|JEE MAIN 2025 | DATE : 24 JAN 2025 (SHIFT-2) EVENING
MATHEMATICS
SECTION 1

[:Q.1] If $\alpha > \beta > \gamma$ 0, then the expression

$$\cot^{-1}\left\{\beta+\frac{\left(1+\beta^2\right)}{(\alpha-\beta)}\right\}+\cot^{-1}\left\{\gamma+\frac{\left(1+\gamma^2\right)}{(\beta-\gamma)}\right\}+\cot^{-1}\left\{\alpha+\frac{\left(1+\alpha^2\right)}{(\gamma-\alpha)}\right\}$$ is equal to :

[:A] $\frac{\pi}{2}-(\alpha+\beta+\gamma)$

[:B] 0

[:C] π

[:D] 3π

[:ANS] **C**

[:SOLN] $\cot^{-1}\left(\frac{\alpha\beta-\beta^2+1+\beta^2}{\alpha-\beta}\right)+\cot^{-1}\left(\frac{\beta\gamma-\gamma^2+1+\gamma^2}{\beta-\gamma}\right)+\cot^{-1}\left(\frac{\alpha\gamma-\alpha^2+1+\alpha^2}{\gamma-\alpha}\right)$

$=\cot^{-1}\left(\frac{\alpha\beta+1}{\alpha-\beta}\right)+\cot^{-1}\left(\frac{\beta\gamma+1}{\beta-\gamma}\right)+\cot^{-1}\left(\frac{\alpha\gamma+1}{\gamma-\alpha}\right)$

$=\tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right)+\pi+\tan^{-1}\left(\frac{\gamma-\alpha}{1+\gamma\alpha}\right)$

$= (\tan^{-1}\alpha - \tan^{-1}\beta) + (\tan^{-1}\beta - \tan^{-1}\gamma) + (\pi + \tan^{-1}\gamma - \tan^{-1}\alpha)$

$= \pi$

[:Q.2] If the equation of the parabola with vertex $V\left(\frac{3}{2},3\right)$ and the directrix $x + 2y = 0$ is

$\alpha x^2 + \beta y^2 - \gamma xy - 30x - 60y + 225 = 0$, then $\alpha + \beta + \gamma$ is equal to:

[:A] 6

[:B] 7

[:C] 9

[:D] 8

[:ANS] **C**

[:SOLN] Equation of axis $y-3=2\left(x-\frac{3}{2}\right)$

$y-2x=0$

foot of directrix

$y-2x=0$

& $\Rightarrow (0,0)$

$2y+x=0$

Focus $=(3,6)$

$PS^2=PM^2$

$(x-3)^2+(y-6)^2=\left(\frac{x+2y}{\sqrt{5}}\right)^2$

$4x^2+y^2-4xy-30x-60y+225=0$

Compare, we get $\alpha=4, \beta=1, \gamma=4 \Rightarrow \alpha+\beta+\gamma=9$

[:Q.3] Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to :

[:A] 8925

[:B] 9100

[:C] 8575

[:D] 8750

[:ANS] **A**

[:SOLN] **Group A** **Group B**

Total Boys 7	Total girls 3	Total Boys 6	Total Girls 5	Total Selection
2	3	2	1	${}^7C_2 \times {}^3C_3 \times {}^6C_2 \times {}^5C_1$
3	2	1	2	${}^7C_3 \times {}^3C_2 \times {}^6C_1 \times {}^5C_2$
4	1	0	3	${}^7C_4 \times {}^3C_1 \times {}^6C_0 \times {}^5C_3$

Total ways = 1575 + 6300 + 1050 = 8925

[:Q.4] Let the points $\left(\frac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides x + y = 11, x + 2y = 16 and

2x + 3y = 29. Then the product of the smallest and the largest values of α is equal to :

[:A] 44

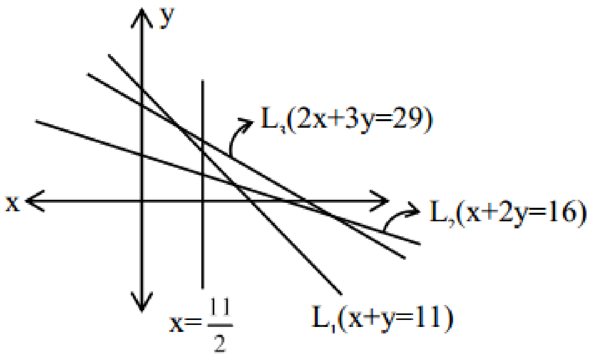
[:B] 55

[:C] 22

[:D] 33

[:ANS] **D**

[:SOLN]



Point of intersection of $x = \frac{11}{2}$ with L_1 & L_3

gives, $\alpha_{min} = \frac{11}{2}$

and $\alpha_{max} = 6$

$\therefore\ \alpha_{min} \cdot \alpha_{max} = \frac{11}{2} \times 6 = 33$

[:Q.5] The area of the region enclosed by the curves $y = e^x$, $y = |e^x - 1|$ and y-axis is:

[:A] $2 \log_e 2 - 1$

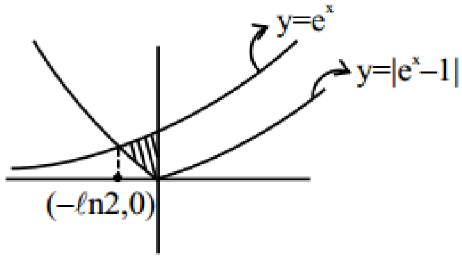
[:B] $1 + \log_e 2$

[:C] $1 - \log_e 2$

[:D] $\log_e 2$

[:ANS] **C**

[:SOLN]



Shaded Area $= \int_{-\ell n2}^{0}\left[e^{x}-\left(1-e^{x}\right)\right]dx$

$\int_{-\ell n2}^{0}\left(2e^{x}-1\right)dx=\left[2e^{x}-x\right]_{-\ell n2}^{0}$

$= (2-(1+\ell n2))$

$= 1-\ell n2$

[:Q.6] Let $A=\left\{x\in(0,\pi)-\left\{\frac{\pi}{2}\right\}:\log_{(2/\pi)}|\sin x|+\log_{(2/\pi)}|\cos x|=2\right\}$ and

$B=\left\{x\geq 0:\sqrt{x}\left(\sqrt{x}-4\right)-3\left|\sqrt{x}-2\right|+6=0\right\}$. Then $n\left(A\cup B\right)$ is equal to:

[:A] 2

[:B] 8

[:C] 6

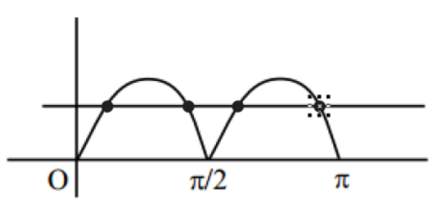
[:D] 4

[:ANS] B

[:SOLN] A : $\log_{2/\pi}|\sin x|+\log_{2/\pi}|\cos x|=2$

$\Rightarrow \log_{2/\pi}(|\sin x\,.\,\cos x|)=2$

$\Rightarrow |\sin 2x|=\frac{8}{\pi^{2}}$



Clearly Now Number of solution 4

B : let $\sqrt{x} = t < 2$

Then $\sqrt{x}\left(\sqrt{x}-4\right)+3\left(\sqrt{x}-2\right)+6=0$

$\Rightarrow t^2 - 4t + 3t - 6 + 6 = 0$ $\Rightarrow t^2 - t = 0$, $t = 0$, $t = 1$

$x = 0$, $x = 1$

again let $\sqrt{x} = t > 2$

then $t^2 - 4t - 3t + 6 + 6 = 0$

$\Rightarrow t^2 - 7t + 12 = 0 \Rightarrow t = 3, 4$

$x = 9, 16$

$\therefore$ Total number of solutions =

$N(A \cup B) = 4 + 4 = 8$

[:Q.7] If the system of equations

$x + 2y - 3z = 2$

$2x + \lambda y + 5z = 5$

$14x + 3y + \mu z = 33$

Has infinitely many solutions, then $\lambda + \mu$ is equal to :

[:A] 11

[:B] 13

[:C] 12

[:D] 10

[:ANS] **C**

[:SOLN] $\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 14 & 3 & \mu \end{vmatrix} = 0$, $\lambda\mu + 42\lambda - 4\mu + 107 = 0$

$\Delta_1 = 2\lambda\mu + 99\lambda - 10\mu + 255$

$\Delta_2 = 13 - \mu$

$\Delta_3 = 5\lambda + 5$

$\Delta_2 = 0 \Rightarrow \mu = 13$ & $\Delta_3 = 0 \Rightarrow \lambda = -1$

[:Q.8] Let (2, 3) be the largest open interval in which the function $f(x) = 2\log_e(x-2) - x^2 + ax + 1$ is strictly increasing and (b, c) be the largest open interval, in which the function $g(x) = (x-1)^3(x+2-a)^2$ is strictly decreasing. Then $100(a + b - c)$ is equal to:

[:A] 360

[:B] 160

[:C] 280

[:D] 420

[:ANS] **A**

[:SOLN] $f'(x) = \dfrac{2}{x-2} - 2x + a \geq 0$

$f''(x) = \dfrac{2}{x-2} - 2x + a \geq 0$

$f'(x)\downarrow$

$f'(x) \geq 0$

$2 - 6 + a \geq 0$

$a \geq 4 \Rightarrow a_{min} = 4$

$g(x) = (x-1)^3(x+2-a)^2$

$g(x) = (x-1)^3(x-2)^2$

$g'(x) = (x-1)^3\,2(x-2) + (x-2)^2\,3(x-1)^2$

$= (x-1)^2(x-2)(2x-2+3x-6)$

$= (x-1)^2(x-2)(5x-8) < 0$

$x \in \left(\dfrac{8}{5}, 2\right)$

$$100(a+b-c) = 100\left(4 + \frac{8}{5} - 2\right) = 360$$

[:Q.9] Suppose A and B are the coefficients of 30^{th} and 12^{th} terms respectively in the binomial expansion of $(1+x)^{2n-1}$. If 2A = 5B, then n is equal to:

[:A] 21

[:B] 19

[:C] 20

[:D] 22

[:ANS] **A**

[:SOLN] Coefficient of 30^{th} term = A

$\therefore$ A $= {}^{2n-1}C_{29}$ and B = ${}^{2n-1}C_{11}$

A/q,

$$2\ {}^{2n-1}C_{29} = 5\ {}^{2n-1}C_{11}$$

$$2\frac{(2n-1)!}{29!(2n-30)!} = 5\frac{(2n-1)!}{(2n-12)!11!}$$

$$\frac{1}{29...12\cdot 5} = \frac{1}{(2n-12)(2n-13)...(2n-29)2}$$

$$\frac{1}{30\cdot 29...12} = \frac{1}{(2n-12)(2n-13)...(2n-29)12}$$

2n – 12 = 30 $\Rightarrow$ n = 21

[:Q.10] Let $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \vec{a} \times (\hat{i} - 2\hat{k})$ and $\vec{c} = \vec{b} \times \hat{k}$. Then the projection of $\vec{c} - 2\hat{j}$ on $\vec{a}$ is:

[:A] $3\sqrt{7}$

[:B] $\sqrt{14}$

[:C] $2\sqrt{14}$

[:D] $2\sqrt{7}$

[:ANS] **C**

[:SOLN] $\vec{b} = \vec{a} \times (\hat{i} - 3\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = 2\hat{i} + 8\hat{j} + \hat{k}$$

$$\vec{c} = \vec{b} \times \hat{k} = 8\hat{i} - 2\hat{j}$$

$$\vec{c} - 2\hat{j} = 8\hat{i} - 4\hat{j}$$

Now, Projection of $(\hat{i} - 2\hat{j})$ on $\vec{a}$

$$\left(\vec{c}-2\hat{j}\right).\hat{a}=\frac{\langle 8,-4,0\rangle.\langle 3,-1,2\rangle}{\sqrt{14}}$$

$$=\frac{28}{\sqrt{14}}=2\sqrt{14}$$

[:Q.11] If 7 = $5+\frac{1}{7}(5+\alpha)+\frac{1}{7^2}(5+2\alpha)+\frac{1}{7^3}(5+3\alpha)+.....\infty$, then the value of α is :

[:A] $\frac{1}{7}$

[:B] $\frac{6}{7}$

[:C] 1

[:D] 6

[:ANS] **D**

[:SOLN]

$$\text{Let } S = 5+\frac{1}{7}(5+\alpha)+\frac{1}{7^2}(5+2\alpha)+\ldots$$

$$\frac{1}{7}S = \quad \frac{1}{7}(5) + \frac{1}{7^2}(5+\alpha) + \ldots\infty$$

$$\frac{6}{7}(S) = 5+\frac{1}{7}\alpha\left(\frac{1}{1-\frac{1}{7}}\right) \Rightarrow \quad 6=5+\frac{\alpha}{6}\Rightarrow\alpha=6$$

[:Q.12] Let A = [a_{ij}] be a square matrix of order 2 with entries either 0 or 1. Let E be the event that A is an invertible matrix. Then the probability P(E) is :

[:A] $\frac{1}{8}$

[:B] $\frac{3}{8}$

[:C] $\frac{5}{8}$

[:D] $\frac{3}{16}$

[:ANS] **B**

[:SOLN] C-I $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \to 4$ ways

C-II $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ & $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \to 2$ ways

Probability P(E) = $\dfrac{\text{favourable}}{\text{total}} = \dfrac{6}{16} = \dfrac{3}{8}$

[:Q.13] Let [x] denote the greatest integer function, and let m and n respectively be the numbers of the points, where the function f(x) = [x] + $|x-2|$, – 2 < x < 3, is not continuous and not differentiable. Then m + n is equal to:

[:A] 7

[:B] 9

[:C] 8

[:D] 6

[:ANS] **C**

[:SOLN] f(x) = [x] + $|x-2|$ $\quad -2 < x < 3$

$$f(x) = \begin{cases} -x, & -2 < x < -1 \\ -x+1, & -1 \le x < 0 \\ -x+2, & 0 \le x < 1 \\ -x+3, & 1 \le x < 2 \\ x, & 2 \le x < 3 \end{cases}$$

So f(x) is not continuous at 4 points and not differentiable at 4 point

So m + n = 4 + 4 = 8

[:Q.14] In an arithmetic progression, if S_{40} = 1030 and S_{12} = 57, then $S_{30} - S_{10}$ is equal to :

[:A] 510

[:B] 505

[:C] 525

[:D] 515

[:ANS] **D**

[:SOLN] $S_{40} = \dfrac{40}{2}[2a + 39d] = 1030$ ………….(1)

$S_{12} = \frac{12}{2}[2a + 11d] = 57$ ....(2)

by (1) & (2)

$a = -\frac{7}{2}$ $d = \frac{3}{2}$

$\therefore S_{30} - S_{10} = \frac{30}{2}[2a + 29d] - \frac{10}{2}[2a + 9d]$

$= 20a - 390d$

$= 515$

[:Q.15] The function f : (–∞, ∞) → (–∞, 1), defined by $f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$ is :

[:A] Both one-one and onto

[:B] Neither one-one nor onto

[:C] One-one but not onto

[:D] Onto but not one-one

[:ANS] **C**

[:SOLN] $f(x) = \frac{2^{2x} - 1}{2^{2x} + 1} = 1 - \frac{2}{2^{2x} + 1}$

Now, $f'(x) = \frac{2}{(2^{2x} + 1)^2} .2.2^{2x}.\ell n2$ i.e. always +ve

So f(x) is ↑ function

$\therefore f(-\infty) = -1$

$f(\infty) = 1$

$\therefore f(x) \in (-1, 1) \neq$ co-domain

so function is one-one but not onto

[:Q.16] The number of real solution(s) of the equation $x^2 + 3x + 2 = \min\{|x-3|, |x+2|\}$ is :

[:A] 0

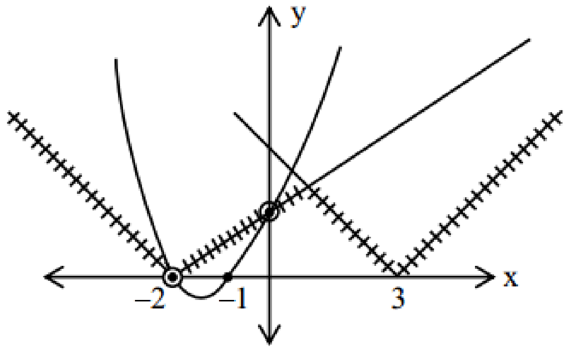
[:B] 1

[:C] 3

[:D] 2

[:ANS] **D**

[:SOLN] From graph



Only 2 solutions.

[:Q.17] Let the position vectors of three vertices of a triangle be $4\vec{p}+\vec{q}-3\vec{r}, -5\vec{p}+\vec{q}+2\vec{r}$ and $2\vec{p}-\vec{q}+2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$ and $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ -respectively, then $\alpha + 2\beta + 5\gamma$ is equal to:

[:A] 3

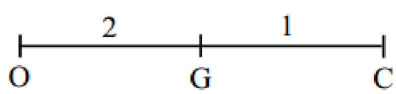
[:B] 1

[:C] 4

[:D] 6

[:ANS] **A**

[:SOLN] We know that



O (orthocentre) $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$

C (circum centre) $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$

C (centroid) = $\frac{\vec{p}+\vec{q}+\vec{r}}{3}$

We know, by relation

$$\Rightarrow\ 2\left(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}\right)+\frac{\vec{p}+\vec{q}+\vec{r}}{4}=3\left(\frac{\vec{p}+\vec{q}+\vec{r}}{3}\right)$$

$\Rightarrow \quad 8(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}) = 3(\vec{p}+\vec{q}+\vec{r})$

$\Rightarrow \quad 8\alpha = 3,\ 8\beta = 3,\ 8\gamma = 3 \Rightarrow \alpha = \frac{3}{8},\ \beta = \frac{3}{8},\ \gamma = \frac{3}{8}$

$\therefore \quad \alpha + 2\beta + 3\gamma = \frac{3}{8}+\frac{6}{8}+\frac{15}{8} = \frac{24}{8} = 3$

[:Q.18] The equation of the chord, of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid-point is (3, 1) is :

[:A] 4x + 122y = 134

[:B] 25x + 101y = 176

[:C] 48x + 25y = 169

[:D] 5x + 16y = 31

[:ANS] **C**

[:SOLN] use equation of chord with given middle point T = S_1

$\Rightarrow \quad \frac{3x}{25}+\frac{y}{16}-1 = \frac{9}{25}+\frac{1}{16}-1 \Rightarrow 48x + 25y = 144 + 25 \Rightarrow 48x + 25y = 169$

[:Q.19] For some a, b, let $f(x) = \begin{vmatrix} a+\frac{\sin x}{x} & 1 & b \\ a & 1+\frac{\sin x}{x} & b \\ a & 1 & b+\frac{\sin x}{x} \end{vmatrix}$, $x \neq 0$, $\lim_{x\to 0} f(x) = \lambda + \mu a + \nu b$.

Then $(\lambda + \mu + v)^2$ is equal to :

[:A] 16

[:B] 9

[:C] 36

[:D] 25

[:ANS] **A**

[:SOLN] $\lim_{x\to 0} f(x) = \begin{vmatrix} a+1 & 1 & b \\ a & 1+1 & b \\ a & 1 & b+1 \end{vmatrix}$

$= (a+1)(2(b+1)-b) + 1(ab - a(b+1)) + ba = (a+1)(b+2) - a + ab$

$\therefore\ b + a + 2 = \lambda + \mu a + \nu b$

$= b + a + 2 = \lambda + \mu a + \nu b$

$\lambda = 2, \mu = 1, \nu = 1 \Rightarrow (\lambda + \mu + \nu)^2 = 16$

[:Q.20] Let f : (0, ∞) → R be a function which is differentiable at all points of its domain and satisfies the condition $x^2f'(x) = 3xf(x) + 3$, with f(1) = 4. Then 2f(2) is equal to :

[:A] 23

[:B] 39

[:C] 29

[:D] 19

[:ANS] **B**

[:SOLN] $x^2f'(x) - 2x\, f(x) = 3$

$$\left(\frac{x^2f'(x) - 2xf(x)}{(x^2)^2}\right) = \frac{3}{(x^2)^2} \Rightarrow \frac{d}{dx}\left(\frac{f(x)}{x^2}\right) = \frac{3}{x^4}$$

Integrating both sides, we get

$$\frac{f(x)}{x^2} = -\frac{1}{x^3} + C$$

$$f(x) = -\frac{1}{x} + Cx^2$$

put x = 1

$4 = -1 + C \Rightarrow C = 5$

$$f(x) = -\frac{1}{x} + 5x^2$$

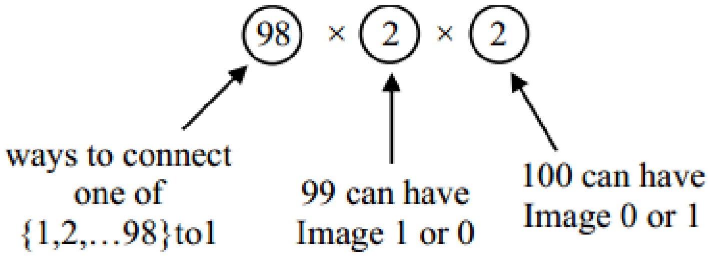
$$\text{Now } 2 \times f(2) = 2 \times \left[-\frac{1}{2} + 5 \times 2^2\right] = 39$$

SECTION 2

[:Q.21] Number of functions f : {1, 2, ……., 100} → {0, 1}, that assign 1 to exactly one of the positive integers less than or equal to 98, is equal to __________.

[:ANS] 392

[:SOLN]

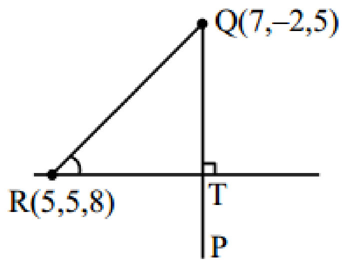


392 Ans.

[:Q.22] Let P be the image of the point Q(7, –2, 5) in the line $L:\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ and R(5, p, q) be a point on L. Then the square of the area of ΔPQR is _________.

[:ANS] 957

[:SOLN]



Let $R(2\lambda + 1, 3\lambda - 1, 4\lambda)$

$2\lambda + 1 = 5$

$\lambda = 2$

$R(5, 5, 8)$

let $T(2\lambda + 1, 3\lambda - 1, 4\lambda)$

$\overrightarrow{QT} = (2\lambda - 6)\hat{i} + (3\lambda + 1)\hat{j} + (4\lambda - 5)\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{QT}.\vec{b} = 0$

$4\lambda - 12 + 9\lambda + 3 + 16\lambda - 20 = 0$

$\lambda = 1$

$T(3, 2, 4)$

$QT = \sqrt{33} \quad RT = \sqrt{29}$

$(\text{area of } \Delta PQR)^2 = \left(\frac{1}{2}\sqrt{29}.2\sqrt{33}\right)^2$

$= 957$

[:Q.23] Let y = y(x) be the solution of the differential equation $2\cos x \frac{dy}{dx} = \sin 2x - 4y \sin x$, $x \in \left(0, \frac{\pi}{2}\right)$.

If $y\left(\frac{\pi}{3}\right) = 0$, then $y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right)$ is equal to _______.

[:ANS] **1**

[:SOLN] $\frac{dy}{dx} + 2y\tan x = \sin x$

$\text{I.F.} = e^{2\int \tan x\, dx} = \sec^2 x$

$y\sec^2 x = \int \frac{\sin x}{\cos^2 x} dx$

$= \int \tan x \sec x\, dx$

$= \sec x + C$

$C = -2$

$y = \cos x - 2\cos^2 x$

$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} - 1$

$y' = -\sin x + 4\cos x \sin x$

$y'\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} + 2$

$y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right) = 1$

[:Q.24] Let $H_1 : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $H_2 : -\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ be two hyperbolas having length of latus rectums $15\sqrt{2}$ and $12\sqrt{5}$ respectively. Let their eccentricities be $e_1 = \sqrt{\frac{5}{2}}$ and e_2 respectively. If the product of the lengths of their transverse axes is $100\sqrt{10}$, then $25e_2^2$ is equal to ________

[:ANS] **55**

[:SOLN] $\frac{2b^2}{a} = 15\sqrt{2}$

$1 + \frac{b^2}{a^2} = \frac{5}{2}$

$a = 5\sqrt{2}$

$b = 5\sqrt{3}$

$\frac{2A^2}{B} = 12\sqrt{5}$

$2a.2B = 100\sqrt{10}$

$2.5\sqrt{2}.2B = 100\sqrt{10}$

$B = 5\sqrt{5}$

$A = 5\sqrt{6}$

$e_2^2 = 1 + \frac{A^2}{B^2}$

$= 1 + \frac{150}{125}$

$e_2^2 = 1 + \frac{30}{25}$

$25e_2^2 = 55$

[:Q.25] If $\int \frac{2x^2+5x+9}{\sqrt{x^2+x+1}} dx = x\sqrt{x^2+x+1} + \alpha\sqrt{x^2+x+1} + \beta \log_e \left| x + \frac{1}{2} + \sqrt{x^2+x+1} \right| + C$, where C is the constant of integration, then $\alpha + 2\beta$ is equal to ________

[:ANS] **16**

[:SOLN] Put, $2x^2 + 5x + 9 = A(x^2 + x + 1) + B(2x + 1) + C$

$$A = 2 \qquad B = \frac{3}{2} \qquad C = \frac{11}{2}$$

$$2\int \sqrt{x^2 + x + 1}\, dx + \frac{3}{2}\int \frac{2x+1}{\sqrt{x^2 + x + 1}}\, dx + \frac{11}{2}\int \frac{dx}{\sqrt{x^2 + x + 1}}$$

$$2\int \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\, dx + 3\sqrt{x^2 + x + 1} + \frac{11}{2}\int \frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$2\left(\frac{x+\frac{1}{2}}{2}\sqrt{x^2 + x + 1} + \frac{3}{8}\ell n\left(x + \frac{1}{2} + \sqrt{x^2 + x + 1}\right)\right) + 3\sqrt{x^2 + x + 1}$$

$$+\frac{11}{2}\ell n\left(x + \frac{1}{2} + \sqrt{x^2 + x + 1}\right) + C$$

$$\alpha = \frac{7}{2} \qquad \beta = \frac{25}{4}$$

$$\alpha + 2\beta = 16$$