|JEE MAIN 2025 | DATE : 24 JAN 2025 (SHIFT-1) MORNING
MATHEMATICS
SECTION 1

[:Q.1] Let $S_n = \frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+....$upto n terms. If the sum of the first six terms of an A.P. with first term –p and common difference p is $\sqrt{2026S_{2025}}$, then the absolute difference between 20^{th} and 15^{th} terms of the A.P. is

[:A] 90

[:B] 20

[:C] 45

[:D] 25

[:ANS] **D**

[:SOLN] $S_n = \sum_{r=1}^{n}\frac{1}{r(r+1)} = \sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right) = 1-\frac{1}{n+1} = \frac{n}{n+1}$

$\sqrt{2026\,S_{2025}} = \frac{6}{2}\{-2p+5p\}$

$\Rightarrow \sqrt{2026\times\frac{2025}{2026}} = 9p \Rightarrow p = 5$

$|t_{20} - t_{15}| = |5p| = 25$

[:Q.2] Let the line passing through the points (–1, 2, 1) and parallel to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ intersect the line $\frac{x+2}{3}=\frac{y-3}{2}=\frac{z-4}{1}$ at the point P. Then the distance of P from the point Q (4, –5, 1) is

[:A] 5

[:B] 10

[:C] $5\sqrt{6}$

[:D] $5\sqrt{5}$

[:ANS] **D**

[:SOLN] Line through (–1, 2, 1) and parallel to

$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ is

$\frac{x+1}{2}=\frac{y-2}{3}=\frac{z-1}{4}=\lambda$ (say)

$\therefore$ P(2λ – 1, 3λ + 2, 4λ + 1)

P lies on the line $\frac{x+2}{3}=\frac{y-3}{2}=\frac{z-4}{1}$

$\therefore \frac{2\lambda-1+2}{3}=\frac{3\lambda+2-3}{2}=\frac{4\lambda+1-4}{1}$

$\Rightarrow$ 4λ + 2 = 9λ – 3 = 24λ – 18

$\Rightarrow$ λ = 1 ⇒ P = (1, 5, 5)

$\therefore$ PQ = $\sqrt{3^2+10^2+4^2}=5\sqrt{5}$

[:Q.3] A and B alternately throw a pair of dice. A wins if he throws a sum of 5 before B throws a sum of 8, and B wins if he throws a sum of 8 before A throws a sum of 5. The probability, that A wins if A makes the first throw, is

[:A] $\frac{9}{19}$

[:B] $\frac{9}{17}$

[:C] $\frac{8}{17}$

[:D] $\frac{8}{19}$

[:ANS] **A**

[:SOLN] E : getting a sum of 5

F : getting a sum of 8

$\therefore$ E = {(1, 4), (2, 3), (3, 2), (4, 1)}

F = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

P(E) = $\frac{4}{36}=\frac{1}{9}$, $P(F)=\frac{5}{36}$

$\therefore$ P(A wins) = P(E) + P(E′F′E) + P(E′F′E′F′E) + ….

$$= \frac{1}{9} + \frac{8}{9} \cdot \frac{31}{36} \cdot \frac{1}{9} + \left(\frac{8}{9}\right)^2 \left(\frac{31}{36}\right)^2 \cdot \frac{1}{9} + \ldots\ldots$$

$$= \frac{\frac{1}{9}}{1 - \frac{8}{9} \cdot \frac{31}{36}} = \frac{9}{19}.$$

[:Q.4] The area of the region $\{(x, y) : x^2 + 4x + 2 \le y \le |x+2|\}$ is equal to

[:A] 24/5

[:B] 5

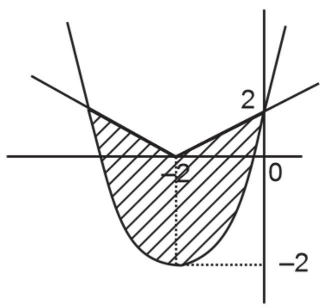
[:C] 7

[:D] 20/3

[:ANS] **D**

[:SOLN] $y = x^2 + 4x + 2 = (x + 2)^2 - 2, \; y = |x+2|$

Required area



$$= 2\int_{-2}^{0} \left(|x+2| - \left((x+2)^2 - 2\right)\right) dx$$

$$= 2\int_{-2}^{0} \left((x+2) - \left(x^2 + 4x + 2\right)\right) dx$$

$$= 2\int_{-2}^{0} \left(-x^2 - 3x\right) dx$$

$$= 2\left[-\frac{x^3}{2} - \frac{3x^2}{2}\right]_{-2}^{0} = -2\left(\frac{8}{3} - 6\right)$$

$$= \frac{20}{3}$$

[:Q.5] Let the product of the focal distance of the point $\left(\sqrt{3},\frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.(a>b)$, be $\frac{7}{4}$. Then the absolute difference of the eccentricities of two such ellipses is

[:A] $\frac{3-2}{3\sqrt{2}}$

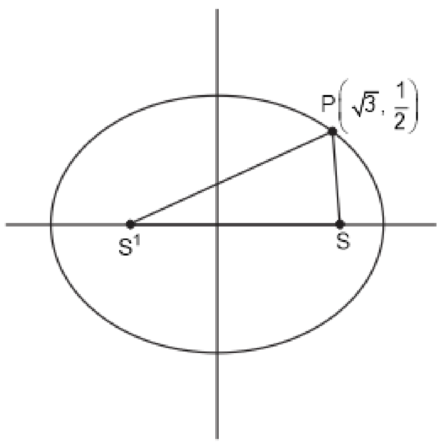
[:B] $\frac{1-2\sqrt{2}}{\sqrt{3}}$

[:C] $\frac{1-\sqrt{3}}{\sqrt{2}}$

[:D] $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

[:ANS] **D**

[:SOLN] $\left(\sqrt{3},\frac{1}{2}\right)$ lies on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



$\Rightarrow \quad \frac{3}{a^2}+\frac{1}{4b^2}=1$ ------------(1)

$PS \cdot PS^1 = \frac{7}{4} \Rightarrow (a + e\cdot\sqrt{3})\left(a-e\sqrt{3}\right)=\frac{7}{4}$

$\Rightarrow \quad a^2 - 3e^2 = \frac{7}{4}$ ------------(2)

(1), $\quad \frac{3}{a^2}+\frac{1}{4a^2\left(1-e^2\right)}=1$

$13 - 12e^2 = 4a^2(1 - e^2) = 4(3e^2 + \frac{7}{4})(1 - e^2)$

$= 7 + 5e^2 - 12e^4$

$12e^4 - 17e^2 + 6 = 0$

$\Rightarrow \quad e^2 = \frac{3}{4} \text{ or } \frac{2}{3}$

$\therefore \quad |e_1 - e_2| = \left|\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{\sqrt{3}}\right| = \frac{3 - 2\sqrt{2}}{2\sqrt{3}}$

[:Q.6] Let circle C be the image of $x^2 + y^2 - 2x + 4y - 4 = 0$ in the line $2x - 3y + 5 = 0$ and A be the point on C such that OA is parallel to x-axis and A lies on the right hand side of the centre O of C. If $B(\alpha, \beta)$, with $\beta < 4$, lies on C such that the length of the are AB is (1/6)th of the perimeter of C, then $\beta - \sqrt{3}\alpha$ is equal to

[:A] $3 + \sqrt{3}$

[:B] 4

[:C] 3

[:D] $4 - \sqrt{3}$

[:ANS] **B**

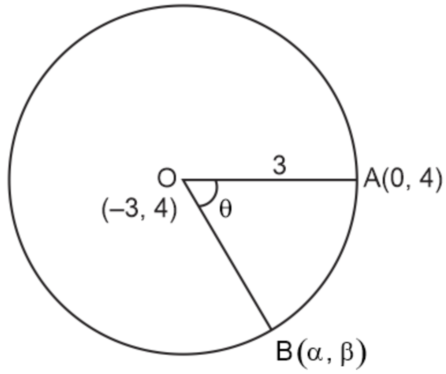
[:SOLN] $(x - 1)^2 + (y + 2)^2 = 3^2$

Image of centre (1, –2) about $2x - 3y + 5 = 0$;

$\frac{h-1}{2} = \frac{k+2}{-3} = \frac{-2(2+6+5)}{13} = -2$

$\Rightarrow \quad h = -3, k = 4 \Rightarrow O = (-3, 4)$

$C : (x + 3)^2 + (y - 4)^2 = 3^2$



$\angle AOB = \left(2\pi \times \frac{1}{6}\right) = \frac{\pi}{3}, \quad A = (0, 4)$

$B(\alpha, \beta) = \left(-3 + 3\cos\left(-\frac{\pi}{3}\right), 4 + 3\sin\left(-\frac{\pi}{3}\right)\right) = \left(-\frac{3}{2}, 4 - \frac{3\sqrt{3}}{2}\right)$

$\beta - \sqrt{3}\alpha = 4 - \frac{3\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} = 4$

[:Q.7] Consider the region $R\left\{(x, y) : x \le y \le 9 - \frac{11}{3}x^2, x \ge 0\right\}$.

The area, of the largest rectangle of sides parallel to the coordinate axes and inscribed in R, is:

[:A] $\frac{730}{119}$

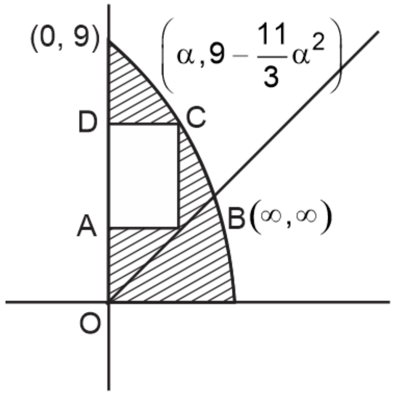
[:B] $\frac{567}{121}$

[:C] $\frac{625}{111}$

[:D] $\frac{821}{123}$

[:ANS] **B**

[:SOLN]



$y \geq x,\ y \leq 9 - \frac{11}{3}x^2,\ x \geq 0$

Let B = (α, α)

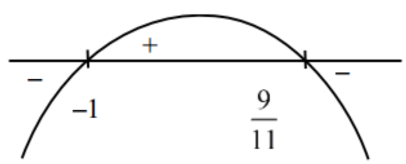
$\therefore$ C = $\left(\alpha, 9 - \frac{11}{3}\alpha^2\right)$

Ar(ABCD) = $\alpha\left(9 - \frac{11}{3}\alpha^2 - \alpha\right)$

$= -\frac{11}{3}\alpha^2 - \alpha^2 + 9\alpha$

$= f(\alpha)$ (say)

$f'(\alpha) = -11\alpha^2 - 2\alpha + 9 = -(11\alpha - 9)(\alpha + 1)$



$\therefore$ Maximum area = $f\left(\frac{9}{11}\right) = \frac{567}{121}$

[:Q.8] If I (m, n) $= \int_0^1 x^{m-1}(1-x)^{n-1}dx, m, n > 0$, then I (9, 14) + I (10, 13) is

[:A] $I(9,1)$

[:B] $I(19,27)$

[:C] $I(9,13)$

[:D] $I(1,13)$

[:ANS] **C**

[:SOLN] I(9, 14) + I(10, 13) = $\int_0^1 \left(x^8(1-x)^{13} + x^9(1-x)^{12}\right)dx$

$= \int_0^1 x^8(1-x)^{12}(1-x+x)dx$

$= \int_0^1 x^8(1-x)^{12}dx = I(9,13)$

[:Q.9] Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If the vector $\vec{c}$ is perpendicular to $\vec{b}$ and $\vec{a}\cdot\vec{c} = 5$, then $|\vec{c}|$ is equal to

[:A] $\frac{1}{3\sqrt{2}}$

[:B] 16

[:C] $\sqrt{\frac{11}{6}}$

[:D] 18

[:ANS] **C**

[:SOLN] $\vec{c} = \lambda(\vec{a}\times\vec{b})\times\vec{b} = \lambda\left((\vec{a}\cdot\vec{b})\vec{b} - (\vec{b}\cdot\vec{b})\vec{a}\right)$

$= \lambda(2\vec{b} - 11\vec{a})$

$\vec{a}\cdot\vec{c} = \lambda(2\vec{a}\cdot\vec{b} - 11\vec{a}\cdot\vec{a}) = 5$

$\Rightarrow \quad \lambda(4 - 11\times 14) = 5 \Rightarrow \lambda = \frac{-1}{30}$

$\therefore \quad \vec{c} = \frac{-1}{30}(6\hat{i} + 2\hat{j} - 2\hat{k} - 11\hat{i} - 22\hat{j} - 33\hat{k})$

$= \frac{5\hat{i} + 20\hat{j} + 35\hat{k}}{30} = \frac{\hat{i} + 4\hat{j} + 7\hat{k}}{6}$

$\therefore \quad |\vec{c}| = \frac{\sqrt{66}}{6} = \sqrt{\frac{11}{6}}$

[:Q.10] Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be a function such that $f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$.

If the $\lim_{x\to 0}\left(\frac{1}{\alpha x}+f(x)\right)=\beta;\alpha\beta\in\mathbb{R}$, then $\alpha+2\beta$ is equal to

[:A] 5

[:B] 3

[:C] 4

[:D] 6

[:ANS] C

[:SOLN] $f(x)-6f\left(\frac{1}{x}\right)=\frac{35}{3x}-\frac{5}{2}$ ------------(1)

Replacing x by $\frac{1}{x}$,

$f\left(\frac{1}{x}\right)-6f(x)=\frac{35x}{3}-\frac{5}{2}$ -------------(2)

(1) + (2) × 6, $-35f(x) = \frac{35}{3x}+70x-\frac{35}{2}$

$\Rightarrow$ $f(x) = \left(-2x-\frac{1}{3x}+\frac{1}{2}\right)$

$\beta = \lim_{x\to 0}\left(\frac{1}{\alpha x}-2x-\frac{1}{3x}+\frac{1}{2}\right)$

$\therefore$ $\alpha = 3,\ \beta = \frac{1}{2}$

$\alpha + 2\beta = 4$

[:Q.11] Let in a ΔABC, the length of the side AC be 6, the vertex B be (1, 2, 3) and the vertices A, C lie on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Then the area (in sq. units) of ΔABC is:

[:A] 21

[:B] 17

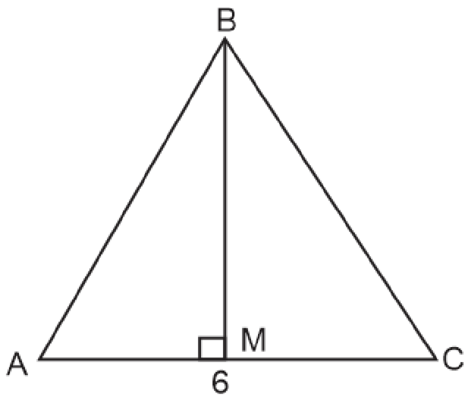
[:C] 42

[:D] 56

[:ANS] A

[:SOLN] $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}=\alpha$

$M = (3\alpha + 6, 2\alpha + 7, -2\alpha + 7)$



$BM \perp AC \Rightarrow 3(3\alpha + 5) + 2(2\alpha + 5) - 2(-2\alpha + 4) = 0$

$\Rightarrow \quad \alpha = -1$

$\therefore \quad M = (3, 5, 9)$

$\therefore \quad BM = \sqrt{2^2 + 3^2 + 6^2} = 7$

$\therefore \quad \text{ar}\,(\Delta ABC) = \frac{1}{2} \times 6 \times 7 = 21$

[:Q.12] If the system of equations

$2x - y + z = 4$

$5x + \lambda y + 3z = 12$

$100x - 47y + \mu z = 212,$

has infinitely many solutions, then $\mu - 2\lambda$ is equal to

[:A] 59

[:B] 57

[:C] 56

[:D] 55

[:ANS] **B**

[:SOLN] $2x - y + z = 4$ ----------(1)

$5x + \lambda y + 3z = 12$ ---------(2)

$100x - 47y + \mu z = 212$ ------(3)

(3) – (1) × 50, $3y + (\mu - 50)z = 12$ -------(4)

(2) × 2 – (1) × 5, $(2\lambda + 5)y + z = 4$ --------(5)

For infinite solution, $\frac{3}{2\lambda+5} = \frac{\mu-50}{1} = \frac{12}{4}$

$\Rightarrow$ $\lambda = -2, \mu = 53$

$\mu - 2\lambda = 57$

[:Q.13] The product of all the rational roots of the equation $(x^2 - 9x + 11)^2 - (x-4)(x-5) = 3$, is equal to

[:A] 14

[:B] 21

[:C] 7

[:D] 28

[:ANS] **A**

[:SOLN] $(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 3$

Let $x^2 - 9x + 11 = y$

$\therefore y^2 - (y + 9) = 3$

$\Rightarrow$ $y^2 - y - 12 = 0$

$\Rightarrow$ $(y - 4)(y + 3) = 0$

$\Rightarrow$ $x^2 - 9x + 7 = 0$ or $x^2 - 9x + 14 = 0$

$\therefore$ rational roots 2 and 7

Product = 14.

[:Q.14] Let the lines $3x - 4y - \alpha = 0, 8x - 11y - 33 = 0$, and $2x - 3y + \lambda = 0$ be concurrent. If the image of the point (1, 2) in the line $2x - 3y + \lambda = 0$ is $\left(\frac{57}{13}, \frac{-40}{13}\right)$, than $|\alpha\lambda|$ is equal to

[:A] 101

[:B] 84

[:C] 91

[:D] 113

[:ANS] **C**

[:SOLN] $\begin{vmatrix} 3 & -4 & -\alpha \\ 8 & -11 & -33 \\ 2 & -3 & \lambda \end{vmatrix} = 0$

$\Rightarrow$ $-\alpha(-24 + 22) + 33(-3 + 8) + \lambda(-33 + 32) = 0$

$\Rightarrow$ $2\alpha - \lambda - 33 = 0$

Mid point of (1, 2) and $\left(\frac{57}{13}, \frac{-40}{13}\right)$ lies on $2x - 3y + \lambda = 0$

$\therefore\ 2\left(\frac{35}{13}\right) - 3\left(\frac{-7}{13}\right) + \lambda = 0 \Rightarrow \lambda = -7$

$\therefore$ $\alpha = 13$

$\therefore$ $|\alpha\lambda| = 91$

[:Q.15] For some $n \neq 10$, let the coefficients of the 5th 6th and 7th terms in the binomial expansion of $(1+x)^{n+4}$ be in A.P. Then the largest coefficient in the expansion of $(1+x)^{n+4}$ is:

[:A] 70

[:B] 10

[:C] 35

[:D] 20

[:ANS] **C**

[:SOLN] ${}^{n+4}C_4, {}^{n+4}C_5, {}^{n+4}C_6$ are in A.P.

$\Rightarrow$ ${}^{n+4}C_4 + {}^{n+4}C_6 = {}^{n+4}C_5 \times 2$

$\Rightarrow$ $\frac{{}^{n+4}C_4}{{}^{n+4}C_5} + \frac{{}^{n+4}C_6}{{}^{n+4}C_5} = 2$

$\Rightarrow$ $\frac{5}{n+4-5+1} + \frac{n+4-6+1}{6} = 2$ $\left(\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right)$

$\Rightarrow$ $\frac{5}{n} + \frac{n-1}{6} = 2$

$\Rightarrow$ $n^2 - n + 30 = 12n \Rightarrow n^2 - 13n + 30 = 0$

$\Rightarrow$ $n = 3$ or 10

But $n \neq 10$ $\therefore\ n = 3$

$\therefore$ Largest coefficient = ${}^7C_3 = 35$

[:Q.16] If α and β are the roots of the equation $2z^2 - 3z - 2i = 0,$ where $i = \sqrt{-1},$ then 16.

$R\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right).\text{Im}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)$ is equal to

[:A] 409

[:B] 312

[:C] 398

[:D] 441

[:ANS] **D**

[:SOLN] $2z^2 - 3z - 2i = 0$

$\Rightarrow \quad (2z^2 - 2i)^2 = (3z)^2$

$\Rightarrow \quad 4z^4 - 8iz^2 - 4 = 9z^2$

$\Rightarrow \quad (4z^4 - 4)^2 = ((9 + 8i)z^2)^2$

$\Rightarrow \quad 16z^8 - 32z^4 + 16 = (17 + 144i)z^4$

$\Rightarrow \quad 16(z^8 + 1) = (49 + 144i)z^4$

Let $S_n = \alpha^n + \beta^n$, $n \in N$

$\therefore \frac{S_{19}+S_{11}}{S_{15}} = \frac{49+144i}{16}$

$\therefore 16 \cdot \text{Re}\left(\frac{S_{19}+S_{11}}{S_{15}}\right) \cdot \text{Im}\left(\frac{S_{19}+S_{11}}{S_{15}}\right)$

$= 16 \cdot \frac{49}{16} \cdot \frac{144}{16} = 441$

[:Q.17] Let y = y(x) be the solution of the differential equation

$\left(xy - 5x^2\sqrt{1+x^2}\right)dx + \left(1+x^2\right)dy = 0, y(0) = 0.$ Then $y\left(\sqrt{3}\right)$ is equal to

[:A] $2\sqrt{2}$

[:B] $\frac{5\sqrt{3}}{2}$

[:C] $\sqrt{\frac{15}{2}}$

[:D] $\sqrt{\frac{14}{3}}$

[:ANS] **B**

[:SOLN] $\frac{dy}{dx}+\frac{xy}{1+x^2}=\frac{5x^2}{\sqrt{1+x^2}}$

I.F. = $e^{\int \frac{x}{1+x^2}dx}$ = $e^{\frac{1}{2}\ln\left(1+x^2\right)}$ = $\sqrt{1+x^2}$

$\therefore$ solution is $y\sqrt{1+x^2}=\int \frac{5x^2}{\sqrt{1+x^2}}\sqrt{1+x^2}dx+C$

i.e. $y\sqrt{1+x^2}=\frac{5x^3}{3}+C$

y(0) = 0 $\Rightarrow$ C = 0

$\therefore\ y\left(\sqrt{3}\right)=\frac{5\sqrt{3}}{\sqrt{4}}=\frac{5\sqrt{3}}{2}$

[:Q.18] Let $f\left(x\right)=\frac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$. Then the value of $8\left(f\left(\frac{1}{15}\right)+f\left(\frac{2}{15}\right)+......+f\left(\frac{59}{15}\right)\right)$ is equal to

[:A] 108

[:B] 118

[:C] 92

[:D] 102

[:ANS] **B**

[:SOLN] $f\left(x\right)=\frac{2^{x+1}+8}{2^{2x}+2^{x+3}+16}$

$f\left(4-x\right)=\frac{2^{5-x}+8}{2^{8-2x}+2^{7-x}+16}$

$=\frac{2^{5+x}+8.2^{2x}}{2^8+2^{7+x}+16.2^{2x}}$

$=\frac{2^{x+1}+2^{2x-1}}{2^{2x}+2^{x+3}+16}$

$\therefore \quad f(x)+f(4-x)=\dfrac{2\cdot 2^{x+1}+2^{2x-1}+8}{2^{2x}+2^{x+3}+16}$

$=\dfrac{2^{x+2}+2^{2x-1}+8}{2\left(2^{2x-1}+2^{x+2}+8\right)}=\dfrac{1}{2}$

$\therefore \quad 8\left(f\left(\dfrac{1}{15}\right)+f\left(\dfrac{59}{15}\right)+\left(f\left(\dfrac{2}{15}\right)+f\left(\dfrac{58}{15}\right)\right)+......\text{to 29 terms } + f(2)\right)$

$=8\left(\dfrac{1}{2}\times 29+\dfrac{16}{64}\right)=118$

[:Q.19] For a statistical data x_1, x_2, ……. , x_{10} of 10 values, a student obtained the mean as 5.5 and $\sum_{i=1}^{10} x_i^2 = 371$ He later found that he had noted two values in the data incorrect as 4 and 5, instead of the correct values of 6 and 8, respectively. The variance of then corrected data is

[:A] 5

[:B] 4

[:C] 9

[:D] 7

[:ANS] **D**

[:SOLN] $\bar{x}_{\text{correct}} = \bar{x}_{\text{incorrect}} + \dfrac{6+8-(4+5)}{10}$

= 5 . 5 + 0.5 = 6

$\left(\sum_{i=1}^{10} x_i^2\right)_{\text{correct}} = \left(\sum_{i=1}^{10} x_i^2\right)_{\text{incorrect}} + \left(6^2+8^2\right)-\left(4^2+5^2\right)$ = 371 + 36 + 64 – 16 – 25

= 480

$\therefore$ correct variance = $\dfrac{430}{10}-6^2$

= 7

[:Q.20] $\lim_{x\to 0} \operatorname{cosec} x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$

[:A] $-\dfrac{1}{2\sqrt{5}}$

[:B] $\frac{1}{\sqrt{15}}$

[:C] $\frac{1}{2\sqrt{5}}$

[:D] 0

[:ANS] **A**

[:SOLN] $\lim_{x\to 0} \operatorname{cosec}x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$

$$=\lim_{x\to 0} \operatorname{cosec}x\frac{\left(2\cos^2 x+3\cos x\right)-\left(\cos^2 x+\sin x+4\right)}{\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}}$$

$$=\lim_{x\to 0}\frac{\cos^2 x+3\cos x-\sin x-4}{\sin x\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$=\lim_{x\to 0}\frac{\left(\cos^2 x-1\right)+3\left(\cos x-1\right)-\sin x}{2\sqrt{5}\sin x}$$

$$=\lim_{x\to 0}\frac{-\sin x-\frac{6\sin^2\frac{x}{2}}{\sin x}-1}{2\sqrt{5}}$$

$$=-\frac{1}{2\sqrt{5}}$$

SECTION 2

[:Q.21] If for some $\alpha\le\beta,\alpha+\beta=8$ and $\sec^2\left(\tan^{-1}\alpha\right)+\operatorname{cosec}^2\left(\cot^{-1}\beta\right)=36$, then $\alpha^2+\beta$ is ________.

[:ANS] **14**

[:SOLN] Let $\tan^{-1}\alpha = \theta$ and $\cot^{-1}\beta = \phi$

$\therefore \quad \sec^2\theta + \operatorname{cosec}^2\phi = 36$

$\Rightarrow \quad 1 + \tan^2\theta + 1 + \cot^2\phi = 36$

$\Rightarrow \quad \alpha^2 + \beta^2 = 34$ ---------(1)

$\alpha + \beta = 8$ ---------(2)

$\therefore \quad (\beta - \alpha)^2 = 2(\alpha^2 + \beta^2) - (\alpha + \beta)^2$

$= 68 - 64 = 4$

$\therefore \quad \beta - \alpha = 2$ ----------(3) $(\alpha \le \beta)$

$\therefore \quad \alpha = 3, \beta = 5$

$\therefore \quad \alpha^2 + \beta = 14$

[:Q.22] Let A be 3×3 matrix such that $X^TAX = O$ all nonzero 3×1 matrices $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. If

$A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, A\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$, and det (adj (2(A + I))) = $2^\alpha 3^\beta 5^\gamma, \alpha, \beta, \gamma \in \mathbb{N}$, then $\alpha^2 + \beta^2 + \gamma^2$ is

_______.

[:ANS] **44**

[:SOLN] let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$$[x \;\; y \;\; z]\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\Rightarrow \quad x(a_1x + a_2y + a_3z) + y(b_1x + b_2y + b_3z) + z(c_1x + c_2y + c_3z) = 0$

$\Rightarrow \quad a_1x^2 + b_2y^2 + c_3z^2 + (a_2 + b_1)xy + (b_3 + c_2)yz + (a_3 + c_1)zx = 0 \qquad \forall x, y, z$

$\therefore \quad a_1 = b_2 = c_3 = a_2 + b_1 = b_3 + c_2 = a_3 + c_1 = 0$

$\therefore \quad A = \begin{bmatrix} 0 & b_1 & c_1 \\ -b_1 & 0 & c_2 \\ -c_1 & -c_2 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & b_1 & c_1 \\ -b_1 & 0 & c_2 \\ -c_1 & -c_2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 4 \\ -5 & -8 \end{bmatrix}$$

$\Rightarrow \quad b_1 + c_1 = 1, 2b_1 + c_1 = 0, -b_1 + c_2 = 4, -c_1 - c_2 = -5, -c_1 - 2c_2 = -8$

$\Rightarrow \quad b_1 = -1, c_1 = 2, c_2 = 3$

$|\text{Adj}(2(A+I))| = |2(A+I)|^2$

$$= 2^6|A+I|^2 = 2^6\begin{vmatrix} 1 & -1 & 2 \\ 1 & 1 & 3 \\ -2 & -3 & 1 \end{vmatrix}^2$$

$= 2^6 \cdot 3^2 \cdot 5^2 = z^{\alpha}\, 3^{\beta}\, 5^{\gamma}$

$\alpha^2 + \beta^2 + \gamma^2 = 6^2 + 2^2 + 2^2 = 44$

[:Q.23] The number of 3-digit numbers, that are divisible by 2 and 3, but not divisible by 4 and 9, is ______.

[:ANS] **50**

[:SOLN] The numbers must be of the form 6k, where $k \in \{17, 18, \ldots\ldots, 166\}$ 2 + k and 3 + k.

So number of such no.

= n{17, 18, ……, 166} – n({18, 20, ………., 166})

– n({18, 21, ………, 165}) + n({18, 24, …….., 162})

= 150 – 75 – 50 + 25 = 50

[:Q.24] Let f be a differentiable function such that $2(x+2)^2 f(x) - 3(x+2)^2 = 10\int_0^x (t+2)f(t)dt, x \geq 0.$

Then f(2) is equal to _______.

[:ANS] **19**

[:SOLN] $2(x+2)^2 f(x) - 3(x+2)^2 f'(x) = 10\int_0^x (t+2)f(t)\,dt$ --------(1)

Defferentiation

$4(x+2)f(x) + 2(x+2)^2 f'(x) - 6(x+2) = 10(x+2)f(x)$

$\Rightarrow \quad f'(x) - \frac{3}{x+2}f(x) = \frac{3}{x+2}$

I.F. = $e^{\int \frac{-3}{x+2}dx} = e^{-3\ln(x+2)} = (x+2)^{-3}$

$\therefore$ solution is

$f(x)(x+2)^{-3} = \int \frac{3}{x+2}(x+2)^{-3}dx + C = -(x+2)^{-3} + C$

$\Rightarrow \quad f(x) = c(x+2)^3 - 1$ ---------(2)

Putting x = 0 in (1),

$8f(0) - 12 = 0 \Rightarrow f(0) = \frac{3}{2}$

$\therefore$ from (2), $\frac{3}{2} = -8c - 1$

$\Rightarrow \quad c = \frac{5}{16}$

$\therefore \quad f(2) = \frac{5}{16} \times 4^3 - 1 = 19$

[:Q.25] Let $S = \{p_1, p_2 \ldots\ldots, p_{10}\}$ be the set of first ten prime numbers. Let A = S U P, where P is the set of all possible products of distinct elements of S. Then the number of all ordered pairs (x, y) $x \in S$, $y \in A$, such that x divides y, is________.

[:ANS] 5120

[:SOLN] No. of elements in A divisible by P_1

= total no. of combinations of $\{P_2, \ldots\ldots\ldots, P_{10}\}$

$= 2^9$

Similarly for P_2, P_3, ……….. , P_{10}

$\therefore$ the no. of ordered pairs = $2^9 \times 10$

= 5120