2500$ nm

For maximum angular separation, the order of diffraction must be highest

$\Rightarrow \sin\theta = \dfrac{n\lambda}{a} < 1$

or $n < \dfrac{a}{\lambda}$ i.e., $n < 4.24$

Maximum order is 4

For λ_1, $\theta_1 = \sin^{-1}\left(\dfrac{4 \times 589}{2500}\right) = 13.627°$

For λ_2, $\theta_2 = \sin^{-1}\left(\dfrac{4 \times 598.6}{2500}\right) = 13.641°$

$\Delta\theta = 9.9'$

[:Q.15] A proton with kinetic energy 1 ke V is shot at a static uranium nucleus. The radius of the uranium nucleus is approximately 15 femto-meter, the range of the strong force is of the oder of a femto-meter and the impact parameter is 50 femto-meter. The angular momentum of the proton with respect to the centre of the Uranium nucleus in units of the reduced Planck’s constant $(\hbar)$ is

[:A] 6.96 [:B] 55.2 [:C] 1.04 [:D] 0.346

[:ANS] **D**

[:SOLN] Angular momentum $L = mvb$

$$L = p \times b = \sqrt{2Km} \times b = hx = \frac{h}{2\pi} \times x$$

$$\Rightarrow x = \frac{2\pi b}{h} \times \sqrt{2km},\ k = 1KeV, b = 50fm$$

= 0.346

D is correct.

[:Q.16] At the interface between two media which are transparent to the incident light, specular reflection with angle of reflection equal to angle of incidence will take place if surface irregularities are small (scale of micro-roughness less $< \frac{1}{100}$ times the wavelength of incident light). Reflectance ρ is the fraction of the incident intensity that is reflected. For light going from a rarer medium to a denser one $\rho_{per} = \left(\frac{n^2 \cos\theta - \sqrt{(n^2 - \sin^2\theta)}}{n^2 \cos\theta + \sqrt{(n^2 - \sin^2\theta)}}\right)^2$ and

$\rho_{per} = \left(\frac{\cos\theta - \sqrt{(n^2 - \sin^2\theta)}}{\cos\theta + \sqrt{(n^2 - \sin^2\theta)}}\right)^2$ are the reflectances respectively for radiation polarized parallel to the interface and perpendicular to theat. Here, n is the refractive index of medium two with respect to medium one. For unpolarised light ρ is the mean of ρ_{per} and ρ_{per}; it is a monotonic function of the angle of incidence θ . Headilights of a faraway car coming towards you are getting reflected off the wet road into your eyes. The emission cone of light is narrow. The brightness of the reflection will be

[:A] greatest when car is farther away

[:B] independent of distance

[:C] greatest when car is nearer

[:D] increases continuously

[:ANS] **A**

[:SOLN] Reflectance is 1 for θ close to 90° *i.e.*, when car is farther away

Since light is unpolarised, the reflectance is the average of the reflectance of radiation polarised parallel to the interface and perpendicular to that

$$\rho = \frac{1}{2}(\rho_{per} + \rho_{per})$$

$$\text{or } \rho = \frac{1}{2}\left[\left(\frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}\right)^2 + \left(\frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}\right)^2\right]$$

[:Q.17] In quantum mechanics the wave function carries all the information about the system. The spin part of the general state of a spin half particle, with spin pointing in the (θ, ϕ) direction in three dimensional space can be written down in matrix from a $|\alpha> = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\theta}\sin\frac{\theta}{2} \end{pmatrix}$. The scalar product between two vectors $|\lambda>$ and $|\rho>$ is given by $<\lambda\|\rho>$ where $<\lambda|$ stands for the complex conjugate transpose of $|\lambda>$. The vector orthogonal to $|\alpha>$ is

[:A] $\begin{pmatrix} \sin\frac{\theta}{2} \\ -e^{-i\theta}\cos\frac{\theta}{2} \end{pmatrix}$ [:B] $\begin{pmatrix} \cos\frac{\theta}{2} \\ -e^{-i\theta}\sin\frac{\theta}{2} \end{pmatrix}$ [:C] $\begin{pmatrix} \sin\frac{\theta}{2} \\ -e^{-i\theta}\cos\frac{\theta}{2} \end{pmatrix}$ [:D] $\begin{pmatrix} \cos\frac{\theta}{2} \\ -e^{-i\theta}\sin\frac{\theta}{2} \end{pmatrix}$

[:ANS] **D**

[:SOLN] $\vec{\alpha} = \cos\frac{\theta}{2}\hat{i} + e^{i\phi}\sin\frac{\theta}{2}\hat{j}$

Let $\vec{\beta}$ is $\perp$ lar to $\vec{\alpha}$

$\Rightarrow \vec{\alpha}.\vec{\beta} = 0$

From the option $\vec{\beta}$ is satisfied by (a)

[:Q.18] The value of $\frac{h}{e^2}$ is SI units is

[:A] 25.9 kΩ [:B] 259 kV [:C] 25.9 kJ/C^2 [:D] 259 kJ/A^2

[:ANS] **A**

[:SOLN] Dimension of $\left[\frac{h}{e^2}\right] = [\text{Resistance}]$

$$\frac{hc}{\lambda} = i^2Rt = \frac{q^2}{t^2}Rt$$

$$\frac{h}{q^2} = \frac{R\lambda}{c \times t}$$

Dimensionally $\left[\frac{h}{q^2}\right] = \left[\frac{R\lambda}{c \times t}\right] = [R]$

$\Rightarrow$ option (A) is correct.

[:Q.19] Three identical convex lenses of focal length f have been placed with common principal axis along the X axis. The separation between two adjacent lenses is f. A laser beam is made incident parallel to principal axis. If the middle lens is now slightly shifted along –ve y direction, the emergent rays from the third lens will

[:A] shift upward in the +ve y direction and stay parallel to principal axis

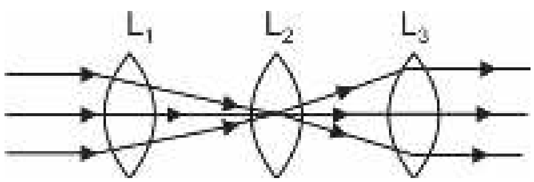
[:B] move upward

[:C] shift downward in the –ve y direction and stay parallel to principal axis

[:D] move downward

[:ANS] **C**

[:SOLN]



If middle lens L_2 is shifted along -ve y direction $(\downarrow)$ the emergent rays the third lens L_3 will also shift along -ve y direction but remains parallel to principal axis.

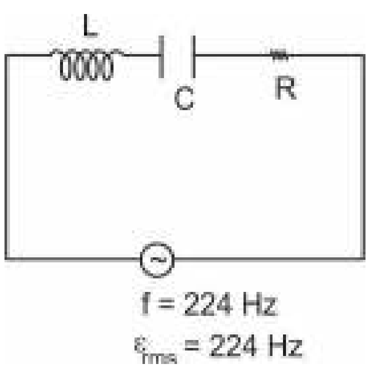
(C) is correct.

[:Q.20] Consider a coil of inductance 22.4 mH is connected in series with a capacitor 22.4 μF. The coil has resistance $22.4\,\Omega$. This combination is connected to an AC source of 224 Hz, rms 22.4 V sinusoidal signal. The rms current in the circuit will be

[:A] 262 mA [:B] 354 mA [:C] 501 mA [:D] 1000 mA

[:ANS] **D**

[:SOLN]



L = 22.4mH as $X_L = X_C$

$C = 22.4\ \mu F \Rightarrow Z = R$

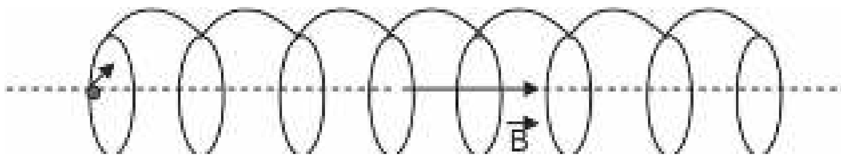
$$i_{rms} = \frac{\varepsilon_{rms}}{z} = \frac{\varepsilon_{rms}}{R} = 1000\,mA$$

[:Q.21] A proton moving with speed of 20 km/s enters a 200 cm long and 2.0 cm radius solenoid close to its axis. Solenoid has magnetic field of 200 gauss. If it leaves from the other end of the solenoid in $200\,\mu s$, how many revolutions can it complete (approximately) while inside the solenoid, (neglect end effects) ?

[:A] 20 [:B] 61 [:C] 102 [:D] 122

[:ANS] **C**

[:SOLN]



Inside the solenoid

$B = 200G = 200 \times 10^{-4} T$

Time period $T = \frac{2\pi m}{eB}$

If proton complete $\hat{n}$. Revolutions in time 't' then

$nT = t$

$$n = \frac{t}{T} = \frac{eBt}{z\pi m}$$

Putting all the values $n \simeq 61$

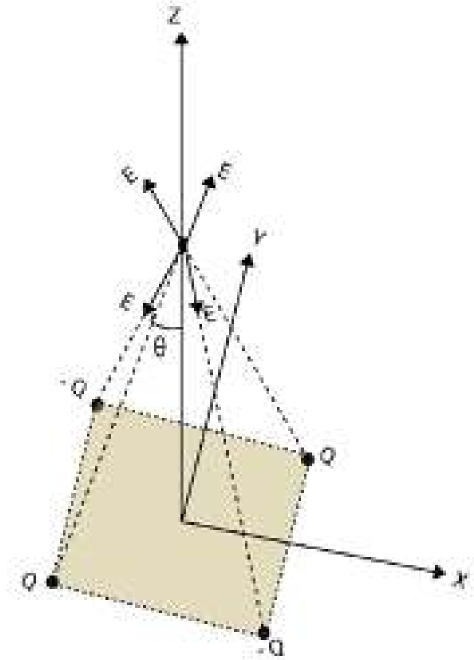
Option c is correct.

[:Q.22] Four charges are placed at points whose Cartesian coordinates are $+Q(-a, -a, 0)$, $-Q(-a, a, 0)$, $+Q(a, a, 0)$ and $-Q(a, -a, 0)$. The number of neutral points where electric field is zero is :

[:A] zero [:B] one [:C] four [:D] infinite

[:ANS] **D**

[:SOLN]



Consider any point on z-axis.

E.F.I due to +ve charges = $2E\cos\theta(\hat{k})$

E.F.I due to –ve charges = $2E\cos\theta(-\hat{k})$

Thus $(E_P)_{net} = 0 \rightarrow$ It implies that EFI is

Zero at every point on z-axis.

[:Q.23] Stars emit radiation approximately like perfectly black bodies. The amount of light energy emitted per unit time is called the luminosity of the star. Luminosity L and radius R of main sequence stars whose masses are neither too high nor too low are related to mass m of star as $L \propto m^{3.5}$ and $R \propto m^{0.8}$. The surface temperature T depends on the mass as

[:A] $T \propto m^{-2.1}$ [:B] $T \propto m^{0.475}$ [:C] $T \propto m^{1.1}$ [:D] $T \propto m^{1.275}$

[:ANS] **B**

[:SOLN] From Stefan-Boltzmann's law -

$$\frac{d\theta}{dt} = \sigma A T^4$$

$$L = \sigma(4\pi R^2)T^4$$

$$\therefore \; T \propto \frac{L^{1/4}}{R^{1/2}}$$

As $L \propto m^{7/2}$ and $R \propto m^{0.8}$

$$\therefore \ T \propto \frac{m^{7/8}}{m^{0.4}}$$

$$T \propto m^{0.875-0.4}$$

$$T \propto m^{0.475}$$

[:Q.24] A box of 22.4 litre is filled with ideal gas at pressure 2.00 atm and temperature 300 K. The box is opened in vacuum chamber of volume 44.8 litres. The final temperature of the gas will be

[:A] 150 K [:B] 200 K [:C] 300 K [:D] 450 K

[:ANS] **C**

[:SOLN] As box is open in chamber, free expansion of gas takes place i.e. w = 0. As well as no transfer of heat takes places i.e. Q = 0

$$\therefore \Delta u = 0$$

$$\Delta T = 0$$

$$T_f = T_i = 300\,K$$

[:Q.25] If the density of a planet is varying with radius as $\rho(r) = \rho_0\left(1-\left(\frac{r}{R}\right)^2\right)$, its gravity will be maximum at

[:A] $r = \frac{5}{9}R$ [:B] $r = \frac{3}{5}R$ [:C] $r = \frac{\sqrt{5}}{3}R$ [:D] $r = \left(\frac{5}{9}R\right)^{\frac{3}{2}}$

[:ANS] **C**

[:SOLN] Applying Gauss law -

$$\phi_g = 4\pi G m_{enclosed}$$

$$g \times r^2 = 4\pi G \int_0^r \rho dv$$

$$g = \frac{G}{r^2}\rho_0 \int_0^r \left(1-\frac{r^2}{R^2}\right) \times 4\pi r^2 dr$$

$$g = 4\pi\rho_0 G\left[\frac{r}{3}-\frac{r^3}{5R^2}\right]$$

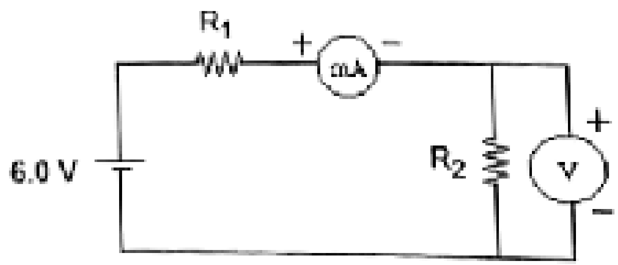
For g to be maximum –

$$\frac{dg}{dr} = 0$$

$$\frac{1}{3}-\frac{3}{5}\frac{r^2}{R^2}=0$$

$$r=\frac{\sqrt{5}}{3}R$$

[:Q.26] In the following circuit, voltmeter has $40\ k\Omega$ resistance and ammeter has $40\ \Omega$ resistance. Their readings are 4.00 V and 40.0 mA respectively. The actual values if the meters were ideal would be



[:A] Larger than 4.00 V, smaller than 40.0 mA

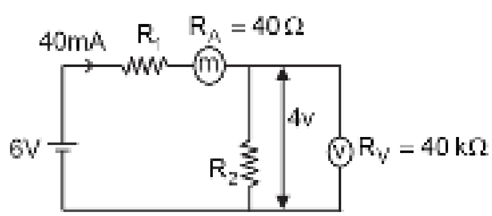
[:B] Smaller than 4.00 V, smaller than 40.0 mA

[:C] Larger than 4.00 V, larger than 40.0 mA

[:D] Smaller than 4.00 V, larger than 40.0 mA

[:ANS] **C**

[:SOLN]



P.d. across ammeter $=40\times40\times10^{-3}=1.6\,\text{V}$

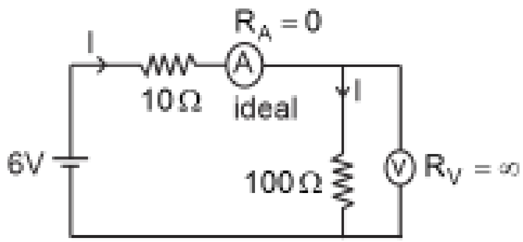
P.d. across ammeter $=6-4-1.6=0.4\ \text{V}$

$$\therefore R_1=\frac{0.4}{4\times10^{-2}}=10\,\Omega$$

Current in voltmeter $=\dfrac{4}{4\times10^{4}}=0.1\,\text{mA}$

Current in $R_2=(40-0.1)\text{mA}=39.9\,\text{mA}$

$$\therefore R_2=\frac{4}{39.9\times10^{-3}}\approx100\,\Omega$$



$\therefore I = \frac{6}{110} = 54.5\,\text{mA}$

$\rightarrow$ Current through ammeter increases.

$V_{R_2} = IR_2$

$= \frac{6}{110} \times 100 = 5.4\,\text{V}$

$\rightarrow$ Voltmeter reading also increases.

[:Q.27] If an atom absorbs red photon of 650 nm and a yellow photon of 580 nm and emits the entire absorbed energy as a single photon, the wavelength of the emitted photon is

[:A] 70.5 nm [:B] 306.5 nm [:C] 613 nm [:D] 1230 nm

[:ANS] **B**

[:SOLN] $E = E_1 + E_2$

$$\frac{1240}{\lambda} = \frac{1240}{650} + \frac{1240}{580}$$

$$\frac{1}{\lambda} = \frac{1}{650} + \frac{1}{580}$$

$$\lambda = 306.5\ \text{nm}$$

[Q.28] The energy radiated per second by star A is twice that radiated per second by star B. Also the distance of star A from Earth is twice to that of star B from Earth. Which of the two stars appears brighter when seen from the Earth, and by how much?

[:A] Star A appears 2 times brighter than star B

[:B] Star B appears 2 times brighter than star A

[:C] Star B appears 4 times brighter than star A

[:D] The observed brightness of the stars A and B are equal

[ANS] **B**

[:SOLN] $\left.\begin{aligned} \left(\frac{d\theta}{dt}\right)_A &= 2\left(\frac{dQ}{dt}\right)_B \\ r_A &= 2r_B \end{aligned}\right\}$ given

Now, intensity -

$I = \frac{dQ}{A\,dt}$ Where $A = 4\pi r^2$

$$\therefore \frac{I_A}{I_B} = \frac{\left(\frac{dQ}{dt}\right)_A}{\left(\frac{dQ}{dt}\right)_B} \times \left(\frac{r_B}{r_A}\right)^2$$

$$= 2 \times \left(\frac{1}{2}\right)^2$$

$$\frac{I_A}{I_B} = \frac{1}{2}$$

[Q.29] How large should the aperture of a telescope be in order to achieve a diffraction limit of 0.001 are second for visible light of wavelength 500 nm?

[:A] 2.4 m [:B] 12.6m [:C] 126m [:D] 24m

[ANS] **C**

[:SOLN] $\theta = 0.001''$

$$\theta = \left(\frac{10^{-3}}{3600}\right)^\circ$$

$$\theta = \frac{10^{-6}}{3.6} \times \frac{\pi}{180} \text{ radian}$$

$$\theta = 4.84 \times 10^{-9} \text{ radian}$$

Resolving limit of telescope

$$\theta = \frac{1.22\lambda}{D}$$

$$D = \frac{1.22\lambda}{\theta\,(\text{in radian})}$$

$$= \frac{1.22 \times 5 \times 10^{-7}}{4.84 \times 10^{-9}}$$

$= 126$ m

[Q.30] A nearby star, Alpha Centauri, subtends a parallax angle 0.7420”. The distance to Alpha Centauri from us is

[:A] 1.35 pc [:B] 13.5 pc [:C] 0.742 pc [:D] 7.42 pc

[ANS] **A**

[:SOLN] $r = \frac{1}{\rho}$

Where –

r = distance of star in parsec

ρ= Parallax angle in arc second

$\therefore r = \frac{1}{0.742}$

$r = 1.35$ Parsec

= 1.35 pc

[Q.31] The spectral lines of two stars in an eclipsing binary system with both stars moving in circular orbits, shift back and forth with a period of 2 years, $T = 6.3 \times 10^7$ second. The lines of one star (star 1) shift twice as far as the lines of the other (star 2). Which one of the following statements is true?

[:A] Masses of star 1 and star 2 are equal

[:B] Mass of star 2 is twice the mass of star 1

[:C] Mass of star 1 is twice the mass of star 2

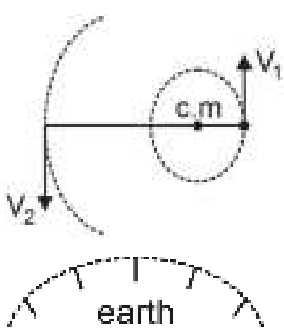
[:D] Nothing can be predicted about the masses of the two stars from the given data

[ANS] **B**

[:SOLN] Doppler shift –

$\Delta\lambda = \frac{V}{C}\lambda$

According to question –



$\Delta\lambda_1 = 2\Delta\lambda_2$

$V_1 = 2V_2$

The orbital velocity of star (1) –

$$V_1 = \left\{\frac{2\pi G}{T(m_1+m_2)^2}\right\}^{\frac{1}{3}} m_2$$

Similarly –

$$V_2 = \left\{\frac{2\pi G}{T(m_1+m_2)^2}\right\}^{\frac{1}{3}} m_1$$

$\because \frac{V_1}{V_2} = \frac{m_2}{m_1}$

$\frac{m_2}{m_1} = 2$

$m_2 = 2m_1$

[Q.32] The reciprocal of Hubble's constant, or $1/H_0$, tells us the age of the universe if the expansion rate has remained constant over time. How would the estimated age of the universe differ if the measured value of H_0 were 44 km/s/Mly rather than 22 km/s/Mly?

[:A] The estimated age will remain the same

[:B] Indeterminate

[:C] The estimated age will be twice the current estimated age

[:D] The estimated age will be half the current estimated age

[ANS] **D**

[:SOLN] Let T = Current estimated age

T' = Measured estimated age

$\frac{T'}{T} = \frac{(H_0)}{(H_0)'} = \frac{22}{44}$

$T' = T/2$

[Q.33] If you live at latitude of 28 degrees North, what is the angle you observe between the northern horizon and North Celestial pole?

[:A] 62 degrees [:B] 90 degrees [:C] 23.5 degrees [:D] 28 degrees

[:ANS] **D**

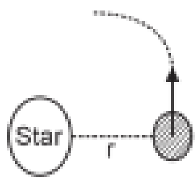
[;SOLN] The angle between the northern horizon and the north Celestial pole is equal to the latitude of the observer, that mean 28°

[Q.34] A new planet is discovered to be orbiting a star which has the same mass as our Sun in an approximately circular orbit. The planet orbits the star every 3 months. Approximately, what is the average distance of the planet from the star?

[:A] 3 AU [:B] 0.4 AU [:C] 0.125 AU [:D] 1.25 AU

[:ANS] **B**

[;SOLN]



For circular orbit

$$T^2 = \frac{4\pi^2}{Gm_s} r^3$$

Given,

$T = 3 \times 30 \times 24 \times 60 \times 60$ Sec

T = 7776000

and $M_{star} = M_{sun} = 1.98 \times 10^{30}$ kg

IAU = 149597870700 m

So, $\gamma = \left[\frac{6.67 \times 10^{-11} \times 1.98 \times 10^{30} \times T^2}{4\pi^2}\right]^{1/3}$

$\Rightarrow \gamma = 27246.743 \times 10^{19/3}$

γ = 0.39 $\simeq$ 0.4 AU

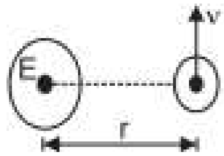
[Q.35] Suppose a future space station orbits the Earth in a circular geosynchronous orbit, 42,000 kilometre from the centre of the Earth. With what speed with respect to Earth, must a spacecraft be launched from this space station so as to escape the Earth?

[:A] 11.2 km/s [:B] 112 km/s [:C] 4.35 km/s [:D] 2.45 km/s

[:ANS] **C**

[;SOLN] Escape velocity from an orbit

Geosynchronous orbit $\rightarrow$ Time period (24-hrs)



To escape

$$\frac{1}{2}mv^2 - \frac{GM_Em}{r} = 0$$

$$\Rightarrow \nu = \sqrt{\frac{2GM_E}{r}} = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 5.9\times 10^{24}}{42000\times 10^3}}$$

$$\nu = \sqrt{1.87}\times 10^{3.5} = 1.367\times 10^{3.5}\,\text{m/s}$$
$$\nu = 4.3\,\text{km/s}$$

[Q.36] Planets orbiting very close to black holes show general relativistic effects; the orbits of such planets are precessing ellipses i.e. the major axis of the ellipse rotates about the occupied focus. A planet is on an elliptical orbit with semi-major axis 1.0 AU around an isolated black hole of one solar mass. A second planet is orbiting the black hole at a distance of 0.3 AU. If observations of this second planet are started when it is closest to the black hole, the minimum length of time over which observations should be carried out so that the precession rate may be ascertained with better accuracy, is a little more than

[:A] 30 day [:B] 60 day [:C] 75 day [:D] 90 day

[:ANS] **B**

[;SOLN] Orbital period of the second planet

$$a = 0.3\text{AU, and IAU} = 1.446\times 10^{11}\text{m}, T_{orbit} = 2\pi\sqrt{\frac{a^3}{GM}}$$

$$T_{orbit} = 2\pi\sqrt{\frac{\left(0.3\times 1.496\times 10^{11}\right)^3}{6.67\times 10^{-11}\times M_{bbcchole}\left(1.989\times 10^{30}\right)}} = 0.165\text{yr} = 60\,\text{day}$$

Precession rate $\Delta W = \dfrac{6\pi GM}{a\left(1-e^2\right)c^2} = 8.6\times 10^{-7}\,\text{rad / orbit}$

For second planet if we take e = 0

$$T_{precession} = \frac{T_{orbit}}{\Delta W} = \frac{0.165}{8.6\times 10^{-7}} = 191{,}860\,\text{yr}$$

Since the precession per obit is extremely small, a practical observation time would be little more than one orbital period (60 days)

[:Q.37] Spectral resolution of a grating is given by $RP = \frac{\lambda}{\delta\lambda}$ where $\delta\lambda$ is the smallest difference in wavelength between two lines that may be resolved as separate at some order after dispersion by the grating. In the binary system consisting of a star and a planet, both the mass orbit around the common centre of mass. The orbital speed of the star will be extremely small due to its comparatively very large mass. The Doppler shift due to this orbital motion will also be extremely small. Doppler spectroscopy is a technique used to detect and estimate the orbital speeds of stars, induced by the presence of exoplanets around them. With what accuracy can the recession speed of a star be determined by simple use of a high resolution spectrograph with RP = 1,00,000 observing a spectral line at 0.3 micron?

[:A] 3.0 m/s [:B] 30.0 m/s [:C] 3.0 km/s [:D] 30.0 km/s

[:ANS] **C**

[;SOLN] $RP = \frac{\lambda}{\delta\lambda} = 100000,\ \lambda = 0.3 \times 10^{-6}$

Here $\delta\lambda = \frac{100000}{0.3 \times 10^{-6}}$, dopper shift, $\frac{\delta\lambda}{\lambda} = \frac{V}{C}$

$$\Rightarrow \nu = C\left(\frac{\delta\lambda}{\lambda}\right) = \frac{3 \times 10^{8}}{10^{5}} = 3 \times 10^{3}\,\text{m/sec} = 3\,\text{km/sec}$$

[:Q.38] Fermions (half – integer spin particles) obey the Pauli Exclusion Principle which says that no two fermions can have all physical characteristics (Like position, momentum, spin, charge, electron number, muon number etc….) the same. As per the uncertainty principle position and momentum can be simultaneously measured only to a precision that satisfies the relation $\Delta x \Delta p_x \geq \frac{h}{2}$. So a 'cell' in phase space (position – momentum space) of size $\left(\frac{h}{2}\right)^3$ can contain only a pair of any one type of neutrino (for example electron neutrino) of opposing spin and all other quantum numbers the same. Consider a spherical dwarf galaxy of a total mass M and radius R consisting of a large amount of dark matter and very little ordinary matter. Suppose dark matter consists of n_f different types of neutrinos all having the same mass m equivalent to 1 eV. Given that the escape speed from the galaxy $v_{esc} = \sqrt{\frac{2GM}{R}}$, a limit may be put on n_f by the relation

[:A] $\frac{\left(\frac{4\pi}{3}\right)^2 (mv_{esc}R)^3}{\left(\frac{\hbar}{2}\right)^3} \times 2n_f \times m < M$

[:B] $\frac{\left(\frac{4\pi}{3}\right)^2 (mv_{esc}R)^3}{\left(\frac{\hbar}{2}\right)^3} \times n_f \times m < M$

[:C] $\dfrac{\left(\dfrac{4\pi}{3}\right)^2 (mv_{esc}R)^3}{\left(\dfrac{\hbar}{2}\right)^3} \times n_f \times m < M$ [:D] No such limit may be extracted

[:ANS] **Bonus**

[:SOLN] $\Delta x \Delta p \geq \dfrac{\hbar}{2}$

$\Delta x \geq \dfrac{\hbar}{2mv}$

$x_{min} > \dfrac{\hbar}{2mv_{esc}}$

Volume of cell = $(x_{min})^3 = \dfrac{\hbar^3}{(2mv_{esc})^3}$

Number of cells = $\dfrac{V}{(x_{min})^3} = \dfrac{\dfrac{4}{3}\pi R^3}{\left(\dfrac{\hbar}{2}\right)^3}(mv_{esc})^3$

Number of neutrinos = $2n_f$

Mass of dark matter = $\dfrac{\dfrac{4}{3}\pi R^3 (mv_{esc})^3 \times 2n_f m}{\left(\dfrac{\hbar}{2}\right)^3}$

$M > \dfrac{\dfrac{4}{3}\pi R^3 (mv_{esc})^3 (2n_f m)}{\left(\dfrac{\hbar}{2}\right)^3}$

[:Q.39] Inverse Compton scattering in the scattering of low energy photon by high energy ultra – relativistic charged particle. The photon gains energy and the charged particle loss energy. The frequency of the photon gets multiplied by a factor equal to the square of the Lorentz factor of the ultra – relativistic particle. The low energy photons of the Cosmic Microwave Background (CMB) can be scattered by (wavelength around 10 cm) get scattered, on the average, into soft X – ray region (wavelength □ nanometer), the Lorentz factor for the electrons involved is

[:A] $\sim 10^4$ [:B] $\sim 10^8$ [:C] $\sim 2.73 \times 10^9$ [:D] ~ 2.73

[:ANS] **A**

[;SOLN] $E_{\text{photon scattered}} \approx r^2 E_{\text{photon,initial}}$

$E_{\text{photon}} = h\nu \;\&\; \nu = {}^{C}\!/\!_{\lambda}$ for $\lambda \sim 10\,\text{cm}$

$$\mu_{CMB} = \frac{C}{\lambda} = 3\times10^{9}\text{HZ}$$

Thus, the energy of a CMB photon is

$$E_{CMB} = h\nu_{CMB} = \left(6.626\times10^{-34}\,\text{J/s}\right)\times3\times10^{9}\,\text{Hz} = 1.987\times10^{-24}\,\text{J}$$

and $E_{x\text{-ray}} = \dfrac{hC}{\lambda_{x\text{-ray}}}$ for $\lambda_{x\text{-ray}} = 1\,\text{nm} = 10^{-9}\,\text{m}$

$\Rightarrow E_{x-ray} = 1.987\times10^{-16}\,\text{J}$

So, for Lorentz factor 'γ'

$$\gamma^2 = \frac{E_{x\text{-ray}}}{E_{CMB}} = 10^{8} \Rightarrow \gamma = 10^{4}$$

[:Q.40] The Saha ionization formula gives the ratio of the number of (n + 1) times ionized ions of an element per unit volume to the number of n times ionized ions of the same element in the same volume in a plasma. Normal stars show absorption spectra due to atoms/ions/ molecules of the atmosphere of the star in a lower energy state absorbing photons of appropriate energy, from the continuum radiation emitted by the photosphere, to go into a higher energy state. Given the Saha ionization formula as $\log\dfrac{n_{i+1}}{n_i} = -0.1761 - \log P_e + \log\dfrac{U_{j+1}(T)}{U_j(T)} + 2.5\log T - \dfrac{5040\,E_j}{T}$ where, $n+1$ is the number density of $(i+1)$ times ionized ion, n_i is the number density of i times ionized ions, P_e is the electron pressure, U.(T) is the partition function and E_j is the ionization of helium = 24.48 eV and of second ionization of helium = 54.17 eV. As the temperature of photosphere decreases from $\sim 50{,}000$ K, in the spectrum

[:A] Strength of lines of ionized helium and hydrogen will decrease together

[:B] Lines of hydrogen will decrease in strength, helium will increase

[:C] Lines of ionized helium will decrease in strength, of helium increase

[:D] Strength of hydrogen lines will decrease, ionized helium will increase

[:ANS] **C**

[:SOLN] As the temperature of photosphere decreases the ionisation of element will also decrease. So lower temperature leads to decrease in the ratio of ionized neutral atoms.

For Hydrogen with ionization energy 13.54 eV and for helium with first two ionization energy 24.48 eV and 54.17 eV respectively, the strength of lines of helium decreases as the temperature drops, while neutral helium lines may increase in strength due to increased number of neutral atoms.

[:Q.41] The observed Doppler shift of spectral lines of galaxies is used to determine their velocity of recession from us using formula $v = cz$ where $z = \frac{\delta\lambda}{\lambda}$ (this may then be used in the Hubble relation v = HD to determine the distance to the galaxy). Spectrometry is expensive since the received light is highly dispersed and only a small number of photons are available in small frequency ranges and so observations have to be integrated over long hours to capture the spectrum. In photometry. Light transmitted through various filters(which transmit light in a small band of wavelengths centred on given wavelengths) is measured. Photometry is much cheaper and faster. The hydrogen surrounding galaxies absorb light with wavelength below 100 nm . Hence the observed brightness of the galaxy will drop drastically below this wavelength of emission. A galaxy was imaged through the 4, g, r, i, z filters in the Sloan Digital Sky Survey. The pivot wavelength of the filters are 354 ,477, 623, 763, and 913 nm respectively. A galaxy is visible when imaged through the last three filters. The galaxy 'drops out' and is not visible through the other filters. The redshift of the galaxy as determined by this 'dropout method' lies between

[:A] 4.77 and 6.23 [:B] 5.77 and 7.23 [:C] 3.77 and 5.23 [:D] 5.23 and 6.63

[:ANS] **C**

[:SOLN] Observed wavelength of dropout corresponds to $\lambda_{obs} = \lambda_{rest} \times (1 + z)$

Where, λ_{rest} = 121.6 nm (Lyman limit)

And, from the data given, dropout occurs between 477 nm and 623 nm filter

$\therefore$ Redshift, $z = \frac{\lambda_{obs}}{\lambda_{rest}} - 1$

$\Rightarrow z_{min} = \frac{477}{121.6} - 1 \approx 3.9$

and $z_{max} = \frac{623}{121.6} - 1 \approx 4.1$

$\therefore$ Redshift range lies between 3.77 and 5.23

[:Q.42] A liquid column of height 1.0 cm and having density 0.8 g/cc, is in a closed container on a horizontal surface inside Chandrayaan – 3 Lander on the surface of Moon. The temperature where the landing took place was around $50\,^{\circ}\mathrm{C}$. What is the hydrostatic pressure on the bottom of the container? Mass of the Moon = $7 \times 10^{22}\,\mathrm{kg}$ and radius of the Moon = 1750 km

[:A] $12.2\,\mathrm{N/m^2}$ [:B] $98.0\,\mathrm{N/m^2}$ [:C] $78.4\,\mathrm{N/m^2}$ [:D] $1.22\,\mathrm{N/m^2}$

[:ANS] **A**

[;SOLN] $$g_{moon} = \frac{GM_{moon}}{r_{moon}^2} = \frac{6.67 \times 10^{-11} \times 7 \times 10^{22}}{(1750)^2 \times 10^6} = 1.524\,\mathrm{m/s^2}$$

Pressure = $\rho gh = 800 \times 1.524 \times \frac{1}{100} = 12.2\,\mathrm{N/m^2}$

[:Q.43] Cerenkov radiation may be defined as " electromagnetic radiation emitted when a charged particle (such as an electron) passes through a dielectric medium (such as distilled water) at a speed greater than the phase velocity (speed of propagation of a wave front in a medium) of light in that medium". Which of the following cosmic ray particles is likely to be detectable by recording of the Cerenkov radiation emitted by it in its passage through the Earth's atmosphere. The lower atmosphere near the Earth ahs a refractive index of 1.002.

[:A] A 13 GeV proton [:B] A 13 MeV electron

[:C] A 25 MeV proton [:D] A 0.51 MeV electron

[:ANS] **B**

[:SOLN] Speed of light in medium, $\frac{c}{\mu} = v = \frac{3 \times 10^8}{1.002}$

$\Rightarrow \quad v = 2.99401197 \times 10^8$ m/s

KE = $mc^2(\gamma - 1)$ and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\left(\frac{KE}{mc^2} + 1\right) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \quad 1 - \frac{v^2}{c^2} = \frac{1}{\left[1+\frac{KE}{mc^2}\right]^2}$$

$$\Rightarrow \quad \left[1 - \frac{1}{\left[1+\frac{KE}{mc^2}\right]^2}\right] c^2 = v^2$$

$$\Rightarrow \quad v = c\left[1 - \frac{1}{\left[1+\frac{\text{KE}}{mc^2}\right]^{-2}}\right]^{1/2}$$

Now putting the values of the energy given for different particles.

For (a) 13 GeV proton, $v \approx 2.992 \times 10^8$ m/s which is less than speed of light in medium.

For (b) 13 MeV electron, $v \approx 2.9978 \times 10^8$ m/s which is greater than speed of light in medium.

So, electron of 13 MeV will be detected.

[:Q.44] Which of the following animals, if they develop their own number system, is likely to write 100 where you count 64 ?

[:A] Spiders [:B] Ants [:C] Crabs [:D] Seahorses

[:ANS] **A**

[:SOLN] 64 in base 10 is equal to 100 in base 8 and a spider has 8 legs so it is likely to write 100

[:Q.45]. The Mercator projection maps the surface of the globe (excluding latitudes close to the two poles) onto a cylinder whose axis coincides with the rotation axis of the globe. The cylinder when cut open with a straight cut parallel to its axis gives a rectangular map which preserves directions. Maps made utilizing this projection are hence used for marine navigation and also by major online street mapping services. On a map of this kind the length of the equator is 40 cm. How many kilometres on the Earth does 1 cm at 60° N latitude on this map correspond to? Assume that the earth is a perfect sphere.

[:A] 866 km [:B] 1732 km [:C] 1000 km [:D] 500 km

[:ANS] **D**

[:SOLN] Length of equator is $L_0 = 2\pi R$

Which is 40 cm, so 1 cm corresponds to $\frac{L_0}{40\text{ cm}} = \frac{2\pi R}{40\text{ cm}}$

Scaling factor $\cos\theta = \frac{1}{2}$

So 1 cm corresponds to $\frac{2\pi R}{40\text{ cm}} \times 1\text{ cm}$

And at 60° N, length = $\frac{2\pi R}{40\text{ cm}} \times 1\text{ cm} \times \frac{1}{2} = \frac{2\pi \times 6400}{80} \approx 500$ km

[:Q.46]. Five identical letters have to be posted to five different addresses and there are two messengers available. The number of ways in which the letters may be assigned for posting is

[:A] 5!/(2!3!) [:B] 2^5 [:C] 5^2 [:D] 6

[:ANS] **B**

[:SOLN] For each address a messenger can be assigned in two ways.

So total no. of ways = 2^5

[:Q.47] Consider the ellipse with equation $\frac{x^2}{9}+\frac{y^2}{b}=1$ with one focus at (2, 0). The equation to the line that passes through the point (0, 2) and is perpendicular to the tangent to the ellipse at (xo, yo) is

[:A] $y=\dfrac{9\left(1-\frac{x_0^2}{9}\right)^{\frac{1}{2}}}{\sqrt{5}x_0}x+2$

[:B] $y=\dfrac{9}{10x_0}x+2$

[:C] $y=-\dfrac{9\sqrt{1-\frac{x_0^2}{9}}}{\sqrt{5}x_0}x+2$

[:D] $y=-\dfrac{9}{10x_0}x+2$

[:ANS] **A or C**

[:SOLN] Focus $=\left(\sqrt{9-b},0\right)=(2,0)$

$\therefore\ 9-b=4\Rightarrow b=5$

Slope of tangent at $(x_0,y_0)=\left.\dfrac{dy}{dx}\right]_{(x_0,y_0)}=\dfrac{-\frac{2x_0}{9}}{\frac{2y_0}{b}}=-\dfrac{5x_0}{9y_0}$

$\therefore$ slope of the line $=\dfrac{9y_0}{5x_0}=\pm\dfrac{9\left(b\left(1-\frac{x_0^2}{9}\right)\right)^{1/2}}{5x_0}=\pm 9\dfrac{\left(1-\frac{x_0^2}{9}\right)^{1/2}}{\sqrt{5}x_0}$

$\therefore$ equation of the line is

$$y-2=\pm\frac{9\left(1-\frac{x_0^2}{9}\right)^{1/2}}{\sqrt{5}x_0}(x-0)$$

[:Q.48].. Consider a planet moving on an elliptical orbit that may be represented by the equation $\frac{x^2}{9}+\frac{y^2}{5}=1$ (in some units) with the parent star at the focus at (-2, 0). The area swept by the radius vector from the focus to the planet in equal intervals of time is a constant. The ratio of the time taken to traverse the portion of the path with $x < -2$ to the time taken to traverse the portion of the path with x >– 2 is

[:A] $\dfrac{\left(\frac{\pi}{2} - A - B\right)}{\left(\frac{\pi}{2} + A - B\right)}$, where $A = \sin^{-1}\frac{2}{3}$ and $B = \frac{2\sqrt{5}}{9}$

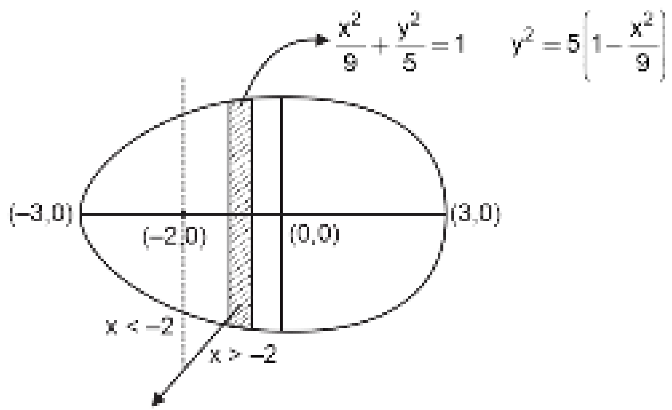
[:B] $\dfrac{\left(\frac{\pi}{2} - \alpha - \beta\right)}{\left(\frac{\pi}{2} + \alpha + \beta\right)}$, where $\alpha = \sin^{-1}\frac{2}{3}$ and $\beta = \frac{5}{27}$

[:C] 1 : 4

[:D] 4 : 1

[:ANS] **A**

[:SOLN]



$$A = \int dA = 2\int_0^{-2} y\,dx = 2x\int_0^{-2} \sqrt{5}\sqrt{1 - \frac{x^2}{9}}\,dx$$

$$A = 2\sqrt{5}\left[\frac{-9\sin^{-1}\left(\frac{2}{3}\right) + 2\sqrt{5}}{6}\right]$$

Ratio of time = $\dfrac{A_1(xC-2)}{A_2(xD-2)} = \dfrac{\frac{\pi(3)(\sqrt{5})}{2} - A}{\frac{\pi(3)(\sqrt{5})}{2} + A}$

Correct option is A.

(NSEA) PART : A–2

ANY NUMBER OF OPTIONS 4, 3, 2 OR 1 MAY BE CORRECT

MARKS WILL BE AWARDED ONLY IF ALL THE CORRECT OPTIONS ARE BUBBLED.

[:Q.49] Calculus was invented by Newton to solve the equations that came up when he examined motion under a central force such as gravity. The force of gravity is the controlling force underlying the laws of Kepler. The possible solutions to the equation were obtained as conic sections. Which of the following is/are possible solutions (x, y, t, r, θ are standard coordinates; A, B, k and K are constants) to the Kepler problem?

[:A] x(t) = A cos kt; y(t) = B sinkt [:B] $r(t) = k; \theta(t) = Kt \text{ modulo } 2\pi$

[:C] $x(t) = A\sec\theta k;\ y(t) = B\tan\theta$ [:D] $\frac{(x-2)}{2} = \frac{(y-3)}{3} = \frac{(z-4)}{4}$

[:ANS] **A, B**

[:SOLN] The Kepler's law gives the solution for closed loop.

Due to central forces the path will be closed.

$$\left.\begin{array}{l}\text{(a) Equation of ellipse}\\ \text{(b) Equation of Circle}\end{array}\right\} \rightarrow \text{Both are closed}$$

Correct answer. A and B

Explanation- Conceptual

[:Q.50] Consider the functions f and g defined by $f(x) = |x-1|,\ x \in R$ and $g(x) = |\sin x|, x \in R$

Define the sum of f and g as $(f+g)(x) = f(x) + g(x), x \in R$ and difference of f and g as

$(f-g)(x) = f(x) - g(x), x \in R$

Which of the following is/are true?

[:A] f + g is differentiable at x = 0 [:B] f - g is not differentiable at x = 0

[:C] f + g is not differentiable at x = 0 [:D] f-g is differentiable at x =1

[:ANS] **B, C**

[:SOLN] $(f+g)(x) = |x-1| + |\sin x|$

At x = 0, $|x-1|$ is differentiable but $|\sin x|$ is not differentiable

$\therefore (f+g)(x)$ is non-differentiable at x = 0

At x = 1 $|x-1|$ is non differentiable but $|\sin x|$ is differentiable

$\therefore$ (f + g) (x) is non-differentiable at x = 1

$(f-g)(x) = |x-1| - |\sin x|$

At x = 0, $|x-1|$ is differentiable but $|\sin x|$ is non-differentiable

$\therefore (f-g)(x)$ is non-differentiable at x = 0

At x = 1 $|x-1|$ is non-differentiable but $|\sin x|$ is differentiable

$\therefore (f-g)(x)$ is non-differentiable at x = 1

[:Q.51] Consider the tangents to the curve y = (x – 1) (x – 2) (x – 3) at the points where it meets the X-axis.

Which of the following is/are true?

[:A] Two of the tangents are parallel to each other

[:B] Two of the tangents are perpendicular to each other

[:C] One of the tangents makes angle 45° with the positive X-axis

[:D] Sum of the Y-intercepts made by the three tangents is –6

[:ANS] **A, C, D**

[:SOLN] $y = (x-1)(x-2)(x-3)$ meets the x-axis at $(1,0),(2,0)$ and $(3,0)$

$$\frac{dy}{dx} = 3x^2 - 12 + 11$$

Slope of tangent at (1, 0) = 2; Equation of tangent $\Rightarrow y = 2x - 2$

Slope of tangent at (2,0) = –1; Equation of tangent $\Rightarrow y = -x + 2$

Slope of tangent at $(3,0) = 2$; Equation of tangent $\Rightarrow y = 2x - 6$

[:Q.52] Consider the LPP

Max Z = x + y subject to the constraints

$x + y \le 6,\ 2x - 3y \le 6,\ -x + y \le 3, x, y \ge 0$

Which of the following is/are true?

[:A] x = 3, y = 3 is an feasible solution of the LPP

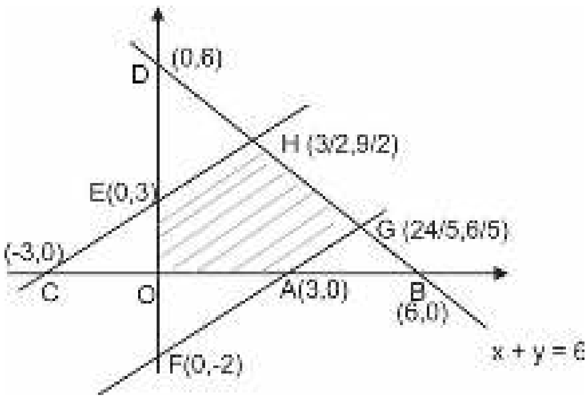
[:B] x = 3, y = 3 is an optimal solution of the LPP

[:C] x = k , y = 6 – k is an optimal solution of the LPP for $1 \le k \le 4$

[:D] The LPP has infinitely many optimal solutions

[:ANS] **A, B, D**

[:SOLN]



OAHGEO is feasible region.

(3, 3) lies on segment GH and so a feasible solution.

corner point	Z = x+ y	
O(0,0)	0	
A(3,0)	3	
G(24/5,6/5)	6	Maximum
H(3/2,9/2)	6	
E(0,3)	3	

$\therefore$ max Z = 6, at all points on line segment GH.

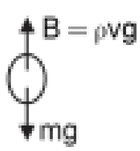
So, (a), (b), (d) are correct

[:Q.53]. There are two whales of exactly the same shape and size. One can just float in a sea somewhat below the arctic circle (say arctic whale) and the other in a tropical sea near the equator (say equatorial whale). Salinity of sea water first increases in going from the equator towards the poles and then decreases in going close to the poles. Which of the following statement(s) is/are true?

[:A] The two whales are of the same mass
[:B] Equatorial whale has smaller mass
[:C] Equatorial whale has larger mass
[:D] Arctic whale has more weight

[:ANS] **B, D**

[:SOLN] Gravity and Salinity both are higher of poles than equator.



For Equation,

$$mg = \rho_{\ell} vg = \text{weight}$$

$\boxed{m = \rho_{\ell} V}$

Option (B) and (D) are Correct.

[:Q.54] A point charge q I .00 μC and mass m = 1.00 μg moves in a potential $V(x) = -\frac{a}{x^2} + \frac{b}{x^3} (x > 0)$ where a = 1000 V x μm^2 and b = 100 V x μm^3. The correct statement(s) is/are

[:A] There is an equilibrium at x = 0.15 gm

[:B] Around x = 0.15 urn, charge can make small oscillations

[:C] The period of small oscillations of charge around x = 0.15 is 0.10 ms

[:D] A negative charge will not experience zero force at x = 0.15 gm

[:ANS] **A, B, C, D**

[:SOLN] $E = \frac{-dv(x)}{dx} = -\left(\frac{2a}{x^3} - \frac{3b}{x^4}\right)$

At equation, E = 0

$$\Rightarrow x = \frac{3b}{2a} = 0.15\,\mu m$$

$$F = q\left[\frac{2a}{(0.15+\delta x)^3} - \frac{3b}{(0.15+\delta x)^4}\right]$$

$$F = -q\left[\frac{2a}{(0.15)^3}\left[1 - \frac{3\delta x}{0.15}\right] - \frac{3b}{(0.15)^4}\left[1 - \frac{4\delta x}{0.15}\right]\right]$$

$$F = \frac{q}{(0.15)^3} - \left[\frac{2a(-3)}{0.15} + \frac{12b}{(0.15)^2}\right]\delta x$$

$$F = \frac{1}{(0.15)^3} - \left[\frac{-2\times 1000}{0.05} + \frac{12\times 100}{(0.15)^2}\right]\delta x = mw^2\delta x$$

Clearly the motion is S.H.M. about $x = 0.15\ \mu m$

$$\Rightarrow \text{Time period } T = \frac{2\pi}{\omega} = 0.1\,ms$$

[:Q.55] A spectrometer can be used for

[:A] determining the of different lines seen in the spectrum

[:B] determining the extent to which specific spectral lines are Doppler shifted

[:C] determining the elements present in the atmosphere of a star

[:D] determining the physical properties of the outer cooler layers of a star.

[:ANS] **A, B, C**

[:SOLN] Conceptual

[:Q.56] Choose all the correct statements pertaining to Earth

[:A] Winter in southern hemisphere is longer than winter in Northern hemisphere

[:B] The Earth moves fastest in its orbit during summer in Northern hemisphere.

[:C] The Earth is closest to the Sun in January

[:D] The orbit of the Earth is farthest from the Sun close to the summer solstice in the Northern hemisphere

[:ANS] **A, B**

[:SOLN] Factual based

[:Q.57] A gas cloud will collapse under self-gravity if the expansive force due to gas pressure is insufficient to counter the self-gravitational force. Such collapse and condensation can lead to the formation of stars. One way of expressing the situation is to note that collapse can take place if the travel time of sound(speed of sound is rms speed of the gas molecules) across the cloud is more than the free fall time of the cloud under its own gravity. The condition for collapse may be specified as

(i) The radius of the cloud > Jeans length = $R_J = \dfrac{C_s}{(G\rho)^{1/2}} = \left(\dfrac{15k_sT}{4\pi G\mu\rho}\right)^{1/2}$ or

(ii) The mass of the cloud > Jeans mass = $\dfrac{4\pi}{3}R_J^3\rho$, which works out as $3\times10^4\left(\dfrac{T^3}{n}\right)^{1/2}$ in unit of solar mass.

Here, μ is the mean molecular mass of the particles of the cloud, ρ is the density, n is the number density i.e., the number of particles per cc and T is the temperature of the cloud.

Which of the following interstellar molecular hydrogen clouds are unstable to collapse ?

[:A] Giant molecular cloud of mass 10^4 solar masses, \with n = 10^5 per cc and T = 10 K

[:B] Molecular cloud core of mass 10 solar masses, with n = 10^5 per cc and T = 10 K

[:C] Molecular cloud core of mass equal to 1 solar mass, radius 0.001 pc and T = 10 K

[:D] A molecular cloud core of radius 10 pc, temperature 10 K with n = 10^4 per cc.

[:ANS] **C, D**

[:SOLN] (a) jeans mass = $3\times10^4\times\left(\dfrac{20^3}{100}\right)^{1/2}$

$= 26.83\times10^4$

Cloud mass < jean mass

Cloud will not collapse

(b) jeans mass = $3\times10^4\times\left(\frac{10^3}{10^5}\right)^{1/2}$

$=3\times10^3$ solar masses

(c) $\rho=\frac{M}{\frac{4}{3}\pi r^3}$

$4\pi\rho=\frac{M}{r^3}$

$$R_J=\left(\frac{15\times1.38\times10^{-23}\times\left(3.086\times10^{16}\times0.001\right)^3}{6.67\times10^{-11}\times2\times10^{-3}\times2\times10^{30}}\right)^{1/2}$$

$R_J=\left(22.8\times10^4\right)$

$\because r>R_J$ cloud will collapse

(c) mass cloud = 1.23×10^{21} solar masses

Jeans mass = $3\times10^4\times\left(\frac{10^3}{10^4}\right)^{1/2}$

$=0.9486\times10^4$ solar masses

$\therefore$ mass of cloud > jeans mass

$\therefore$ cloud collapses

[:Q.58] Consider a spherical satellite of mass m and radius r, held intact by its own gravity, moving in a circular orbit around a planet of mass M and radius R. Consider the following possible critical distance d_R (say), between the planet and the satellite. When the distance d is less than d_R the tidal force on the satellite, which is stretching the satellite along the line from the planet to the satellite, is greater than the self-gravitational force holding the satellite together. Hence if the planet-satellite separation d becomes less than d_R, the satellite disintegrates. Now consider the rotating reference frame in which both the planet and the satellite are stationary with respect to each other. In this frame, at the side of the satellite that is closest to the planet, the tidal acceleration towards the planet is given by $\frac{2GMr}{d^3}$. The magnitude of the acceleration due to self gravity of the satellite is $\frac{GM}{r^2}$. The most appropriate value of d_R may be obtained as

[:A] $R\left(\frac{2\rho_M}{\rho_m}\right)^{1/3}$ [:B] $R\left(\frac{\rho_M}{\rho_m}\right)^{1/3}$ [:C] $r\left(\frac{2M}{m}\right)^{1/3}$ [:D] $r\left(\frac{2\rho_M}{\rho_m}\right)^{1/2}$

[:ANS] **A, C**

[:SOLN] $\frac{2GMr}{d^3} = \frac{Gm}{r^2}$

$$d = \left(2r^3\frac{M}{m}\right)^{1/3} = r\left(\frac{2M}{m}\right)^{1/3}$$

$$\Rightarrow 2G\rho_m\frac{4}{3}\pi R^3(R^3) = G\rho_m\frac{4}{3}\pi R^3 d^3$$

$$\Rightarrow d = \left(\frac{2R^3\rho_M}{\rho_m}\right)^{1/3} = R\left(\frac{2\rho_M}{\rho_m}\right)^{1/3}$$

Correct answer A and C.

[:Q.59] Two bodies are moving under their mutual gravitational force in circular orbits. Their mass ratio is 2 : 5.

The ratio of their

[:A] Speeds of the two masses will be in ration of 5 : 2

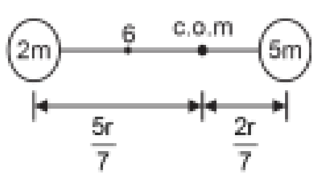
[:B] Angular speeds about COM will be in ratio of 4 : 25

[:C] Angular momenta about the COM will be I : 1

[:D] Kinetic energies will be 5 : 2

[:ANS] **A, D**

[:SOLN]



$$F = \frac{G(2m)(5m)}{r^2} = \frac{5mv_2^2}{\left(\frac{2r}{7}\right)} = \frac{2mv_2^2}{\left(\frac{5r}{7}\right)}$$

$$\Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{4}{25}} = \frac{2}{5}$$

Angular speed is same for both bodies w.r./c.o.m

Angular momenta will be different about the c.o.m

$$K \cdot E_1 = \frac{1}{2} 2m v_1^2 \text{ , } K \cdot E_2 = \frac{1}{2} 5m v_2^2 \text{ ,}$$

$$\frac{K \cdot E_1}{K \cdot E_2} = \left(\frac{2}{5}\right)\left(\frac{v_1}{v_2}\right)^2 = \left(\frac{2}{5}\right)\left(\frac{5}{2}\right)^2 = \frac{5}{2}$$

Correct answer A and D

[:Q.60] Assume that the Moon's orbit around the Earth is an almost circular ellipse and that neither its orbit nor its rotation axis processes. From the Earth we can see more than 50% of the surface of the Moon because

[:A] rotation period and revolution period of the Moon are not equal

[:B] the Moon's speed of revolution increases and decreases along the elliptical orbit

[:C] inclination of the Moon's orbit with the ecliptic plane makes the Moon alternately show more northern or more southern part

[:D] other side of the Moon is also visible sometimes

[:ANS] **B, C**

[:SOLN] (a) The rotational and revolution periods are equal.

(b) Because of elliptical path, speed increases and decreases

(c) This causes liberation in latitude letting us see beyond the near side at poles.

(d) Because of locking we can never see more than 50%.