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|JEE MAIN 2025 | DATE : 22 JAN 2025 (SHIFT-1) MORNING

MATHEMATICS

SECTION 1

[:Q.1] Let the triangle PQR be the image of the triangle with vertices (1,3), (3,1) and (2, 4) in the line $x+2y=2$. IF the centroid of ΔPQR is the point (α,β), then $15(\alpha-\beta)$ is equal to:

[:A] 22

[:B] 24

[:C] 21

[:D] 19

[:ANS] **A**

[:SOLN] Let 'G' be the centroid of Δ formed by (1, 3) (3, 1)

& (2, 4)

$$G \cong \left(2, \frac{8}{3}\right)$$

Image of G w.r.t. $x + 2y - 2 = 0$

$$\frac{\alpha-2}{1} = \frac{\beta-\frac{8}{3}}{2} = -2\frac{\left(2+\frac{16}{3}-2\right)}{1+4}$$

$$= \frac{-2}{5}\left(\frac{16}{3}\right)$$

$$\Rightarrow \alpha = \frac{-32}{15}+2 = \frac{-2}{15},\ \beta = \frac{-32\times 2}{15}+\frac{8}{3} = \frac{-24}{15}$$

$$15(\alpha-\beta) = -2+24 = 22$$

[:Q.2] Let the foci of a hyperbola be (1, 14) and (1, –12). If it passes through the point (1, 6), then the length of its latus-rectum is:

[:A] $\frac{288}{5}$

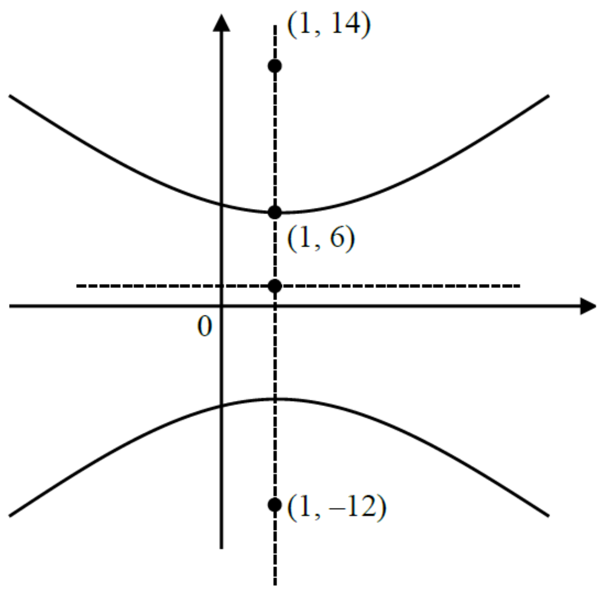
[:B] $\frac{144}{5}$

[:C] $\frac{24}{5}$

[:D] $\frac{25}{6}$

[:ANS] **A**

[:SOLN]



$$be = 13,\ b = 5$$

$$a^2 = b^2(e^2 - 1)$$

$$= b^2e^2 - b^2$$

$$= 169 - 25 = 144$$

$$\ell(LR) = \frac{2a^2}{b} = \frac{2\times 144}{5} = \frac{288}{5}$$

[:Q.3] Let $x = x(y)$ be the solution of the differential equation $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. If x(1) = 1, then $x\left(\frac{1}{2}\right)$ is:

[:A] 3 + e

[:B] $\frac{1}{2} + e$

[:C] $\frac{3}{2} + e$

[:D] 3 – e

[:ANS] **D**

[:SOLN]

$$\frac{dx}{dy} + \left(\frac{1}{y^2}\right)x = \frac{1}{y^3}$$

$$\text{I.F.} = e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$$

$$\Rightarrow x.e^{-\frac{1}{y}} = \int \left(e^{-\frac{1}{y}}\right).\frac{1}{y^3}dy$$

$$\text{Put } -\frac{1}{y} = t$$

$$+\frac{1}{y^2}dy = dt$$

$$x.e^{-\frac{1}{y}} = -\int t.e^t dt$$

$$x.e^{-\frac{1}{y}} = -te^t + e^t + C$$

$$x.e^{-\frac{1}{y}} = \frac{+1}{y}e^{-\frac{1}{y}} + e^{-\frac{1}{y}} + C$$

$x = 1, y = 1$

$$\frac{1}{e} = \frac{1}{e} + \frac{1}{e} + C$$

$$\Rightarrow C = -\frac{1}{e}$$

Put $y = \frac{1}{2}$

$$\frac{x}{e^2} = \frac{2}{e^2} + \frac{1}{e^2} - \frac{1}{e}$$

$x = 3 - e$

[:Q.4] A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point (2, 5) and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta - 2\alpha$ is equal to:

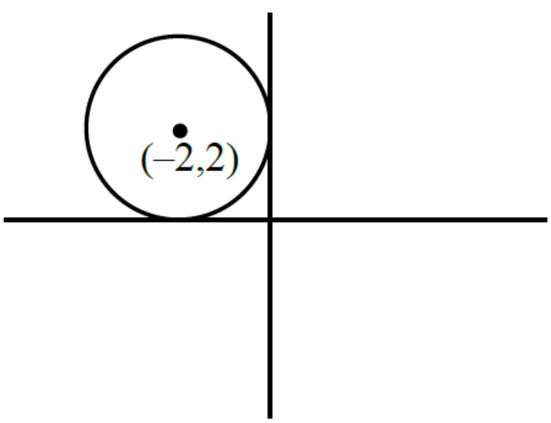
[:A] 14

[:B] 12

[:C] 10

[:D] 15

[:ANS] **D**



[:SOLN]

$$S_1 : (x+2)^2 + (y-2)^2 = 2^2$$
$$S_2 : (x-2)^2 + (y-5)^2 = r^2$$

Both circle intersect at two points

$\therefore\ |r_1 - r_2| < c_1c_2 < r_1 + r_2$

$|r - 2| < 5 < 2 + r$

$\Rightarrow 3 < r < 7$

$r \in (3, 7)$

$\alpha = 3, \beta = 7$

$3\beta - 2\alpha = 15$

[:Q.5] Let $a_1, a_2, a_3, \ldots\ldots\ldots$ be a G.P. of increasing positive terms. If $a_1a_5 = 28$ and $a_2a_4 = 29$, then a_6 is equal to:

[:A] 812

[:B] 784

[:C] 628

[:D] 526

[:ANS] **B**

[:SOLN] $a_1.a_5 = 28 \Rightarrow a.ar^4 = 28 \Rightarrow a^2r^4 = 28 \qquad \ldots(1)$

$a_2 + a_4 = 29 \Rightarrow ar + ar^3 = 29$

$\Rightarrow\ ar(1 + r^2) = 29$

$\Rightarrow a^2r^2(1 + r^2)^2 = (29)^2 \qquad \ldots(2)$

By Eq. (1) & (2)

$$\frac{r^2}{(1+r^2)^2} = \frac{28}{29 \times 29}$$

$$\Rightarrow \frac{r}{1+r^2} = \frac{\sqrt{28}}{29} \Rightarrow r = \sqrt{28}$$

$\because\ a^2r^4 = 28 \Rightarrow a^2 \times (28)^2 = 28$

$$\Rightarrow\ a = \frac{1}{\sqrt{28}}$$

$$\therefore a_6 = ar^5 = \frac{1}{\sqrt{28}} \times (28)^2 \sqrt{28} = 784$$

[:Q.6] The number of non-empty equivalence relations on the set {1, 2, 3} is :

[:A] 7

[:B] 6

[:C] 5

[:D] 4

[:ANS] **C**

[:SOLN] Let R be the required relation

$A = \{(1, 1)\ (2, 2), (3, 3)\}$

(i) $|R| = 3$, when $R = A$

(ii) $|R| = 5$, e.g. $R = A \cup \{(1, 2), (2, 1)\}$

Number of R can be [3]

(iii) $R = \{1, 2, 3\} \times \{1, 2, 3\}$

[:Q.7] Let $f: R \to R$ be a twice differentiable function such that f(x + y) = f (x) f(y) for all x, y $\in$R. If $f'(0) = 4a$ and f satisfies $f'(x) - 3af'(x) - f(x) = 0, a > 0,$ then the area of the region $R = \{(x, y) \mid 0 \le y \le f(ax), 0 \le x \le 2\}$ is:

[:A] $e^4 + 1$

[:B] $e^4 - 1$

[:C] $e^2 + 1$

[:D] $e^2 - 1$

[:ANS] **D**

[:SOLN] f(x + y) = f(x).f(y)

$\Rightarrow f(x) = e^{\lambda x} \quad f'(0) = 4a$

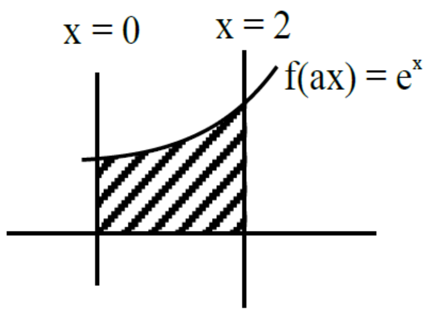
$\Rightarrow f'(x) = \lambda e^{\lambda x} \Rightarrow \lambda = 4a$

So, $f(x) = e^{4ax}$

$f''(x) - 3af'(x) - f(x) = 0$

$$\Rightarrow \lambda^2 - 3a\lambda - 1 = 0$$

$$\Rightarrow 16a^2 - 12a^2 - 1 = 0 \Rightarrow 4a^2 = 1 \Rightarrow \boxed{a = \frac{1}{2}}$$



$$F(x) = e^{2x}$$

$$\text{Area} = \int_0^2 e^x dx = \boxed{e^2 - 1}$$

[:Q.8] Using the principal values of the inverse trigonometric functions, the sum of the maximum and the minimum values of $16\left((\sec^{-1}x)^2 \_ (\operatorname{cosec}^{-1}x)^2\right)$ is:

[:A] $22\pi^2$

[:B] $31\pi^2$

[:C] $24\pi^2$

[:D] $18\pi^2$

[:ANS] **B**

[:SOLN] $16(\sec^{-1}x)^2 + (\operatorname{cosec}^{-1}x)^2$

$$\text{Sec}^{-1}x = a \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$$

$$\operatorname{cosec}^{-1}x = \frac{\pi}{2} - a$$

$$= 16\left[a^2 + \left(\frac{\pi}{2} - a\right)^2\right] = 16\left[2a^2 - \pi a + \frac{\pi^2}{4}\right]$$

$$\max]_{a=\pi} = 16[2\pi^2 - \pi^2 + \pi\frac{2}{4}] = 20\pi^2$$

$$\min]_{a=\frac{\pi}{4}} = 16\left[\frac{2\times\pi^2}{16} - \frac{\pi^2}{4} + \frac{\pi^2}{4}\right] = 2\pi^2$$

$\text{Sum} = 22\pi^2$

[:Q.9] The are of the region, inside the circle $\left(x-2\sqrt{3}\right)^2 + y^2 = 12$ and outside the parabola $y^2 = 2\sqrt{3}x$ is :

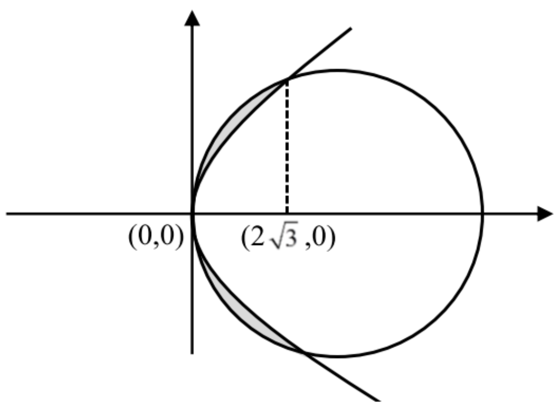
[:A] $3\pi + 8$

[:B] $3\pi - 8$

[:C] $6\pi - 8$

[:D] $6\pi - 16$

[:ANS] **D**



[:SOLN]

$y^2 = 2\sqrt{3}x$

$\left(x-2\sqrt{3}\right)^2 + y^2 = \left(2\sqrt{3}\right)^2$

$A = \frac{\pi r^2}{2} - 2\int_0^{2\sqrt{3}} \sqrt{2\sqrt{3}x}\, dx$

$\frac{\pi(12)}{2} - 2\sqrt{2\sqrt{3}}\frac{\left(x^{3/2}\right)_0^{2/3}}{3/2}$

$= 6\pi - 16$

[:Q.10] Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2?

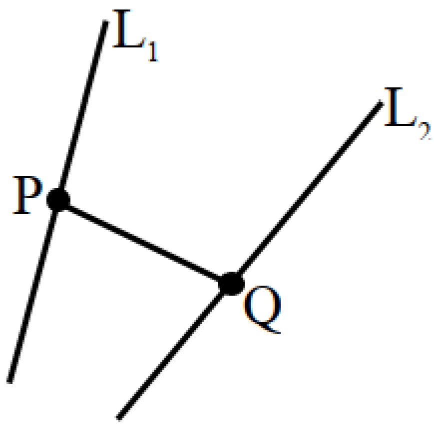
[:A] $\left(-\frac{5}{7}, -7, 1\right)$

[:B] $\left(2, 3, \frac{1}{3}\right)$

[:C] $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

[:D] $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$

[:ANS] **C**



[:SOLN]

$P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$ on L_1

$Q(3\mu + 2, 4\mu + 4, 5\mu + 5)$ on L_2

Dr's of PQ = $3\mu - 2\lambda + 1, 4\mu - 3\lambda + 2, 5\mu - 4\lambda + 2$

$PQ \perp L_1$

$\Rightarrow (3\mu - 2\lambda + 1)2 + (4\mu - 3\lambda + 2)3 + (5\mu - 4\lambda + 2)4 = 0$

$38\mu - 29\lambda + 16 = 0$ ...(1)

$PQ \perp L_2$

$\Rightarrow (3\mu - 2\lambda + 1)3 + (4\mu - 3\lambda + 2)4 + (5\mu - 4\lambda + 2)5 = 0$

$50\mu - 38\lambda + 21 = 0 \qquad \ldots(2)$

By (1) & (2)

$\lambda = \frac{1}{3}; \ \mu = \frac{-1}{6}$

$\therefore \ P\left(\frac{5}{3}, 3, \frac{13}{3}\right) \ \& \ Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$

Line PQ

$$\frac{x - \frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{z - \frac{13}{3}}{1}$$

$\text{Point}\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

lies on the line PQ

[:Q.11] Let A = {1, 2, 3, ….., 10} and $B = \left\{\frac{m}{n}; m, n \in A, m < n \text{ and } \gcd(m, n) = 1\right\}$. Then n(B) equal to:

[:A] 36

[:B] 29

[:C] 31

[:D] 37

[:ANS] **C**

[:SOLN] $A = \{1, 2, \ldots .10\}$

$B \ \{\frac{m}{n} = m, n \in A, m < n, \gcd(m, n) = 1\}$

n(B)

$n = 2 \qquad \left\{\frac{1}{2}\right\}$

$n = 3 \qquad \left\{\frac{1}{3}, \frac{2}{3}\right\}$

$n = 4 \qquad \left\{\frac{1}{4}, \frac{3}{4}\right\}$

$n = 5$ $\left\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right\}$

$n = 6$ $\left\{\frac{1}{6}, \frac{5}{6}\right\}$

$n = 7$ $\left\{\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}\right\}$

$n = 8$ $\left\{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\right\}$

$n = 9$ $\left\{\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}\right\}$

$n = 10$ $\left\{\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}\right\}$

n(B) = 31

[:Q.12] Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where gcd (m, n) = 1, then m + n is equal to:

[:A] 4

[:B] 11

[:C] 13

[:D] 14

[:ANS] **D**

[:SOLN]

$$P = \frac{\frac{6}{10} \times \frac{5}{9}}{\frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{5}{9}} = \frac{5}{9}$$

m = 5, n = 9

m + n = 14

[:Q.13] Let for $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$, $I_1 = \int_0^{\pi/4} f(x)\,dx$ and $I_2 = \int_0^{\pi/4} x\,f(x)\,dx$. Then $7I_1 + I2I_2$ is equal to:

[:A] π

[:B] 2

[:C] 1

[:D] 2π

[:ANS] **C**

[:SOLN] $f(x) = (7\tan^6 x - 3\tan^2 x)(\sec^2 x)$

$$I_1 = \int_0^{\pi/4} (7\tan^6 x - 3\tan^2 x)(\sec^2 x)\,dx$$

Put tanx = t

$$I_1 = \int_0^1 (7t^6 - 3t^2)\,dt = \left[t^7 - t^3\right]_0^1 = 0$$

$$I_2 = \int_0^{\pi/4} \underset{I}{x}\underbrace{(7\tan^6 x - 3\tan^2 x)(\sec^2 x)}_{II}\,dx$$

$$= \left[x\left(\tan^7 x - \tan^3 x\right)\right]_0^{\pi/4} - \int_0^{\pi/4} (\tan^7 x - \tan^3 x)\,dx$$

$$= 0 - \int_0^{\pi/4} \tan^3 x\left(\tan^2 x - 1\right)\left(1 + \tan^2 x\right)dx$$

Put tanx = t

$$= -\int_0^1 \left(t^5 - t^3\right)dt = -\left[\frac{t^6}{6} - \frac{t^4}{4}\right] = \frac{1}{12}$$

$7I_1 + 12I_2 = 1$

[:Q.14] Let z_1, z_2 and z_3 be three complex numbers on the circle $|z| = 1$ with are $(z_1) = \frac{-\pi}{4}$, are $(z_2) = 0$ and are $(z_3) = \frac{\pi}{4}$. If $|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2 = \alpha + \beta\sqrt{2}, \alpha, \beta \in Z$, then the value of $x^2 + \beta^2$ is:

[:A] 29

[:B] 24

[:C] 41

[:D] 31

[:ANS] **A**

[:SOLN] $Z_1 = e^{-i\pi/4}$, $Z_2 = 1$, $Z_3 = e^{i\pi/4}$

$$|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2 = \left|e^{-i\frac{\pi}{4}} \times 1 + 1 \times e^{-i\frac{\pi}{4}} + e^{i\frac{\pi}{4}} \times e^{i\frac{\pi}{4}}\right|^2$$

$$\left|e^{-i\frac{\pi}{4}} + e^{-i\frac{\pi}{4}} + e^{i\frac{\pi}{4}}\right|^2$$

$$= \left|2e^{-i\frac{\pi}{4}} + i\right|^2 = = |\sqrt{2} - \sqrt{2}i + i|^2$$

$$= \left(\sqrt{2}\right)^2 + \left(1 - \sqrt{2}\right)^2 = 2 + 1 + 2 - 2\sqrt{2} = 5 - 2\sqrt{2}$$

$\alpha = 5, \beta = -2$

$\Rightarrow \alpha^2 + \beta^2 = 29$

[:Q.15] If $\sum_{r=1}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{T_r}\right)$ is equal to:

[:A] $\frac{1}{3}$

[:B] $\frac{2}{3}$

[:C] 1

[:D] 0

[:ANS] **B**

[:SOLN] $T_n = S_n - S_{n-1}$

$$\Rightarrow T_n = \frac{1}{8}(2n-1)(2n+1)(2n+3)$$

$$\Rightarrow \frac{1}{T_n} = \frac{8}{(2n-1)(2n+1)(2n+3)}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{T_r} = \lim_{n\to\infty} 8\sum_{r=1}^{n}\frac{1}{(2n-1)(2n+1)(2n+3)}$$

$$= \lim_{n\to\infty}\frac{8}{4}\sum\left(\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right)$$

$$= \lim_{n\to\infty} 2\left[\left(\frac{1}{1.3} - \frac{1}{3.5}\right) + \left(\frac{1}{3.5} - \frac{1}{5.7}\right) + \ldots\right]$$

$$= \frac{2}{3}$$

[:Q.16] A coin is tossed three times. Let X denote the number of times a tail follow a head. If μ and σ^2 denote the mean and variance of X, then the value of $64\left(\mu+\sigma^2\right)$ is:

[:A] 64

[:B] 51

[:C] 32

[:D] 48

[:ANS] **D**

[:SOLN] $HHH \to 0$

$HHT \to 0$

$HTH \to 1$

$HTT \to 0$

$THH \to 1$

$THT \to 1$

$TTH \to 1$

$TTT \to 0$

Probability distribution

x_i	0	1
$P(x_i)$	$1/2$	$1/2$

$\mu = \sum x_i p_i = \frac{1}{2}$

$\sigma^2 = \sum x_i^2 p_i - \mu^2$

$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$64(\mu + \sigma^2) = 64\left(\frac{1}{2} + \frac{1}{4}\right) = 48$

[:Q.17] Let f (x) be a real differentiable function such that $f(0) = 1$ and $f(x+y) = f(x)f'(y) + f'(x)f(y)$ for all $x, y, \in R$. Then $\sum_{n=1}^{100} \log_e f(n)$ is equal to:

[:A] 5220

[:B] 2525

[:C] 2384

[:D] 2384

[:ANS] **B**

[:SOLN] $f(x+y) = f(x)\,f'(y) + f'(x)\,f(x)$

Put = x = y = 0

$f(0) = f(0)f'(0) + f'(0)f(0)$

$f'(0) = \frac{1}{2}$

Put $y = 0$

$f(x) = f(x)\,f'(0) + f'(x)f(0)$

$f(x) = \frac{1}{2}f(x) + f'(x)$

$f'(x) = \frac{f(x)}{2}$

$\frac{dy}{dx} = \frac{y}{2} \Rightarrow \int \frac{dy}{y} = \int \frac{dx}{2}$

$\Rightarrow \ell ny = \frac{x}{2} + c$

[:Q.18] From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle latter is 'M', is:

[:A] 6084

[:B] 14950

[:C] 5148

[:D] 4356

[:ANS] **C**

[:SOLN] $= \underbrace{{}^{12}C_2} \times \underbrace{{}^{13}C_2} = 5148$

[:Q.19] Let the parabola $y = x^2 | px - 3$, meet the corrdinate axes at the points, P, Q and R. If the circle C with centre at (–1, – 1) passes through the points, P, Q and R, then the area of ΔPQR is:

[:A] 6

[:B] 4

[:C] 7

[:D] 5

[:ANS] **A**

[:SOLN] $y = x^2 + px - 3$

Let $P(\alpha, 0)$, $Q(\beta, 0)$, $R(0, -3)$

Circle with centre $(-1, -1)$ is $(x + 1)^2 + (y + 1)^2 = r^2$

Passes through $(0, -3)$

$1^2 + (-2)^2 = r^2$]

$r^2 = 5$

$(x + 1)^2 + (y + 1)^2 = 5$

Put $y = 0$

$(x + 1)^2 = 5 - 1$

$(x + 1)^2 = 4$

$x + 1 = \pm 2$

$x = 1$ or $x = -3$

$\therefore P(1, 0)$ and $Q(-3,0)$

$$\text{Area of } \Delta PQR = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ -3 & 0 & 1 \\ 0 & -3 & 1 \end{vmatrix} = 6$$

[:Q.20] The product of all solutions of the equation $e^{5(\log_e x)^2+3} = x^8, x > 0,$ is:

[:A] e

[:B] $e^{8/5}$

[:C] $e^{6/5}$

[:D] e^2

[:ANS] **B**

[:SOLN] $e^{5(\ell nx)^2+3} = x^8$

$\Rightarrow \ell n\, e^{5(\ell nx)^2+3} = \ell n\, x^8$

$\Rightarrow 5(\ell nx)^2 + 3 = 8\ell nx$

$(\ell nx = t)$

$\Rightarrow 5t^2 - 8t + 3 = 0$

$t_1 + t_2 = \frac{8}{5}$

$\ell n x_1 x_2 = \frac{8}{5}$

$x_1 x_2 = e^{8/5}$

SECTION 2

[:Q.21] Let A be a square matrix of order 3 such that det (A) = –2 and det (3adj (–6adj (3A))) = $2^{m+n}.3^{mm}, m > n$. Then 4 m + 2n is equal to________.

[:ANS] **34**

[:SOLN] $|A| = -2$

$\det(3adj(-6adj(3A)))$

$= 3^3 \det(adj(-adj(3A)))$

$= 3^3(-6)^6(\det(3A))^4$

$= 3^{21} \times 2^{10}$

$m + n = 10$

$mn = 21$

$m = 7\ ;\ n = 3$

[:Q.22] Let the function,

$$f(x) = \begin{cases} -3\,ax^2 - 2, & x < 1 \\ a^2 + bx, & x \geq 1 \end{cases}$$

be differentiable for all $x \in R$, where a>1, $b \in R$. If the area of the region enclosed by $y = f(x)$ and the line y = – 20 is $\alpha + \beta\sqrt{3}, \alpha\beta \in Z$, then the value of $\alpha + \beta$ is ________.

[:ANS] **34**

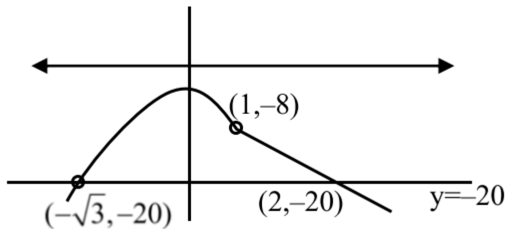
[:SOLN] f(x) is continuous and differentiable

at $x = 1$; LHL = RHL, LHD = RHD

$-3a - 2 = a^2 + b, -6a = b$

$a = 2, 1;\ b = -12$

$$f(x)=\begin{cases}-6x^2-2 & ; \quad x<1\\ 4-12x & ; \quad x\geq 1\end{cases}$$



$$\text{Area}=\int_{-\sqrt{3}}^{1}\left(-6x^2-2+20\right)dx+\int_{1}^{2}(4-12x+20)\,dx$$

$$16+12\sqrt{3}+6=22+12\sqrt{3}$$

[:Q.23] Let : $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and : $\frac{x-2}{2}=\frac{y}{0}=\frac{z+4}{\alpha}\ \alpha\in R,$ be two lines, which intersect at the point B. If P is the foot of perpendicular from the point A (1, 1, –1) on L_2, then then value of 26 $\alpha(PB)^2$ is________.

[:ANS] **216**

[:SOLN] Point B

$(3\lambda+1, -\lambda+1, -1)\equiv(2\mu+2, 0, \alpha\mu-4)$

$3\lambda+1=2\mu+2$

$-\lambda+1=0$

$-1=\alpha\mu-4$

$\lambda=1, \mu=1, \alpha=3$

B(4, 0, –1)

Let Point 'P' is $(2\delta+2, 0, 3\delta-4)$

Dr's of AP $<2\delta+1, -1, 3\delta-3>$

$AP\perp L_2\Rightarrow\ \delta=\frac{7}{13}$

$P\left(\frac{40}{13},0,\frac{-31}{13}\right)$

$26\alpha(PB)^2=26\times3\times\left(\frac{144}{169}+\frac{324}{169}\right)$

$= 216$

[:Q.24] If $\sum_{r=0}^{5} \frac{^{11}C_{2r+1}}{2r+2} = \frac{m}{n}$; $\gcd(m,n) = 1$, then , m – n is equal to ________.

[:ANS] **2035**

[:SOLN]

$$\int_0^1 (1+x)^{11} dx = \left[C_0 x + \frac{C_1 x^2}{2} + \frac{C_2 x^3}{3} + ...\right]_0^1$$

$$\frac{2^{12}-1}{12} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \frac{C_3}{4} + ...$$

$$\int_{-1}^{0} (1+x)^{11} dx = \left[C_0 x + \frac{C_1 x^2}{2} + \frac{C_2 x^3}{3} + ...\right]_{-1}^{0}$$

$$\frac{1}{12} = C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + ...$$

$$\frac{2^{12}-2}{12} = 2\left(\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + ...\right)$$

$$\frac{C_1}{2} + \frac{C_3}{4} - \frac{C_5}{6} + ... = \frac{2^{11}-1}{12} = \frac{2047}{12}$$

[:Q.25] Let $\vec{C}$ be the projection vector of $\vec{b} = \lambda\hat{i} + 4\hat{k}, \lambda > 0$, on the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$. If $\left|\vec{a} + \vec{c}\right| = 7$, then the area of the parallelogram formed by the vectors $\vec{b} + \vec{c}$ is __________.

[:ANS] **16**

[:SOLN]

$$\vec{c} = \left(\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}\right)\frac{\vec{a}}{\|\vec{a}|}$$

$$= \left(\frac{\lambda+8}{9}\right)\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$$

$$|\vec{a} + \vec{c}| - = 7 \Rightarrow \lambda = 4$$

Area of parallelogram

$$= \left|\vec{b} \times \vec{c}\right| = \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \\ 4 & 0 & 4 \end{vmatrix} \right|$$

$= 16$