|JEE MAIN 2025 | DATE : 23 JAN 2025 (SHIFT-2) EVENING
MATHEMATICS
SECTION 1

[:Q.1] If $I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$, then $\int_0^{2I}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$ equals:

[:A] $\frac{\pi^2}{8}$

[:B] $\frac{\pi^2}{16}$

[:C] $\frac{\pi^2}{4}$

[:D] $\frac{\pi^2}{12}$

[:ANS] **B**

[:SOLN] Applying Property 4,

$$2I=\int_0^{\frac{\pi}{2}}\frac{x\cdot\sin x\cdot\cos x}{\sin^4 x+\cos^4 x}dx$$ applying prop. 4 and adding

$$I_2=\frac{\pi}{4}\int_0^{\frac{\pi}{2}}\frac{\tan x\cdot\sec^2 x dx}{\tan^4 x+1},$$ putting $\tan^2 x = t$

$$\Rightarrow \quad \frac{\pi}{8}\int_0^{\infty}\frac{dxt}{t^2+1}\Rightarrow\frac{\pi}{8}\cdot\frac{\pi}{2}=\frac{\pi^2}{16}$$

[:Q.2] Let the range of the function f (x) = 6 + 16 $\cos x\cdot\cos\left(\frac{\pi}{3}-x\right)\cdot\cos\left(\frac{\pi}{3}+x\right)\cdot\sin^3 x\cdot\cos 6x,\ x\in R$ be $[\alpha,\beta]$. Then the distance of the point (α,β) from the line $3x+4y+12=0$ is:

[:A] 10

[:B] 8

[:C] 9

[:D] 11

[:ANS] **D**

[:SOLN] f(x) = 6 + 16 $\left(\frac{\cos 3x}{4}\right)$. Sin3x . cos6x

= 6 + 4 · cos3x · sin3x · cos6x

= 6 + sin 12x $\quad \therefore \quad f(x) \in [5, 7] \quad \therefore\ \alpha = 5, \beta = 7$

Distance = $\left|\frac{15+28+12}{5}\right| = 11$

[:Q.3] Let $\int x^2 \sin x dx = g(x) + C$, where C is the constant of integration. If

$8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \alpha\pi^2 + \beta\pi^2 + \gamma, \alpha, \beta, \gamma \in Z$, then $\alpha + \beta - \gamma$ equals:

[:A] 48

[:B] 55

[:C] 62

[:D] 47

[:ANS] **B**

[:SOLN] $\int x^3 \sin x dx = -x^3 \cdot \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + c$

$\therefore \quad g(x) = -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x$

$g\left(\frac{\pi}{2}\right) = \frac{3\pi^2}{4} - 6, \qquad g'\left(\frac{\pi}{2}\right) = \frac{\pi^3}{8}$

$\therefore \quad 8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \pi^3 + 6\pi^2 - 48$

$\therefore \quad \alpha + \beta - \gamma = 55$

[:Q.4] The number of complex numbers z, satisfying $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$, is

[:A] 4

[:B] 6

[:C] 10

[:D] 8

[:ANS] **D**

[:SOLN] Put z = e^{iQ}

$$\left(\frac{z}{\bar{z}}\right) = e^{iz\theta} \qquad \therefore \quad \text{Questions} = \left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$$

$$\Rightarrow \quad \left|e^{iz\theta} + e^{-iz\theta}\right| = 1$$

$$|\cos 2\theta| = \frac{1}{2}$$

$\therefore$ 8

[:Q.5] Let the point A divide the line segment joining the points $P(-1,-1,2)$ and $Q(5,5,10)$ internally in the ratio r : 1 (r > 0). If O is the origin and $\left(\overrightarrow{OQ}\cdot\overrightarrow{OA}\right) - \frac{1}{5}\left|\overrightarrow{OP}\times\overrightarrow{OA}\right|^2 = 10$, then the value of r is:

[:A] 7

[:B] 14

[:C] 3

[:D] $\sqrt{7}$

[:ANS] **A**

[:SOLN] $A = \left(\frac{5r-1}{r+1}, \frac{5r-1}{r+1}, \frac{10r+2}{r+1}\right)$

$$\overrightarrow{OQ}\cdot\overrightarrow{OA} - \frac{\left|\overrightarrow{OP}\times\overrightarrow{OA}\right|^2}{5} = 10 \qquad \text{-------------(1)}$$

$$\overrightarrow{OQ}\cdot\overrightarrow{OA} = \frac{5}{r+1}(30r+2)$$

$$\left|\overrightarrow{OP}\times\overrightarrow{OA}\right|^2 = \frac{r^2}{(r+1)^2}\cdot(800) \text{ from equation (1)}$$

$$\frac{10}{r+1}(15r+1) - \frac{1}{5}\frac{\left(r^2 \cdot 800\right)}{(r+1)^2} = 10$$

$\therefore$ r = 7

[:Q.6] Let $X = R \times R$. Define a relation R on X as:

$$(a_1, b_1) R (a_2, b_2) \Leftrightarrow b_1 = b_2.$$

Statement I : R is an equivalence relation.

Statement II : For some (a, b) $\in X$, the set $S=\{(x,y)\in X:(x,y)R(a,b)\}$ represent a line parallel to y = x.

In the light of the above statements, choose the correct answer from the options given below:

[:A] Statement I is true but Statement II is false

[:B] Statement I is false but statement II is true

[:C] Both Statement I and Statement II are true

[:D] Both Statement I and statement II are false

[:ANS] **A**

[:SOLN] Reflexive : $(a_1, b)\ R\ (a_1, b_1) \Rightarrow b_1 = b_1$ True

Symmetric : $(a_1, b_1)\ R\ (a_2, b_2) \Rightarrow b_1 = b_2$

$(a_2, b_2)\ R\ (a_1, b_1) \Rightarrow b_2 = b_1$

Transitive : $(a_1, b_1)\ R\ (a_2, b_2) \Rightarrow b_1 = b_2$

& $(a_2, b_2)\ R\ (a_3, b_3) \Rightarrow b_2 = b_3$

$\therefore$ Equivalence Relation

$\Rightarrow$ $(a_1, b_1)\ R\ (a_3, b_3) \Rightarrow$ True

But Statement – II y = b, so false.

[:Q.7] $\lim\limits_{x\to\infty}\dfrac{(2x^2-3x+5)(3x-1)^{\frac{x}{2}}}{(3x^2+5x+4)\sqrt{(3x+2)^x}}$ is equal to:

[:A] $\dfrac{2}{\sqrt{3e}}$

[:B] $\dfrac{2e}{\sqrt{3}}$

[:C] $\dfrac{2e}{3}$

[:D] $\dfrac{2}{3\sqrt{e}}$

[:ANS] **D**

[:SOLN] $\lim_{n\to\infty} \dfrac{\left(2-\frac{3}{x}+\frac{5}{x^2}\right)\left(1-\frac{1}{3x}\right)^{x/2}}{\left(3+\frac{5}{x}+\frac{4}{x^2}\right)\cdot\left(1+\frac{2}{3x}\right)^{x/2}} = \dfrac{2}{3}\dfrac{e^{x/2\left(1-\frac{1}{3x}-1\right)}}{e^{x/2\left(1+\frac{2}{3x}-1\right)}}$

$= \dfrac{2}{3}\dfrac{e^{-\frac{1}{6}}}{e^{\frac{1}{3}}} = \dfrac{2}{3}\cdot e^{-1/2}$

[:Q.8] A rod of length eight units moves such that its ends A and B always lie on the lines x – y + 2 = 0 and y + 2 = 0, respectively. If the locus of the point P, that divides the rod AB internally in the ratio 2 : 1 is $9\left(x^2+xy^2+\beta xy+\gamma x+28y\right)-76=0$, then $\alpha-\beta-\gamma$ is equal to:

[:A] 21

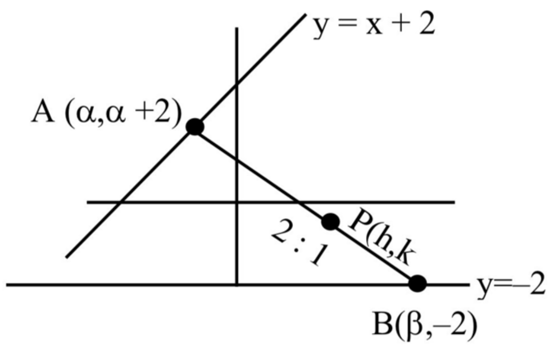
[:B] 22

[:C] 23

[:D] 24

[:ANS] **C**

[:SOLN]



$h = \dfrac{3\beta+\alpha}{3}$

$k = \dfrac{-4+\alpha+2}{3}$

AB = 8

$(\alpha-\beta)^2+(\alpha+4)^2=64$

$\left(\dfrac{9k-3h+6}{4}\right)^2+(3k+6)^2=64 \Rightarrow 9(x^2+13y^2-6xy-4x+28y)=76$

$\therefore \alpha - \beta - \gamma = 13 + 6 + 4 = 23$

[:Q.9] The system of equations

$x + y + z = 6,$

$x + 2y + 5z = 9,$

$x + 5y + \lambda z = \mu,$

Has no solution if

[:A] $\lambda = 17, \mu = 18$

[:B] $\lambda \neq 17, \mu \neq 18$

[:C] $\lambda = 15, \mu \neq 17$

[:D] $\lambda = 17, \mu \neq 18$

[:ANS] **D**

[:SOLN] $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 5 & \lambda \end{vmatrix} = 0 \quad \therefore \quad \lambda = 17$

$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 9 \\ 1 & 5 & M \end{vmatrix} \neq 0 \quad \therefore \quad \mu \neq 18$

[:Q.10] If the area of the region $\left\{(x, y) : 1 \le x \le 1, 0 \le y \le a + e^{|x|} - e^{-x}, a > 0\right\}$ is $\frac{e^2 + 8e + 1}{e}$, then the value of a is:

[:A] 5

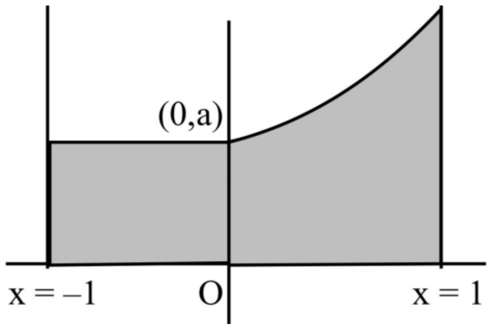
[:B] 7

[:C] 6

[:D] 8

[:ANS] **A**

[:SOLN]



Area = $a + \int_0^1 \left(a + e^x - e^{-x}\right)dx$

$\Rightarrow$ $2a + e^{-1} + e^{-1} - 1 = e + 8 + \frac{1}{e}$

$\therefore$ $a = 5$

[:Q.11] Let x = x(y) be the solution of the differential equation $y = \left(x - y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right), y > 0$ and $x(1) = \frac{\pi}{2}$. Then cos (x(2)) is equal to:

[:A] $1 - 2(\log_e 2)^2$

[:B] $1 - 2(\log_e 2)$

[:C] $2(\log_e 2) - 1$

[:D] $2(\log_e 2)^2 - 1$

[:ANS] **D**

[:SOLN] ydy = (xdy – ydx) $\sin\left(\frac{x}{4}\right)$

$\frac{dy}{y} = \frac{xdy - ydx}{y^2}\sin\left(\frac{x}{4}\right)$

$\frac{dy}{y} = \sin\left(\frac{x}{4}\right)d\left(-\frac{x}{y}\right)$ $\therefore$ lny = $\cos\frac{x}{y} + C$

as $x(1) = \frac{\pi}{2}$ $\therefore$ C = 0, $\log_e y = \cos\frac{x}{y}$

but $y = 2 \Rightarrow \cos\frac{x}{2} = \log_e 2$

$\cos x = 2\left(\log_e 2\right)^2 - 1$

[:Q.12] If the square of the shortest distance between the lines $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{-3}$ and $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then m + n is equal to:

[:A] 9

[:B] 6

[:C] 21

[:D] 14

[:ANS] **A**

[:SOLN] $\vec{a} = (2,1,-3)$

$\vec{b} = (-1,-3,-5)$

$$\vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

$= 2\hat{i} - \hat{j}$

$\vec{b} - \vec{a} = -3\hat{i} - 4\hat{j} - 2\hat{k}$

$$S_d = \frac{\left|(\vec{b}-\vec{a}).(\vec{p}\times\vec{q})\right|}{\left|\vec{p}\times\vec{q}\right|}$$

$= \frac{2}{\sqrt{5}}$

$(S_d)^2 = \frac{4}{5}$

$m = 4, n = 5 \Rightarrow m + n = 9$

[:Q.13] The length of the chord of the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$, whose mid-point is $\left(1, \frac{1}{2}\right)$, is

[:A] $\frac{1}{3}\sqrt{15}$

[:B] $\sqrt{15}$

[:C] $\frac{5}{3}\sqrt{15}$

[:D] $\frac{2}{3}\sqrt{15}$

[:ANS] **D**

[:SOLN] $T = S_1$

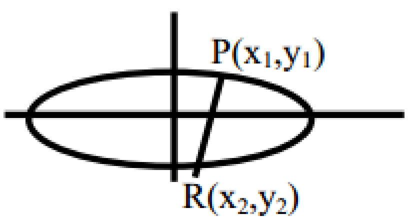
$$\frac{x.1}{4} + \frac{y.\frac{1}{2}}{2} = \frac{1}{4} + \frac{1}{8}$$

$$x + y = \frac{3}{2}$$

solve with ellipse

$$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{2}|x_2 - x_1|$$



$$y_2 = \frac{3}{2} - x_2$$

$$y_1 = \frac{3}{2} - x_1$$

$$y_2 - y_1 = x_2 - x_1$$

$$x^2 + 2y^2 = 4$$

$$x^2 + 2\left(\frac{3}{2} - x\right)^2 = 4$$

$$6x^2 - 12x + 1 = 0$$

$$x_1 + x_2 = 2$$

$$x_1 x_2 = 1/6$$

$$|x_2 - x_1| = \sqrt{(x_2 + x_1)^2 - 4x_1x_2}$$

$$= \sqrt{4 - 4/6}$$

$$PR = \sqrt{2}.2.\frac{\sqrt{5}}{\sqrt{2}\sqrt{3}} = \frac{2}{3}\sqrt{15}$$

[:Q.14] A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm, the ice-cream melts at the rate of 81 cm^3/ min and the thickness of the ice-cream layer decrease at the rate of $\frac{1}{4\pi}$ cm/min. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is:

[:A] 196π

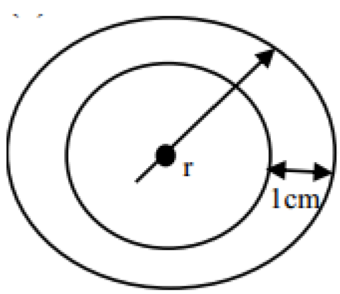
[:B] 256π

[:C] 225π

[:D] 128π

[:ANS] **B**

[:SOLN]



$$v = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$81 = 4\pi r^2 \times \frac{1}{4\pi}$$

$$r^2 = 81$$

$$r = 9$$

$$\text{surface area of chocolate} = 4\pi(r-1)^2 = 256\pi$$

[:Q.15] If in the expansion of $(1+x)^p(1-x)^q$, the coefficients of x and x^2 are 1 and -2, respectively, then $p^2 + q^2$ is equal to:

[:A] 20

[:B] 13

[:C] 18

[:D] 8

[:ANS] **B**

[:SOLN]

$$(1+x)^p(1-x)^q = \left({}^pC_0 + {}^pC_1x + {}^pC_2x^2 + \ldots\right)\left({}^qC_0 - {}^qC_1x + {}^qC_2x^2 + \ldots\right)$$

$$\text{coff of } x \equiv {}^pC_0\,{}^qC_1 - {}^pC_1\,{}^qC_0 = 1$$

$$p - q = 1$$

$$\text{coff of } x^2 \equiv {}^pC_0\,{}^qC_2 - {}^pC_1\,{}^qC_1 + {}^pC_2\,{}^qC_0 = -2$$

$$\frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -2$$

$$q^2 - q - 2pq + p^2 - p = -4$$

$$(p-q)^2 - (p+q) = -4$$

$$p + q = 5$$

$$p = 3$$

$$q = 2$$

so $p^2 + q^2 = 13$

[:Q.16] Let the shortest distance from (a, 0), a > 0, to the parabola $y^2 = 4x$ be 4. Then the equation of the circle passing through the point (a, 0) and the focus of the parabola, and having its centre on the axis of the parabola is:

[:A] $x^2 + y^2 - 8x + 7 = 0$

[:B] $x^2 + y^2 - 6x + 5 = 0$

[:C] $x^2 + y^2 - 4x + 3 = 0$

[:D] $x^2 + y^2 - 10x + 9 = 0$

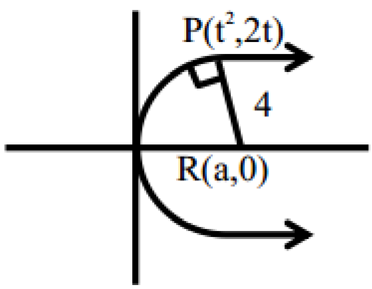
[:ANS] **B**

[:SOLN] Normal at P

$y + tx = 2t + t^3$

$\uparrow$

(a, 0)



$at = 2t + t^3$

$a = 2 + t^2$

$R\ (2 + t^2, 0)$

$PR\ = 4 \Rightarrow 4 + 4t^2 = 16$

$4t^2 = 12 \Rightarrow t^2 = 3$

$a = 5\ ,\ \ R(5, 0)$

Focus (1, 0)

(1, 0) & (5, 0) will be the end points of diameter

$\Rightarrow$ Eq^n of circle is

$(x-1)(x-5)+y^2=0$

$x^2+y^2-6x+5=0$

[:Q.17] The distance of the line $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ from the point (1, 4, 0) along the line $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ is:

[:A] $\sqrt{15}$

[:B] $\sqrt{13}$

[:C] $\sqrt{14}$

[:D] $\sqrt{17}$

[:ANS] **C**

[:SOLN] Let the parallel line is

$$\frac{x-1}{1}=\frac{y-4}{2}=\frac{z-0}{3}$$

so their point of intersection is

$(\lambda+1, 2\lambda+4\ 3\lambda)=(2t+2, 3t+6, 4t+3)$

$\lambda=2t+1$

$2\lambda+4=3t+6 \Rightarrow t=0$

so POI is (2,6,3)

so distance $=\sqrt{(2-1)^2+(6-4)^2+(3-0)^2} = \sqrt{14}$

[:Q.18] Let A = [a_{ij}] be a 3×3 matrix such that $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}, A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$ and $A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals:

[:A] –1

[:B] 1

[:C] 0

[:D] 2

[:ANS] **A**

[:SOLN]

$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

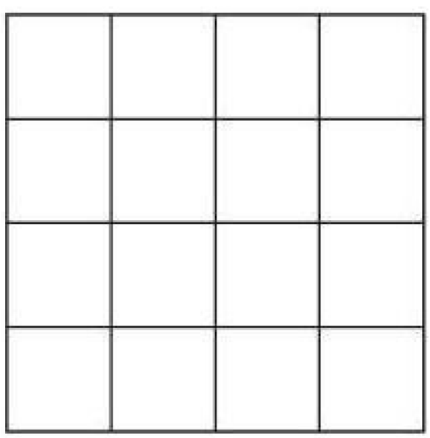
$$A\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} a_{22} = 0; a_{12} = 0 \\ a_{32} = 1 \end{array}$$

$$A\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} 4a_{11} + a_{12} + 3a_{13} = 0 \\ 4a_{21} + a_{22} + 3a_{23} = 1 \Rightarrow 4a_{21} + 3a_{23} = 1 \\ 4a_{31} + a_{32} + 3a_{33} = 0 \end{array}$$

$$A\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} 2a_{11} + a_{12} + 2a_{13} = 1 \\ 2a_{21} + a_{22} + 2a_{23} = 0 \Rightarrow a_{21} + a_{23} = 0 \\ 2a_{31} + a_{32} + 2a_{33} = 0 \end{array}$$

$$-4a_{23} + 3a_{23} = 1 \Rightarrow a_{23} = -1$$

[:Q.19] A board has 16 squares as shown in the figure:



Out of these 16 squares, two squares are chosen at random. The probability that they have bo side in common is:

[:A] $\frac{7}{10}$

[:B] $\frac{23}{30}$

[:C] $\frac{4}{5}$

[:D] $\frac{3}{5}$

[:ANS] **C**

[:SOLN] Total ways for selecting any two squares = $^{16}C_2 = 120$

Total ways for selecting common side squares

$$= \underbrace{3\times4}_{\text{Horizontal side}} + \underbrace{3\times4}_{\text{vertical side}} = 24$$

so required probability

$$= 1-\frac{24}{120}$$

$$= \frac{4}{5}$$

[:Q.20] Let $A=\{(x,y)\in R\times R:|x+y|\geq 3\}$ $B=\{(x,y)\in R\times R:|x|+|y|\leq 3\}$.

If $C=\{(x,y)\in A\cap B: x=0 \text{ or } y=0\}$, then $\sum_{(x,y)\in C}|x+y|$ is:

[:A] 12

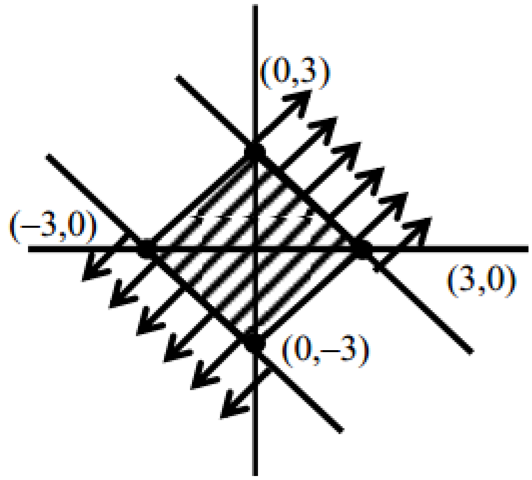
[:B] 24

[:C] 18

[:D] 15

[:ANS] **A**

[:SOLN]



$C = \{(3,0), (-3,0), (0,3), (0,-3)\}$

$\Sigma|x + y| = 12$

SECTION 2

[:Q.21] The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5. If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and $\lambda_2, \lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to _________.

[:ANS] **15**

[:SOLN] $y^2 = 4(x + 4)$

Equation of circle

$(x + 3)^2 + y^2 = 25$

Passes through the point of intersection of two lines $3x - y = 0$ and $x + \lambda y = 4$ which is

$\left(\frac{4}{3\lambda+1}, \frac{12}{3\lambda+1}\right)$, after solving with circle,

we get

$$\lambda = -\frac{7}{6}, 1$$

$$12\lambda_1 + 29\lambda_2$$

$$-14 + 29 = 15$$

[:Q.22] The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is___________.

[:ANS] **17280**

[:SOLN] A : number of ways that all boys

sit together $= 5! \times 5!$

B : number of ways if no 2 boys

sit together $= 4! \times 5!$

$$A \cap B = \phi$$

Required no. of ways $= 5! \times 5! + 4! \times 5! = 17280$

[:Q.23] The variance of the numbers 8, 21, 34, 47, ….., 320 is __________.

[:ANS] **8788**

[:SOLN] Var(8, 21, 34, 47, ..., 320)

Var(0, 13, 26, 39, ..., 312)

13^2.Var(0, 1, 2, ..., 24)

13^2.Var(1, 2, 3, ..., 25)

So, $\sigma^2 = 13^2 \times \left(\frac{25^2 - 1}{12}\right) = 8788$

Alternate solution

$8+(n-1)13=320$

$13n=325$

$n=25$

no. of terms $=25$

$$\text{mean}=\frac{\Sigma x_i}{n}=\frac{8+21+\ldots+320}{25}=\frac{\frac{25}{2}(8+320)}{25}$$

$$\text{variance } \sigma^2=\frac{\Sigma x_i^2}{n}-(\text{mean})^2$$

$$=\frac{8^2+21^2+\ldots.+320^2}{13}-(164)^2$$

$=8788$

[:Q.24] The roots of the quadratic equation $3x^2-px+q=0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then q – 2p is equal to ________.

[:ANS] **474**

[:SOLN]

$$S_{11}=\frac{11}{2}(2a+10d)=88$$

$a+5d=8$

$$a=8-5\times\frac{3}{2}=\frac{1}{2}$$

Roots are

$$T_{10}=a+9d=\frac{1}{2}+9\times\frac{3}{2}=14$$

$$T_{11}=a+10d=\frac{1}{2}+10\times\frac{3}{2}=\frac{31}{2}$$

$$\frac{p}{3} = T_{10} + T_{11} = 14 + \frac{31}{2} = \frac{59}{2}$$

$$p = \frac{177}{2}$$

$$\frac{q}{3} = T_{10} \times T_{11} = 7 \times 31 = 217$$

$$q = 651$$

$$q - 2p$$

$$= 651 - 177$$

$$= 474$$

[:Q.25] Let α, β be the roots of the equation $x^2 - ax - b = 0$ with $\text{Im}(\alpha) < \text{Im}(\beta)$. Let $P_n = \alpha^n - \beta^n$. If $P_3 = -5\sqrt{7}i, P_4 = -3\sqrt{7}i, P_5 = 11\sqrt{7}i$ and $P_6 = 45\sqrt{7}i$, then $|\alpha^4 + \beta^4|$ is equal to __________ .

[:ANS] **31**

[:SOLN] $\alpha + \beta = a \quad \alpha\beta = -b$

$$P_6 = aP_5 + bP_4$$

$$45\sqrt{7}i = a \times 11\sqrt{7}i + b\left(-3\sqrt{7}\right)i$$

$$45 = 11a - 3b \qquad ...(1)$$

and

$$P_5 = aP_4 + bP_3$$

$$11\sqrt{7}i = a\left(-3\sqrt{7}i\right) + b\left(-5\sqrt{7}i\right)$$

$$11 = -3a - 5b \qquad ...(2)$$

$$a = 3,\ b = -4$$

$$|\alpha^4 + \beta^4| = \sqrt{\left(\alpha^4 - \beta^4\right)^2 + 4\alpha^4\beta^4}$$

$$= \sqrt{-63 + 4.4^4}$$

$$= \sqrt{-63 + 1024} = \sqrt{961} = 31$$