|JEE MAIN 2025 | DATE : 22 JAN 2025 (SHIFT-2) EVENING
MATHEMATICS
SECTION 1

[:Q.1] For a 3×3 matrix M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A|=\frac{1}{2}$ and trace (A) = 3. If B = adj (adj(2A)), then the value of $|B|$ + trace (B) equals:

[:A] 174

[:B] 280

[:C] 132

[:D] 56

[:ANS] **B**

[:SOLN] $|A|=\frac{1}{2}$, trace(A) = 3, B = adj(adj(2A))=$|2A|^{n-2}(2A)$

n = 3, B = |2A|(2A) = 2^3.|A|(2A) = 8A

|B| = |8A| = 8^3.|A| = 2^8 = 256

trace(B) = 8 trace(A) = 24

|B| + trace(B) = 280

[:Q.2] Let α_θ and β_θ be the distinct roots of $2x^2+(\cos\theta)x-1=0, \theta\in(0,2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4+\beta_\theta^4$, then 16(M + m) equals:

[:A] 17

[:B] 27

[:C] 25

[:D] 24

[:ANS] **C**

[:SOLN] $(\alpha^2+\beta^2)^2-2\,\alpha^2\beta^2$

$[(\alpha+\beta)^2-2\alpha\beta]^2-2(\alpha\beta)^2$

$$\left[\frac{\cos^2\theta}{4}+1\right]^2-2.\frac{1}{4}$$

$$\left(\frac{\cos^2\theta}{4}+1\right)^2-\frac{1}{2}$$

$$M=\frac{25}{16}-\frac{1}{2}=\frac{17}{16}$$

$$m=\frac{1}{2},\ 16(M+m)=25$$

[:Q.3] If the system of linear equations:

$x+y+2z=6,$

$2x+3y+az=a+1,$

$-x-3y+bz=2b,$

where a, b $\in R$, has infinitely many solutions, then 7a + 3b is equal to:

[:A] 12

[:B] 9

[:C] 22

[:D] 16

[:ANS] **D**

[:SOLN]

$$\Delta=\begin{vmatrix}1 & 1 & 2\\ 2 & 3 & a\\ -1 & -3 & b\end{vmatrix}=0$$

$$\Rightarrow 2a+b-6=0 \qquad \ldots\ldots.(1)$$

$$\Delta_1=\begin{vmatrix}1 & 1 & 6\\ 2 & 3 & a+1\\ -1 & -3 & 2b\end{vmatrix}=0$$

$$\Rightarrow a+b-8=0 \qquad \ldots\ldots.(2)$$

Solving (1) + (2)

$a=-2,\ b=10$

$\Rightarrow 7a+3b=16$

[:Q.4] Let α, β, γ and δ be the coefficient of x^7, x^5, x^3 and x respectively in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5, x>1$. If u and v satisfy the equations

$\alpha u+\beta v=18,$

$\gamma u+\delta v=20,$

Then u + v equals:

[:A] 4

[:B] 5

[:C] 3

[:D] 8

[:ANS] **B**

[:SOLN] $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$= 2\{{}^5C_0.x^5 + {}^5C_2.x^3(x^3-1) + {}^5C_4.x(x^3-1)^2\}$

$= 2\{5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x\}$

$\Rightarrow \alpha = 10, \beta = 2, \gamma = -20, \delta = 10$

Now, $10u + 2v = 18$

$-20u + 10v = 20$

$\Rightarrow u = 1, v = 4$

$u + v = 5$

[:Q.5] The sum of all values of $\theta \in [0, 2\pi]$ satisfying $2\sin^2\theta = \cos 2\theta$ and $2\cos^2\theta = 3\sin\theta$ is

[:A] 4π

[:B] $\frac{5\pi}{6}$

[:C] π

[:D] $\frac{\pi}{2}$

[:ANS] **C**

[:SOLN] $2\sin^2\theta = \cos 2\theta$

$2\sin^2\theta = 1 - 2\sin^2\theta$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$2\cos^2\theta = 3\sin\theta$

$2 - 2\sin^2\theta + 3\sin\theta - 2 = 0$

$(2\sin\theta - 1)(2\sin\theta - 2) = 0$

$\sin\theta = \frac{1}{2}$

so common equation which satisfy both equations

is $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad (\theta \in [0, 2\pi])$

$\text{Sum} = \pi$

[:Q.6] Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{3}$. If $\lambda\vec{a} + 2\vec{b}$ and $3\vec{a} - \lambda\vec{b}$ are perpendicular to each other, then the number of values of λ in $[-1, 3]$ is:

[:A] 0

[:B] 3

[:C] 1

[:D] 2

[:ANS] **A**

[:SOLN] $\hat{a}.\hat{b} = \frac{1}{2}$

Now $\left(\lambda\hat{a} + 2\hat{b}\right).\left(3\hat{a} - \lambda\hat{b}\right) = 0$

$3\lambda\hat{a}.\hat{a} - \lambda^2\hat{a}.\hat{b} + 6\hat{a}.\hat{b} - 2\lambda\hat{b}.\hat{b} = 0$

$$3\lambda - \frac{\lambda^2}{2} + 3 - 2\lambda = 0$$

$$\lambda^2 - 2\lambda - 6 = 0$$

$$\lambda = 1 \pm \sqrt{7}$$

$\Rightarrow$ number of values = 0

[:Q.7] The perpendicular distance, of the line $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$ from the point $P(2,-10,1)$, is:

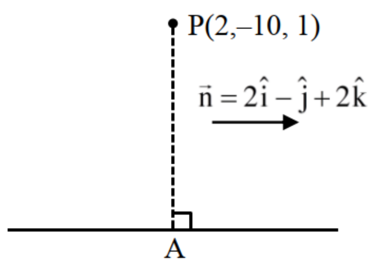
[:A] $3\sqrt{5}$

[:B] 6

[:C] $4\sqrt{3}$

[:D] $5\sqrt{2}$

[:ANS] **A**



[:SOLN]

$$\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2} = \lambda \text{ (let)}$$

$$(2\lambda + 1, -\lambda - 2, 2\lambda - 3)$$

$$\because \overrightarrow{PA}.\vec{n} = 0$$

$$\Rightarrow (2\lambda - 1)2 + (-\lambda + 8)(-1) + (2\lambda - 4)2 = 0$$

$$\Rightarrow 4\lambda - 2 + \lambda - 8 + 4\lambda - 8 = 0$$

$$\Rightarrow 9\lambda - 18 = 0 \Rightarrow \lambda = 2$$

$$\therefore A(5, -4, 1)$$

$$\therefore AP = \sqrt{3^2 + 6^2 + 0^2} = \sqrt{45} = 3\sqrt{5}$$

[:Q.8] Suppose that the number of terms in an A.P. is 2k, $k \in N$. IF the sum of all odd terms of the A.P. is 40, the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27, then k is equal to:

[:A] 6

[:B] 5

[:C] 4

[:D] 8

[:ANS] **B**

[:SOLN] $a_1, a_2, a_3, \ldots\ldots, a_{2k} \rightarrow$ A.P.

$$\sum_{r=1}^{k} a_{2r-1} = 40, \ \sum_{r=1}^{k} a_{2r} = 55, \ a_{2k} - a_1 = 27$$

$$\frac{k}{2}[2a_1 + (k-1)2d] = 40, \ \frac{k}{2}[2a_2 + (k-1)2d] = 55,$$

$$d = \frac{27}{2k-1}$$

$$a_1 = \frac{40}{k} - (k-1)d = \frac{55}{k} - kd$$

$$d = \frac{15}{k} \Rightarrow \frac{27}{2k-1} = \frac{15}{k} \Rightarrow 9k = 10k - 5$$

$$\therefore k = 5$$

[:Q.9] The area of the region enclosed by the curves $y = x^2 - 4x + 4$ and $y^2 = 16 - 8x$ is:

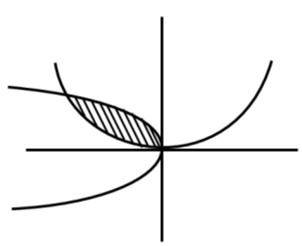
[:A] $\frac{4}{3}$

[:B] 5

[:C] $\frac{8}{3}$

[:D] 8

[:ANS] **C**



[:SOLN]

$y = (x-2)^2, y^2 = 8(x-2)$

$y = x^2, y^2 = -8x$

$= \frac{16ab}{3} = \frac{16 \times \frac{1}{4} \times 2}{3} = \frac{8}{3}$

[:Q.10] Let A = {1, 2, 3, 4} and B = {1, 4, 9, 16}. Then the number of many-one functions f : $A \to B$ such that $1 \in f(A)$ is equal to:

[:A] 127

[:B] 151

[:C] 163

[:D] 139

[:ANS] **B**

[:SOLN] Total $= 4^4$

One-one $= 4!$

Many-one $= 256 - 24 = 232$

Many-one which $1 \notin f(A)$

$= 3.3.3.3 = 81$

$232 - 81 = 151$

[:Q.11] Let $P(4, 4\sqrt{3})$ be a point on the parabola $y^2 = 4ax$ and PQ be a focal chord of the parabola. If M and N are the foot of perpendicular drawn from P and Q respectively on the directrix of the parabola, then the area of the quadrilateral PQMN is equal to:

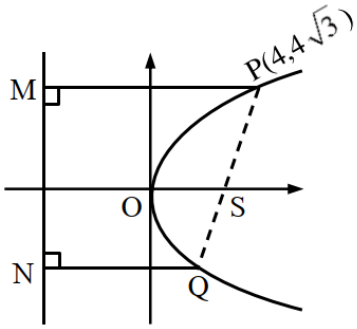
[:A] $\frac{34\sqrt{3}}{3}$

[:B] $17\sqrt{3}$

[:C] $\frac{343\sqrt{3}}{8}$

[:D] $\frac{263\sqrt{3}}{8}$

[:ANS] **C**



[:SOLN]

$(4, 4\sqrt{3})$ lies on $y^2 = 4ax$

$\Rightarrow 48 = 4a.4$

$4a = 12$

$\Rightarrow y^2 = 12x$ is equation of parabola

Now, parameter of P is $t_1 = \frac{2}{\sqrt{3}} \Rightarrow$ Parameters of Q

is $t_2 = -\frac{\sqrt{3}}{2} \Rightarrow Q\left(\frac{9}{4}, -3\sqrt{3}\right)$

Area of trapezium PQNM

$= \frac{1}{2}MN.(PM + QN)$

$= \frac{1}{2}MN.(PS + QS)$

$= \frac{1}{2}MN.\ PQ$

$= \frac{1}{2}7\sqrt{3}.\frac{49}{4} = (343)\frac{\sqrt{3}}{8} = 3$

[:Q.12] If $x = f(y)$ is the solution of the differential equation $\left(1+y^2\right)+\left(x-2e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0, y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with $f(0) = 1$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is equal to:

[:A] $e^{\pi/3}$

[:B] $e^{\pi/12}$

[:C] $e^{\pi/6}$

[:D] $e^{\pi/4}$

[:ANS] **C**

[:SOLN] $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{2e^{\tan^{-1}y}}{1+y^2}$

$\text{I.F.} = e^{\tan^{-1}y}$

$xe^{\tan^{-1}y}=\int\frac{2(e^{\tan^{-1}y})^2dy}{1+y^2}$

Put $\tan^{-1}y=t,\ \frac{dy}{1+y^2}=dt$

$xe^{\tan^{-1}y}=\int 2e^{2t}dt$

$xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+c$

$x=e^{\tan^{-1}y}+ce^{-\tan^{-1}y}$

$\because\ y=0,\ x=1$

$1=1+c\Rightarrow c=0$

$y=\frac{1}{\sqrt{3}},\ x=e^{\pi/6}$

[:Q.13] Let a line pass through two distinct points P(–2, –1, 3) and Q, and be parallel to the vector $3\hat{i}+2\hat{j}+2\hat{k}$. If the distance of the point Q from the point R (1, 3, 3) is 5, then the square of the area ΔPQR is equal to:

[:A] 140

[:B] 148

[:C] 136

[:D] 144

[:ANS] **C**

[:SOLN] $\overrightarrow{PQ}$ parallel to $3\hat{i}+2\hat{j}+2\hat{k}$, R(1, 3, 3)

$\Rightarrow Q(3\lambda-2, 2\lambda-1, 2\lambda+3),\ \lambda\in R-\{0\}$

$|\overrightarrow{QR}|=5=\sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2}$

$\therefore\ 17\lambda^2-34\lambda+25=25\Rightarrow\lambda=2(\because\ \lambda\neq 0)$

$\therefore\ Q(4, 3, 7), P(-2, -1, 3), R(1, 3, 3)$

$\text{Area of }\Delta PQR=[PQR]=\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|$

$$[PQR] = \frac{1}{2}\left\|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 4 \\ 3 & 4 & 0 \end{vmatrix}\right\| = \left\|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & 4 & 0 \end{vmatrix}\right\|$$

$$[PQR] = |-8\hat{i} + 6\hat{j} + 6\hat{k}| = \sqrt{136}$$

$$\therefore [PQR]^2 = 136$$

[:Q.14] In a group of 3 girls and 4 boys, there are two boys B_1 and B_2. The number of ways, in which these girls and boys can stand in a queue such that all the girls stand together, all the boys stand together, but B_1 and B_2 are not adjacent to each other, is:

[:A] 72

[:B] 144

[:C] 96

[:D] 120

[:ANS] **B**

[:SOLN] Total – when B_1 and B_2 are together

= 2!(3! 4!) – 2! (3!(3! 2!)) = 144

[:Q.15] Let $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ and $H: \frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$. Let the distance between the foci of E and the foci of H be $2\sqrt{3}$. If $a - A = 2$, and the ratio of the eccentricities of E and H is $\frac{1}{3}$, then the sum of the lengths of their latus rectums is equal to:

[:A] 10

[:B] 9

[:C] 8

[:D] 7

[:ANS] **C**

[:SOLN] $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ foci are (ae, 0) and (–ae, 0)

$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ foci are (Ae′, 0) and (–Ae′, 0)

[:Q.16] If $\lim_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e} - \frac{x}{1+x}\right)\right)^x = \alpha$, then the value of $\frac{\log_e \alpha}{1+\log_e \alpha}$ equals:

[:A] e

[:B] e^2

[:C] e^{-1}

[:D] e^{-2}

[:ANS] **1**

[:SOLN] $\alpha = \lim_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)\right)^x$ (1^∞ form)

$\therefore \alpha = e^L$

Where $L = \lim_{x\to\infty} x\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)-1\right)$

$\Rightarrow L = \lim_{x\to\infty}\left(\frac{e}{1-e}\right)x\left(\frac{1}{e}-\frac{x}{1+x}-\left(\frac{1-e}{e}\right)\right)$

$\Rightarrow L = \frac{e}{1-e}\lim_{x\to\infty} x\left(1-\frac{x}{1+x}\right)$

$\Rightarrow L = \frac{e}{1-e}\lim_{x\to\infty}\frac{x}{x+1}$

$\Rightarrow L = \frac{e}{1-e}.1$

$\Rightarrow L = \frac{e}{1-e}$

$\therefore \alpha = e^{\frac{e}{1-e}} \Rightarrow \log\alpha = \frac{e}{1-e}$

$\therefore$ Required value $= \frac{\frac{e}{1-e}}{1+\frac{e}{1-e}} = e$

[:Q.17] Let $f(x) = \int_0^{x/2} \frac{t^2-8t+15}{e^t}\,dt, x \in R$. Then the numbers of local maximum and local minimum points of f, respectively, are:

[:A] 3 and 2

[:B] 2 and 3

[:C] 1 and 3

[:D] 2 and 2

[:ANS] **B**

[:SOLN] $f'(x) = \left(\frac{x^4 - 8x^2 + 15}{e^{x^2}}\right)(2x)$

$= \frac{(x^2-3)(x^2-5)(2x)}{e^{x^2}}$

$= \frac{(x-\sqrt{3})(x+\sqrt{3})(x-\sqrt{5})(x+\sqrt{5})2x}{e^{x^2}}$

$$\begin{array}{cccccccccccc} - & & + & & - & & + & & - & & + \\ & -\sqrt{5} & & -\sqrt{3} & & 0 & & \sqrt{3} & & \sqrt{5} & \end{array}$$

Maxima at $x \in \{-\sqrt{3}, \sqrt{3}\}$

Minima at $x \in \{-\sqrt{5}, 0, \sqrt{5}\}$

2 points of maxima and 3 points of minima.

[:Q.18] Let the curve $z(1+i)+\bar{z}(1-i)=4, z \in C$, divide the region $|z-3| \le 1$ into two parts of areas α and β. Then $|\alpha-\beta|$ equals:

[:A] $1+\frac{\pi}{4}$

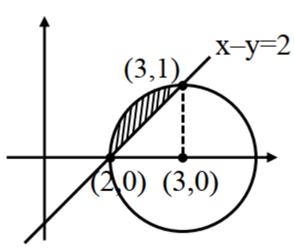
[:B] $1+\frac{\pi}{3}$

[:C] $1+\frac{\pi}{2}$

[:D] $1+\frac{\pi}{6}$

[:ANS] **C**

[:SOLN]



Let $z = x + iy$

$(x + iy)(1 + i) + (x - iy)(1 - i) = 4$

$x + ix + iy - y + x - ix - iy - y = 4$

$2x - 2y = 4$

$x - y = 2$

$|z - 3| \le 1$

$(x - 3)^2 + y^2 \le 1$

Area of shaded region $= \dfrac{\pi.1^2}{4} - \dfrac{1}{2}.1.1 = \dfrac{\pi}{4} - \dfrac{1}{2}$

Area of unshaded region inside the circle

$= \dfrac{3}{4}\pi.1^2 + \dfrac{1}{2}.1.1 = \dfrac{3\pi}{4} + \dfrac{1}{2}$

$\therefore$ difference of area $= \left(\dfrac{3\pi}{4} + \dfrac{1}{2}\right) - \left(\dfrac{\pi}{4} - \dfrac{1}{2}\right)$

$= \dfrac{\pi}{2} + 1$

[:Q.19] If $\int e^x \left(\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}} + \dfrac{\sin^{-1}x}{(1-x^2)^{3/2}} + \dfrac{x}{1-x^2} \right) dx = g(x) + C$, where C is the constant of integration, then $g\left(\dfrac{1}{2}\right)$ equals:

[:A] $\dfrac{\pi}{4}\sqrt{\dfrac{e}{3}}$

[:B] $\dfrac{\pi}{6}\sqrt{\dfrac{e}{3}}$

[:C] $\dfrac{\pi}{6}\sqrt{\dfrac{e}{2}}$

[:D] $\dfrac{\pi}{4}\sqrt{\dfrac{e}{2}}$

[:ANS] **B**

[:SOLN] $\because \dfrac{d}{dx}\left(\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}\right) = \dfrac{\sin^{-1}x}{(1-x^2)^{3/2}} + \dfrac{x}{1-x^2}$

$\Rightarrow \int e^x \left(\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}} + \dfrac{\sin^{-1}x}{(1-x^2)^{3/2}} + \dfrac{x}{1-x^2} \right) dx$

$= e^x.\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}} + c = g(x) + C$

Note : assuming $g(x) = \frac{xe^x \sin^{-1} x}{\sqrt{1-x^2}}$

$$g(1/2) = \frac{e^{1/2}}{2} \cdot \frac{\frac{\pi}{6} \times 2}{\sqrt{3}} = \frac{\pi}{6}\sqrt{\frac{e}{3}}$$

Comment : In this question we will not get a unique function g(x), but in order to match the answer we will have to assume

$$g(x) = \frac{xe^x \sin^{-1} x}{\sqrt{1-x^2}}.$$

[:Q.20] If A and B are two events such that $P(A \cap B) = 0.1$, and $P(A|B)$ and $P(B|A)$ are the roots of the equation $12x^2 - 7x + 1 = 0$, then the value of $\frac{P(\bar{A} \cup \bar{B})}{P(\bar{A} \cap \bar{B})}$ is:

[:A] $\frac{7}{4}$

[:B] $\frac{5}{3}$

[:C] $\frac{9}{4}$

[:D] $\frac{4}{3}$

[:ANS] **C**

[:SOLN] $12x^2 - 7x + 1 = 0$

$$x = \frac{1}{3}, \frac{1}{4}$$

$$\text{Let } P\left(\frac{A}{B}\right) = \frac{1}{3} \ \& \ P\left(\frac{B}{A}\right) = \frac{1}{4}$$

$$\frac{P(A \cap B)}{P(B)} = \frac{1}{3} \ \& \ \frac{P(A \cap B)}{P(A)} = \frac{1}{4}$$

$$\Rightarrow P(B) = 0.3$$

$$\& \ P(A) = 0.4$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.3 + 0.4 - 0.1 = 0.6$$

Now $\frac{P(\bar{A} \cup \bar{B})}{P(\bar{A} \cap \bar{B})} = \frac{P(\overline{A \cap B})}{P(\overline{A \cup B})}$

$= \frac{1-P(A \cap B)}{1-P(A \cup B)} = \frac{1-0.1}{1-0.6} = \frac{9}{4}$

SECTION 2

[:Q.21] Let A = {1, 2, 3}. The number of relations on A, containing (1,2) and (2,3), which are reflexive and transitive but not symmetric, is________.

[:ANS] **3**

[:SOLN] Transitivity

$(1, 2) \in R, (2, 3) \in R \Rightarrow (1, 3) \in R$

For reflexive $(1, 1), (2, 2)\ (3, 3) \in R$

Now $(2, 1), (3, 2), (3, 1)$

$(3, 1)$ cannot be taken

(1) (2, 1) taken and (3, 2) not taken

(2) (3, 2) taken and (2, 1) not taken

(3) Both not taken therefore 3 relations are possible.

[:Q.22] If $\sum_{r=1}^{30} \frac{r^2 \left({}^{30}C_r\right)^2}{{}^{30}C_{r-1}} = \alpha \times 2^{29}$, then α is equal to _________.

[:ANS] **465**

[:SOLN] $\sum_{r=1}^{30} \frac{r^2 ({}^{30}C_r)^2}{{}^{30}C_{r-1}}$

$= \sum_{r=1}^{30} r^2 \left(\frac{31-r}{r}\right) . \frac{30!}{r!(30-r)!}$

$\left(\because \frac{{}^{30}C_r}{{}^{30}C_{r-1}} = \frac{30-r+1}{r} = \frac{31-r}{r}\right)$

$= \sum_{r=1}^{30} \frac{(31-r)30!}{(r-1)!(30-r)!}$

$= 30\sum_{r=1}^{30} \frac{(31-r)29!}{(r-1)!(30-r)!}$

$$= 30\sum_{r=1}^{30}(30-r+1)^{29}C_{30-r}$$

$$= 30\left(\sum_{r=1}^{30}(31-r)^{29}C_{30-r} + \sum_{r=1}^{30}{}^{29}C_{30-r}\right)$$

$$= 30(29 \times 2^{28} + 2^{29}) = 30(29+2)2^{28}$$

$$= 15 \times 31 \times 2^{29}$$

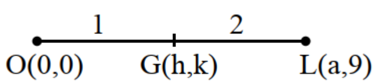
$$= 465(2^{29})$$

$$\alpha = 465$$

[:Q.23] Let A (6, 8), B$(10\cos\alpha, -10\sin\alpha)$ and $C(-10\sin\alpha, 10\cos\alpha)$, be the vertices of a triangle. If L (a, 9) and G(h, k) be its orthocenter and centroid respectively, then $(5a - 3h + 6k + 100\sin 2\alpha)$ is

[:ANS] **145**

[:SOLN] All the three points A, B, C lie on the circle $x^2 + y^2 = 100$ so circumcentre is (0, 0)



$$\frac{a+0}{3} = h \Rightarrow a = 3h$$

and $\frac{9+0}{3} = k \Rightarrow k = 3$

also centroid $\frac{6+10\cos\alpha-10\sin\alpha}{3} = h$

$\Rightarrow 10(\cos\alpha - \sin\alpha) = 3h - 6$ ....(i)

and $\frac{8+10\cos\alpha-10\sin\alpha}{3} = k$

$\Rightarrow 10(\cos\alpha - \sin\alpha) = 3k - 8 = 9 - 8 = 1$ ....(ii)

on squaring $100(1 - \sin 2\alpha) = 1$

$\Rightarrow 100\sin 2\alpha = 99$

from equ. (i) and (ii) we get $h = \frac{7}{3}$

Now $5a - 3h + 6k + 100\sin 2\alpha$

$= 15h - 3h + 6k + 100\sin 2\alpha$

$= 12 \times \frac{7}{3} + 18 + 99$

$= 145$

[:Q.24] Let y = f (x) be the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^6 + 4x}{\sqrt{1 - x^2}}, -1 < x < 1$ such that f (0) = 0. If $6\int_{-1/2}^{1/2} f(x)dx = 2\pi - \alpha$ then α^2 is equal to ________.

[:ANS] **27**

[:SOLN] I.F. $e^{-\frac{1}{2}\int\frac{2x}{1-x^2}dx} = e^{-\frac{1}{2}\ell n(1-x^2)} = \sqrt{1-x^2}$

$y \times \sqrt{1-x^2} = \int (x^6 + 4x)dx = \frac{x^7}{7} + 2x^2 + c$

Given $y(0) = 0 \Rightarrow c = 0$

$$y = \frac{\frac{x^7}{7} + 2x^2}{\sqrt{1-x^2}}$$

Now, $6\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\frac{x^7}{7} + 2x^2}{\sqrt{1-x^2}} dx = 6\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{2x^2}{\sqrt{1-x^2}} dx$

$= 24\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}} dx$

Put $x = \sin\theta$

$dx = \cos\theta \, d\theta$

$= 24\int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos\theta} \cos\theta d\theta$

$= 24\int_0^{\frac{\pi}{6}} \left(\frac{1-\cos 2\theta}{2}\right) d\theta = 12\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{6}}$

$= 12\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

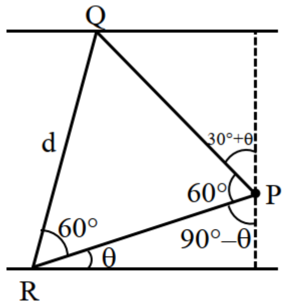
$= 2\pi - 3\sqrt{3}$

$\alpha^2 = (3\sqrt{3})^2 = 27$

[:Q.25] Let the distance between two parallel lines be 5 units and a point P lie between the lines at a unit distance from one of them. An equilateral triangle PQR is formed such that Q lies on one of the parallel lines, while R lies on the other. Then $(QR)^2$ is equal to _________.

[:ANS] **28**

[:SOLN]



$PR = \text{cosec}\theta,\ PQ = 4\sec(30 + \theta)$

For equilateral
$d = PR = PQ$

$\Rightarrow \cos(\theta + 30°) = 4\sin\theta$

$$\Rightarrow \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta = 4\sin\theta$$

$$\Rightarrow \tan\theta = \frac{1}{3\sqrt{3}}$$

$QR^2 = d^2 = \text{cosec}^2\theta = 28$