|JEE MAIN 2025 | DATE : 23 JAN 2025 (SHIFT-1) MORNING

MATHEMATICS

SECTION 1

[:Q.1] Let $R\{(1,2),(2,3),(3,3)\}$ be a relation defined on the set {1, 2, 3, 4}. Then the minimum number of elements, needed to be added in R so that R becomes an equivalence relation, is:

[:A] 10

[:B] 7

[:C] 8

[:D] 9

[:ANS] **2**

[:SOLN] To make the given relation reflexive we have to add (1,1), (2,2), (4,4) then to make symmetric we have to add (2,1), (3,2) and to ensure that it is transitive we have to add (1,3), (3,1). Therefore, minimum number of ordered pairs to be added is 7.

[:Q.2] Let f (x) = $\log_e$ x and $g(x)\dfrac{x^2-2x^3+3x^2-2x+2}{2x^2-2x+1}$.

Then the domain of fog is

[:A] $(0,\infty)$

[:B] $[1,\infty)$

[:C] $[0,\infty)$

[:D] $\mathbb{R}$

[:ANS] **4**

[:SOLN] $D_{fog}=\{x:g(x)\in D_f\}$

$$\therefore\ \frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}>0$$

i.e $\dfrac{(x^2+1)(x^2-2x+2)}{2x^2-2x+1}>0$

True $\forall\ x\in R$

[:Q.3] If the line 3x – 2y + 12 = 0 intersects the parabola $4y = 3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to

[:A] $\tan^{-1}\left(\frac{9}{7}\right)$

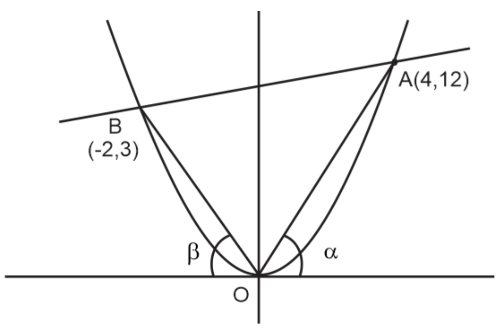
[:B] $\tan^{-1}\left(\frac{4}{5}\right)$

[:C] $\tan^{-1}\left(\frac{11}{9}\right)$

[:D] $\frac{\pi}{2} - \tan^{-1}\left(\frac{3}{2}\right)$

[:ANS] **A**

[:SOLN] Solving the parabola $x^2 = \frac{4}{3}y$ with



The line $3x - 2y + 12 = 0$ we

get $A \equiv (4,12)$ & $B = (-2,3)$

required angle = $\pi - (\alpha + \beta)$

$$= \pi - \left(\tan^{-1} 3 + \tan^{-1} \frac{3}{2}\right)$$

$$= \tan^{-1} \frac{9}{7}$$

[:Q.4] Let a curve y = f(x) pass through the points (0, 5) and ($\log_e 2$, k). If the curve satisfies the differential equation $2(3 + y)e^{2x}dx - (7 + e^{2x})dy = 0$, then k is equal to

[:A] 8

[:B] 16

[:C] 32

[:D] 4

[:ANS] **A**

[:SOLN] $2(3+y)e^{2x}dx-(7+e^{2x})dy=0$

$$\frac{2(3+y)e^{2x}dx-e^{2x}dy}{(3+y)^2}=\frac{7dy}{(3+y)^2}$$

$$d\left(\frac{e^{2x}}{3+y}\right)=\frac{7dy}{(3+y)^2}$$

$$\frac{e^{2x}}{3+y}=\frac{-7}{3+y}+C$$

Curve passes through (0, 5)

$\therefore$ C = 1

$\therefore$ the curve is $\frac{e^{2x}}{3+y}=-\frac{7}{3+y}+1$

It has to pass through $(\log_e 2, k)$

$\Rightarrow k=8$

[:Q.5] Let the are AC of a circle subtend a right angle at the centre O. If the point B on the are AC, divides the arc AC such that $\frac{\text{length of are AB}}{\text{length of are BC}}=\frac{1}{5}$, and $\overrightarrow{OC}=\alpha\overrightarrow{OA}+\beta\overrightarrow{OB}$, then $\alpha+\sqrt{2}(\sqrt{3}-1)\beta$ is equal to

[:A] $2-\sqrt{3}$

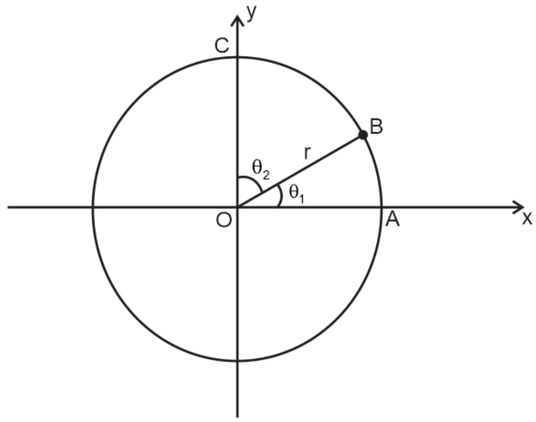
[:B] $2\sqrt{3}$

[:C] $2+\sqrt{3}$

[:D] $5\sqrt{3}$

[:ANS] **1**

[:SOLN] We can take the given situation as shown in the figure



Given Condition $\dfrac{r\theta_1}{r\theta_2}=\dfrac{1}{5}$

$\theta_1+\theta_2=\dfrac{\pi}{2}$

$\therefore \quad \theta_1=\dfrac{\pi}{12}$ & $\theta_2=\dfrac{5\pi}{12}$

$\overrightarrow{OC}=\alpha\,\overrightarrow{OA}+\beta\,\overrightarrow{OB}$

$r\hat{j}=\alpha r\hat{i}+\beta\left(r\cos\theta_1\hat{i}+r\cos\theta_2\hat{j}\right)$ (where r is radius of the circl)

$\Rightarrow(\alpha+\beta\cos\theta_1)\hat{i}+(\beta\cos\theta_2-1)\hat{j}=\vec{O}$

$\therefore \beta=\dfrac{1}{\cos\theta_2}=\dfrac{2\sqrt{2}}{\sqrt{3}-1}=\sqrt{2}\left(\sqrt{3}+1\right)$ and $\alpha+\beta\cos\theta_1=O\Rightarrow\alpha=-\beta\dfrac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore \quad \alpha+\sqrt{2}\left(\sqrt{3}-1\right)\beta=\left(\sqrt{2}\left(\sqrt{3}-1\right)-\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right)\sqrt{2}\left(\sqrt{3}+1\right)=4-\left(2+\sqrt{3}\right)=2-\sqrt{3}$

[:Q.6] If $\dfrac{\pi}{2}\le x\le\dfrac{3\pi}{4}$, then $\cos^{-1}\left(\dfrac{12}{13}\cos x+\dfrac{5}{13}\sin x\right)$ is equal to

[:A] $x+\tan^{-1}\dfrac{4}{5}$

[:B] $x+\tan^{-1}\dfrac{5}{12}$

[:C] $x-\tan^{-1}\dfrac{5}{12}$

[:D] $x - \tan^{-1}\frac{4}{3}$

[:ANS] **C**

[:SOLN] $\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{13}\sin x\right)$

$\cos^{-1}(\cos(x-\alpha))$ where $\alpha = \tan^{-1}\frac{5}{12}$

$= x - \alpha$

$= x - \tan^{-1}\frac{5}{12}$

[:Q.7] If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

[:A] –1200

[:B] –1020

[:C] –1080

[:D] –120

[:ANS] **C**

[:SOLN] $5(3+(3+d)+(3+2d)+(3+3d)) = ((3+4d)+(3+5d)+(3+6d)+(3+7d))$

$60 + 30d = 12 + 22d$

$d = -6.$

Now, $S_{20} = \frac{20}{2}[6+(20-1)(-6)]$

$= -1080$

[:Q.8] One die has two faces marked 1, two face marked 2, one face marked 3 and one face marked 4. Another die has one face marked 1, two face marked 2, two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5, when both the dice are thrown together, is

[:A] $\frac{1}{2}$

[:B] $\frac{4}{9}$

[:C] $\frac{2}{3}$

[:D] $\frac{3}{5}$

[:ANS] **A**

[:SOLN] Die – 1 $\equiv$ 1, 1, 2, 2, 3, 4

Die – 2 $\equiv$ 1, 2, 2, 3, 3, 4

$$P(4)=\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{6}\times\frac{1}{6}=\frac{1}{4}$$

$$P(5)=\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{6}\times\frac{1}{3}+\frac{1}{6}\times\frac{1}{6}=\frac{1}{4}$$

$\therefore$ required prob $=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

[:Q.9] Let the position vectors of the vertices A, B and C of a tetrahedron ABCD be $\hat{i}+2\hat{j}+\hat{k}, \hat{i}+3\hat{j}-2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is

[:A] $\frac{1}{6}\left(12\hat{i}+12\hat{j}+\hat{k}\right)$

[:B] $\frac{1}{6}\left(7\hat{i}+12\hat{j}+\hat{k}\right)$

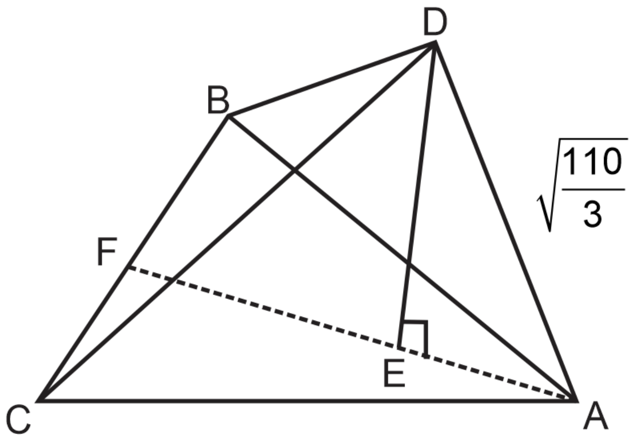
[:C] $\frac{1}{2}\left(\hat{i}+4\hat{j}+7\hat{k}\right)$

[:D] $\frac{1}{12}\left(7\hat{i}+4\hat{j}+3\hat{k}\right)$

[:ANS] **B**

[:SOLN] $\frac{1}{3}\text{ar}(\Delta ABC)\times DE=\frac{\sqrt{805}}{6\sqrt{2}}$

$$\frac{1}{3}\times\frac{1}{2}\sqrt{35}.DE=\frac{\sqrt{35\times 23}}{6\sqrt{2}}$$



$$\therefore DE = \sqrt{\frac{23}{2}}$$

$$AE = \sqrt{AD^2 - DE^2} = \sqrt{\frac{13}{18}} = \frac{\sqrt{26}}{6}$$

Now Position vector of F (mid point of BC) = $\frac{3}{2}\hat{i} + 2\hat{j} - \frac{3}{2}\hat{k}$

$$\therefore AF = \left|\vec{AF}\right| = \frac{\sqrt{26}}{2}$$

$$\therefore EF = \frac{\sqrt{26}}{2} - \frac{\sqrt{26}}{6} = 2.\frac{\sqrt{26}}{6}$$

$$\therefore EF : AE :: 2 : 1$$

$\therefore$ Position vector of Point $E = \dfrac{2\left(\hat{i} + 2\hat{j} + \hat{k}\right) + \left(\frac{3}{2}\hat{i} + 2\hat{j} - \frac{3}{2}\hat{k}\right)}{2+1}$

$$= \frac{1}{6}\left(7\hat{i} + 12\hat{j} + \hat{k}\right)$$

[:Q.10] The value of

$$\int_{e^2}^{e^4} \frac{1}{x}\left(\frac{e^{\left((\log_e x)^2+1\right)^{-1}}}{e^{\left((\log_e x)^2+1\right)^{-1}} + e^{\left((6-\log_e x)^2+1\right)^{-1}}}\right) dx \text{ is}$$

[:A] e^2

[:B] 1

[:C] 2

[:D] $\log_e 2$

[:ANS] **B**

[:SOLN] Put $\log_e x = t$

$$\Rightarrow \frac{1}{x}dx = dt$$

$$I = \int_2^4 \frac{e^{\frac{1}{1+t^2}}}{e^{\frac{1}{1+t^2}} + e^{\frac{1}{1+(6-t)^2}}} dt$$

$$I = \int_2^4 \frac{e^{\frac{1}{1+(e-t)^2}}}{e^{\frac{1}{1+(6-t)^2}} + e^{\frac{1}{1+t^2}}} dt$$

$$\therefore 2I = \int_2^4 dt = 2 \Rightarrow I = 1$$

[:Q.11] Let $I(x) = \int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$. If $I(37) - I(24) = \frac{1}{4}\left(\frac{1}{b^{\frac{1}{13}}} - \frac{1}{c^{\frac{1}{13}}}\right)$, $b, c \in \mathbb{N}$, then 3(b + c) is equal to

[:A] 26

[:B] 22

[:C] 39

[:D] 40

[:ANS] **C**

[:SOLN] $I(x) = \int \frac{dx}{(x-11)^{11/13}(x+15)^{15/13}} = \int \frac{dx}{(x-11)^2\left(\frac{x+15}{x-11}\right)^{15/13}}$

Put $\frac{x+15}{x-11} = t \Rightarrow -\frac{26}{(x-11)^2}dx = dt;$

Integration becomes $-\frac{1}{26}\int \frac{dt}{t^{15/13}}$

$$\therefore I(x) = \frac{1}{4}\left(\frac{x+15}{x-11}\right)^{-2/13} + C$$

$\therefore I(37)-I(24)=\frac{1}{4}\left[2^{-2/13}-3^{-2/13}\right]=\frac{1}{4}\left[\frac{1}{4^{1/13}}-\frac{1}{9^{1/13}}\right]$

$\therefore 3(b+c)=3(4+9)=39.$

[:Q.12] The number of words, which can be formed using all the letters of the word “DAUGHTER”, so that all the vowels never come together, is

[:A] 37000

[:B] 36000

[:C] 34000

[:D] 35000

[:ANS] **B**

[:SOLN] Required number of ways = Total words – words with all vowel together

$=8!-6!\times 3!$

= 36000

[:Q.13] If A, B and $\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)$ are non-singular matrices of same order, then the inverse of $A\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)^{-1}B$, is equal to

[:A] $AB^{-1}+A^{-1}B$

[:B] $\frac{AB^{-1}}{|A|}+\frac{A^{-1}B}{|B|}$

[:C] $\frac{1}{|AB|}\left(\text{adj}(B)+\text{adj}(A)\right)$

[:D] $\text{adj}\left(B^{-1}\right)+\text{adj}\left(A^{-1}\right)$

[:ANS] **C**

[:SOLN] Let $P=AC^{-1}B$

Where C = $\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)$

$\therefore \quad P^{-1}=B^{-1}CA^{-1}$

$=B^{-1}\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)A^{-1}$

$=B^{-1}\left(\text{adj}\left(A^{-1}\right)A^{-1}+B^{-1}\text{adj}\left(B^{-1}\right)\right)A^{-1}$

$= B^{-1}\text{adj}\left(A^{-1}\right)A^{-1} + B^{-1}\text{adj}\left(B^{-1}\right)A^{-1}$

$= \frac{B^{-1}}{|A|} + \frac{A^{-1}}{|B|} = \frac{1}{|A||B|}$ (adj B + adj A).

[:Q.14] Let P be the foot of the perpendicular from the point $Q(10,-3,-1)$ on the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2}$. Then the area of the right angled triangle PQR where R is the point (3, –2, 1), is

[:A] $3\sqrt{30}$

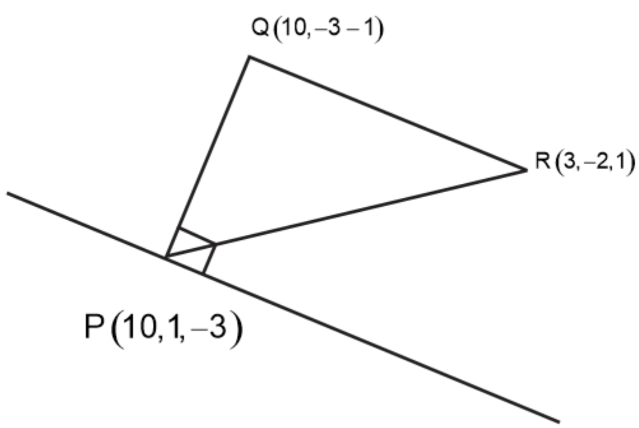
[:B] $9\sqrt{15}$

[:C] $\sqrt{30}$

[:D] $8\sqrt{15}$

[:ANS] **1**

[:SOLN] $P \equiv (3+7\lambda, 2-\lambda, -1-2\lambda)$



Direction ratios of PQ :

$\langle 7-7\lambda, -5+\lambda, 2\lambda \rangle$

PQ is perpendicular to given line

$\therefore$ $7(7-7\lambda) - 1(-5+\lambda) - 2(2\lambda) = 0$.

$\Rightarrow \lambda = 1$

$\therefore$ $P \equiv (10,1,-3)$

$\therefore$ $PQ = \sqrt{20}, PR = \sqrt{74}$ & $QR = \sqrt{54}$

Clearly $PQ^2 + QR^2 = PR^2$

$\therefore$ required area $= \frac{1}{2} PQ.QR = 3\sqrt{30}$

[:Q.15] Let $\left|\frac{\bar{z}-1}{2\bar{z}+i}\right| = \frac{1}{3}; z \in C,$ be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points (0, 0), C and $(\alpha, 0)$ is 11 square units, then α^2 equals:

[:A] 100

[:B] 50

[:C] $\frac{81}{25}$

[:D] $\frac{121}{25}$

[:ANS] **A**

[:SOLN] $\frac{|x - i(y+1)|}{|2x + i(1-2y)|} = \frac{1}{3}$

$\Rightarrow 9\left(x^2 + (y+1)^2\right) = 4x^2 + (1-2y)^2$

$\Rightarrow 5x^2 + 5y^2 + 22y + 8 = 0$

$x^2 + y^2 + \frac{22}{5}y + \frac{8}{5} = 0.$

$\therefore C \equiv \left(0, -\frac{11}{5}\right)$

Area $= \frac{1}{2}|\alpha| . \frac{11}{5} = 11 \Rightarrow |\alpha| = 10$

$\therefore \alpha^2 = 100$

[:Q.16] The value of $(\sin 70°)(\cot 10° \cot 70° - 1)$ is

[:A] 1

[:B] 3/2

[:C] 0

[:D] 2/3

[:ANS] **A**

[:SOLN] $\sin 70°\left(\dfrac{\cos 10°\cos 70° - \sin 10°\sin 70°}{\sin 10°\sin 70°}\right)$

$= \dfrac{\cos 80°}{\sin 10°} = 1$

[:Q.17] Let the area of ΔPQR with vertices P(5, 4), Q(–2, 4) and R(a, b) be 35 square units. If its orthocenter and centroid as $O\left(2, \dfrac{14}{5}\right)$ and C(c, d) respectively, then c + 2d is equal to

[:A] 1

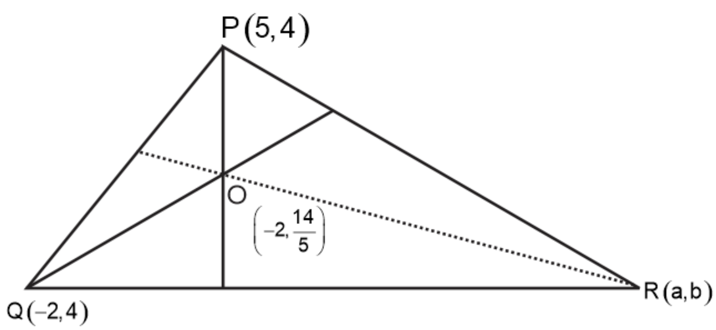
[:B] $\dfrac{7}{3}$

[:C] $\dfrac{8}{3}$

[:D] 2

[:ANS] **1**

[:SOLN] $m_{PR} = -\dfrac{1}{m_{OQ}} = \dfrac{10}{3}$



PR : $y - 4 = \dfrac{10}{3}(x - 5)$

$10x - 3y = 38$ ......(1)

$m_{QR} = -\dfrac{1}{m_{OP}} = -\dfrac{5}{2}$

$y - 4 = -\dfrac{5}{2}(x + 2)$

$5x + 2y = -2$ ......(2)

Solving (1) and (2) $R \equiv (2, -6)$

$\therefore \quad C \equiv \left(\frac{5}{3}, \frac{2}{3}\right)$

$\therefore \quad c + 2d = \frac{5}{3} + \frac{4}{3} = 3.$

[:Q.18] Maks obtained by all the students of class 12 are presented in a frequency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12. If the number of students whose marks are less than 12 is 18, then the total number of students is

[:A] 52

[:B] 44

[:C] 40

[:D] 48

[:ANS] **B**

[:SOLN] Median $= L + \frac{\frac{N}{2} - C}{f} \times h$

$$14 = 12 + \frac{\frac{N}{2} - 18}{12} \times 6$$

$\therefore$ N = 44

[:Q.19] If the system of equations

$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$

$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$

$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$

Has infinitely many solutions, then $\lambda^2 + \lambda$ is equal to

[:A] 12

[:B] 20

[:C] 6

[:D] 10

[:ANS] **A**

[:SOLN] $D=\begin{vmatrix}\lambda-1 & \lambda-4 & \lambda \\ \lambda & \lambda-1 & \lambda-4 \\ \lambda+1 & \lambda+2 & -(\lambda+2)\end{vmatrix}$

Now, D = 0 $\Rightarrow (6-2\lambda)(2\lambda+1)=0$

$\Rightarrow \lambda=3\ \&\ \lambda=-\frac{1}{2}$

For $\lambda=3$ equations are $2x-y+3z=5$ ..... (1)

$3x+2y-z=7$ ..... (2)

$4x+5y-5z=9$ .... (3)

Clearly eq. (3) is $2\times(2)-(1)$ $\therefore$ infinite solutions.

$\therefore\ \lambda^2+\lambda=12$

[:Q.20] If the function

$$f(x)=\begin{cases}\frac{2}{x}\{\sin(k_1+1)x+\sin(k_2-1)x\}, x<0 \\ 4, \quad x=0 \\ \frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right),\ x>0\end{cases}$$

is continuous at x = 0, then $k_1+k_2^2$ is equal to

[:A] 8

[:B] 5

[:C] 20

[:D] 10

[:ANS] **D**

[:SOLN] $\lim_{x\to0^-} f(x)=f(0)=\lim_{x\to0^+} f(x)$

$$\lim_{x\to0^-}\frac{2}{x}\{\sin(k_1+1)x+\sin(k_2-1)x\}=4=\lim_{x\to0^+}\frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right)$$

$$\lim_{x\to0^-}\frac{4.\sin\frac{k_1+k_2}{2}x\cos\frac{k_1+k_2}{2}x}{x}=4=\lim_{x\to0^+}2.\frac{\frac{2+k_1x}{2+k_2x}-1}{x}$$

$2(k_1+k_2)=4=k_1-k_2$

$\therefore k_1 + k_2 = 2$ and $k_1 - k_2 = 4$

$\Rightarrow k_1 = 3,\ \ k_2 = -1$

$\therefore k_1^2 + k_2^2 = 10.$

SECTION 2

[:Q.21] If the area of the large portion bounded between the curves $x^2 + y^2 = 25$ and $y = |x-1|$ is $\frac{1}{4}(b\pi + c), b, c \in \mathbb{N}$, then b + c is equal to ________.

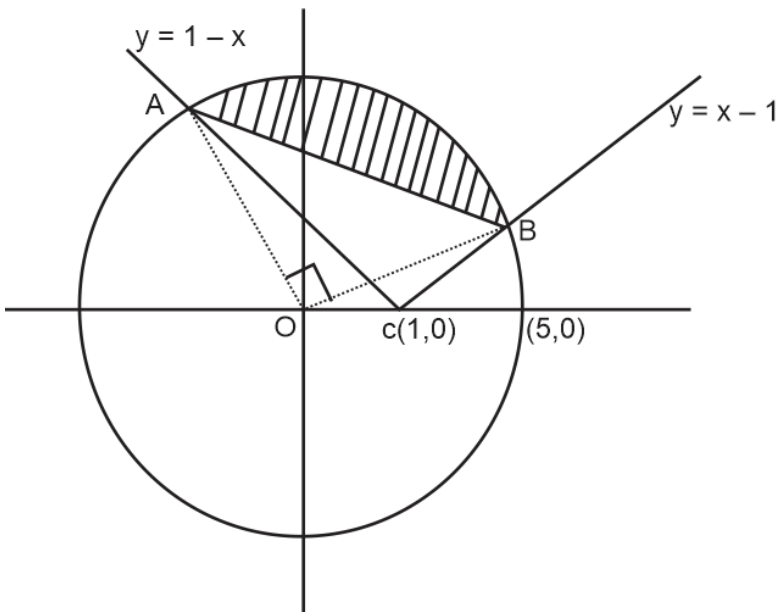
[:ANS] **77**

[:SOLN] Solving $y = |x-1|$ with

$x^2 + y^2 = 25$

$A \equiv (-3, 4)$ and $B \equiv (4, 3)$

$\therefore \Delta AOB = \frac{\pi}{2}$



Area of shaded portion $= \frac{1}{2} 5^2 \frac{\pi}{2} - \frac{25}{2}$

$\therefore$ Area of shaded portion + $\text{Ar.}(\Delta ABC) = \frac{25}{2} . \frac{\pi}{2} - \frac{25}{2} + \frac{1}{2} \left| \begin{vmatrix} 1 & 1 & 1 \\ -3 & 1 & 4 \\ 4 & 0 & 3 \end{vmatrix} \right|$

$$=\frac{25}{2}\frac{\pi}{2}-\frac{25}{2}+12$$

$\therefore$ required area = $25\pi-\left(\frac{25}{2}\frac{\pi}{2}-\frac{25}{2}+12\right)$

$$=\frac{75\pi}{4}+\frac{1}{2}=\frac{1}{4}(75\pi+2)$$

[:Q.22] If the set of all values of a, for which the equation $5x^3-15x-a=0$ has three distinct real roots, is the interval (α,β) then $\beta-2\alpha$ is equal to _____.

[:ANS] **30**

[:SOLN] $f(x)=5x^2-15x-a$

$f'(x)=15x^2-15$

$f'(x)=0\Rightarrow x=-1,1$

Now, $f(-1)f(1)<0$

$(10-a)(-10-a)<0$

$\Rightarrow -10<a<10$

$\therefore \lambda=-10$ and $\beta=10$

$\therefore \beta-2\lambda=30$.

[:Q.23] If the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ has equal roots, where a + c = 15 and $b=\frac{36}{5}$, then a^2+c^2 is equal to ___________

[:ANS] **117**

[:SOLN] $a(b-c)x^2+b(c-a)x+c(a-b)=0$

Sum of coeff, = 0 $\therefore$ x = 1 is a root

Now, roots are equal, therefore, second root is also 1

$\therefore \frac{c(a-b)}{a(b-c)}=1\Rightarrow ac-bc=ab-ac$

$\therefore 2ac=b(a+c)$

$$=\frac{36}{5}\times 15=108$$

$\therefore a^2 + c^2 = (a+c)^2 - 2ac$

= 225 – 108

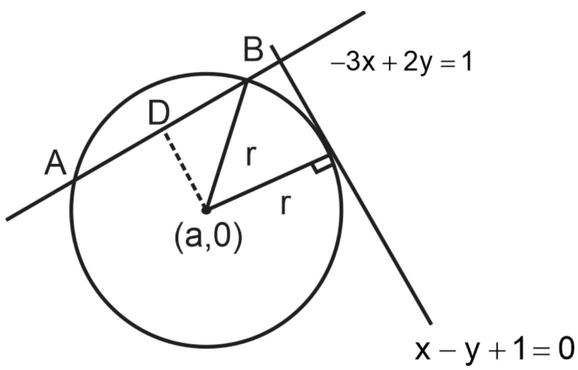
= 117

[:Q.24] Let the circle C touch the line x – y + 1 = 0, have the centre on the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x+2y=1$. Let H be the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then $2\alpha^2+3\beta^2$ is equal to _______.

[:ANS] **19**

[:SOLN] Let center be (a, 0) where a > 0

Now $\left|\frac{a+1}{\sqrt{2}}\right| = \sqrt{\frac{4}{13}+\frac{(3a+1)^2}{13}}$



$\frac{(a+1)^2}{2} = \frac{4+(3a+1)^2}{13}$

$\Rightarrow 5a^2 - 14a - 3 = 0$

$(5a+1)(a-3) = 0$

as $a > 0$, a = 3

$\therefore$ Center $=(3,0); r = 2\sqrt{2}$

Now, $2\alpha = 2r \Rightarrow \alpha = r$

and $\alpha e = 3$

$\therefore\ 2\alpha^2 + 3\beta^2 = 3(\alpha^2+\beta^2) - \alpha^2 = 3\alpha^2 e^2 - r^2 = 3.9 - 8 = 19$

[:Q.25] The sum of all rational terms in the expansion of $\left(1+2^{1/3}+3^{1/2}\right)^6$ is qual to ________ .

[:ANS] **612**

[:SOLN] $\left(1+2^{1/3}+3^{1/2}\right)^6$

General Term $=\dfrac{6!}{r_1!r_2!r_3!}2^{\frac{r_2}{3}}.3^{\frac{r_3}{2}}$

$r_1, r_2, r_3, \in W;\ r_1, r_2, r_3, \le 6$ and $r_1+r_2+r_3=6.$

r_1	r_2	r_3	Term
6	0	0	$\dfrac{6!}{6!0!0!}2^{\circ}3^{\circ}=1$
4	0	2	$\dfrac{6!}{4!0!2!}2^{\circ}3=45$
2	0	4	$\dfrac{6!}{2!0!4!}2^{\circ}.3^2=135$
0	0	6	$\dfrac{6!}{0!0!6!}2^{\circ}.3^{\circ}=27$
3	3	0	$\dfrac{6!}{3!3!0!}2^1.3^{\circ}=40$
1	3	2	$\dfrac{6!}{1!3!2!}2.3=360$
0	6	0	$\dfrac{6!}{0!6!0!}2^2=4$

Sum of these Terms = 612