JEE (ADVANCED) 2025 PAPER-1

[PAPER ANSWER KEY WITH SOLUTION]

HELD ON SUNDAY 18TH MAY 2025

MATHEMATICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.

[:Q.1] Let R denote the set of all real numbers. Let $a_i, b_i \in R$ for $i \in \{1,2,3\}$.

Define the functions $f: R \to R$, $g: R \to R$, and $h: R \to R$ by

$$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4,$$

$$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4,$$

$$h(x) = f(x+1) - g(x+2).$$

If $f(x) \neq g(x)$ for every $x \in R$, then the coefficient of x^3 in h (x) is

[:A] 8 [:B] 2 [:C] –4 [:D] –6

[:ANS] **C**

[:SOLN] $h(x) = f(x+1) - g(x+2)$

Coeff. Of x^3 in h(x) = $\dfrac{h'''(0)}{3!} = \dfrac{f'''(1)}{3!} - \dfrac{g'''(2)}{3!}$

$= (a_3 + 4) - (b_3 + 8)$

$= a_3 - b_3 - 4$ (1)

Further, f(x) $\neq$ g(x) $\forall$ $x \in R$

$\Rightarrow f(x) - g(x) \neq 0 \quad \forall x \in R$

$\Rightarrow a_3 = b_3$

$\therefore$ required coff = - 4

[:Q.2] Three students S_1, S_2 and S_3 are given a problem to solve. Consider the following events:

U: At least one of S_1, S_2 and S_3 can solve the problem,

V : S_1 can solve the problem, given that neither S_2 nor S_3 can solve the problem,

W : S_2 can solve the problem and S_3 cannot solve the problem,

T : S_3 can solve the problem

For any event E, let $P(E)$ denote the probability of E. If

$P(U) = \frac{1}{2}, P(V) = \frac{1}{10}$, and $P(W) = \frac{1}{12}$,

Then P(T) is equal to

[:A] $\frac{13}{36}$ [:B] $\frac{1}{3}$ [:C] $\frac{19}{60}$ [:D] $\frac{1}{4}$

[:ANS] **A**

[:SOLN] Let $P(S_1) = x, P(S_2) = Y \;\&\; P(S_3) = Z$

Now, $P(S_1 \cup S_2 \cup S_3) = P(\cup) = \frac{1}{2}$

$\Rightarrow P(S_1' \cap S_2' \cap S_3') = 1 - P(S_1 \cup S_2 \cup S_3) = \frac{1}{2}$

$\Rightarrow (1-x)(1-y)(1-z) = \frac{1}{2}$ .....(1)

$P(V) = P(S_1 / S_2' \cap S_3') = \frac{P(S_1 \cap S_2' \cap S_3')}{P(S_2' \cap S_3')} = \frac{x(1-y)(1-z)}{(1-y)(1-z)}$

$\therefore x = 1/10$ ......(2)

From (1) and (2) $(1-y)(1-z) = \frac{5}{9}$

$= (1-z) - y(1-z) = \frac{5}{9}$

$\Rightarrow 1 - z = \frac{5}{9} + P(W) = \frac{5}{9} + \frac{1}{12} = \frac{23}{36} \Rightarrow Z = 13/36$

[:Q.3] Let R denote the set of all real numbers. Define the function $f: R \to R$ by

$$f(x)=\begin{cases} 2-2x^2-x^2\sin\frac{1}{x} & \text{if } x\neq 0,\\ 2 & \text{if } x=0.\end{cases}$$

Then which one of the following statements is TRUE?

[:A] The function f is NOT differentiable at x = 0

[:B] There is a positive real number δ, such that f is a decreasing function on the interval $(0,\delta)$

[:C] For any positive real number δ, the function f is NOT an increasing function on the interval $(-\delta,0)$

[:D] x = 0 is a point of local minima of f

[:ANS] **C**

[:SOLN] $f(x)=\begin{cases} 2-2x^2-x^2\sin\frac{1}{x}, & x\neq 0\\ 2 & , x=0\end{cases}$

$$f'(o)=\lim_{x\to 0}\frac{f(x)-f(0)}{x}=\lim_{x\to 0}\frac{-2x^2-x^2\sin 1/x}{x}$$

$$\lim_{x\to 0}\left(-2x-x\sin\frac{1}{x}\right)=0$$

When $x\neq 0$, $f'(x)=-4x-2x\sin\frac{1}{x}+\cos\frac{1}{x}$, when x approaches zero the value of $f'(x)$ will be decided by $\cos\frac{1}{x}$ which lies in [-1,1], therefore, B is false but C is true, further in the deleted nbd. of x = 0

f(0) – f(x) = 2 – f(x) = $2x^2+x^2\sin\frac{1}{x}$

as $-1\le\sin\frac{1}{x}\le 1$, we have

$$x^2\le 2x^2+x^2\sin\frac{1}{x}\le 3x^2$$

$\therefore 2-f(x)\ge 0 \quad \forall x$ in nbd. of x = 0

$\therefore$ D is false

[:Q.4] Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Let the transpose of a matrix X be denoted by X^T. Then the number of 3 × 3 invertible matrices Q with integer entries, such that

$Q^{-1} = Q^T$ and $PQ = QP$, is

[:A] 32 [:B] 8 [:C]16 [:D]24

[:ANS] **C**

[:SOLN] Let $Q = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Now $PQ = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} 2a_1 & 2a_2 & 2a_3 \\ 2b_1 & 2b_2 & 2b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix}$

& $QP = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2a_1 & 2a_2 & 3a_3 \\ 2b_1 & 2b_2 & 3b_3 \\ 2c_1 & 2c_2 & 3c_3 \end{bmatrix}$

as PQ = QP = we have $a_3 = b_3 = c_1 = c_2 = 0$

So $Q = \begin{bmatrix} a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}$, further $Q^{-1} = Q^T$

$\Rightarrow QQ^T = I$ i.e $\begin{bmatrix} a_1^2 + a_2^2 & a_1b_1 + a_2b_2 & 0 \\ a_1b_1 + a_2b_2 & b_1^2 + b_2^2 & 0 \\ 0 & 0 & c_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow a_1^2 + a_2^2 = 1, b_1^2 + b_2^2 = 1, a_1b_1 + a_2b_2 = 0,\ C_3^2 = 1$

So, Q can be formed with $(a_1, a_2) \equiv (\pm 1, 0), (b_1, b_2) \equiv (0, \pm 1), c_3 = \pm 1$

Or $(a_1, a_2) \equiv (0, \pm 1), (b_1, b_2) \equiv (\pm 1, 0), C_3 = \pm 1$

Hence the required number of matrices = 16

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

  choosing ONLY (B) will get +1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option (i.e. the question is unanswered) will get 0 marks; and

  choosing any other combination of options will get −2 marks.

[:Q.5] Let L_1 be the line of intersection of the planes given by the equations

$2x + 3y + z = 4$ and $x + 2y + z = 5$

Let L_2 be the line passing through the point $P(2, -1, 3)$ and parallel to L_1. Let M denote the plane given by the equation

$2x + y - 2z = 6$.

Suppose that the line L_2 meets the plane M at the point Q. Let R be the foot of the perpendicular drawn from P to the plane M.

Then which of the following statements is (are) TRUE?

[:A] The length of the line segment PQ is $9\sqrt{3}$

[:B] The length of the line segment QR is 15

[:C] The area of ΔPQR is $\frac{3}{2}\sqrt{234}$

[:D] The acute angle between the line segments PQ and PR is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

[:ANS] **A,C**

[:SOLN] A vector along $L_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i} - \hat{j} + \hat{k}$

$\therefore L_1 : \dfrac{x-2}{1} = \dfrac{y+1}{-1} = \dfrac{z-3}{1}$

Let Q be $(2+\alpha, -1-\alpha, 3+\alpha)$ as it lies on plane

M i.e, $2x + y - 2Z = 6$, we have

$2(2+\alpha) + (-1-\alpha) - 2(3+\alpha) = 6$

$\Rightarrow \alpha = -9$

$\Rightarrow Q \equiv (-7, 8, -6)$

Now R is foot of perpendicular from $P(2, -1, 3)$ on to

Plane M

$\therefore \dfrac{x-2}{2} = \dfrac{y+1}{1} = \dfrac{z-3}{-2} = \dfrac{-(4-1-6-6)}{9}$

$\therefore R \equiv (4, 0, 1)$

$\therefore \overrightarrow{PQ} = -9\hat{i} + 9\hat{j} - 9\hat{k}$

$\overrightarrow{QR} = 11\hat{i} - 8\hat{j} + 7\hat{k}$

$\overrightarrow{PR} = 2\hat{i} + \hat{j} - 2\hat{k}$

Now, $PQ = \left|\overrightarrow{PQ}\right| = 9\sqrt{3}$

$QR = \left|\overrightarrow{QR}\right| = \sqrt{234}$

ar $(\Delta PQR) = \dfrac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \dfrac{1}{2}\sqrt{PQ^2.PR^2 - \left(\overrightarrow{PQ} \cdot \overrightarrow{PR}\right)^2}$

$= \dfrac{1}{2}\sqrt{243 \times 9 - 81} = \dfrac{9}{2}\sqrt{26} = \dfrac{3}{2}\sqrt{234}$

$\cos\theta = \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PR}\right|} = \dfrac{1}{3\sqrt{3}}$

[:Q.6] Let N denote the set of all natural numbers, and Z denote the set of all integers. Consider the functions $f : N \to Z$ and $g : Z \to N$ define by

$$f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd,} \\ (4-n)/2 & \text{if } n \text{ is even,} \end{cases}$$

And

$$g(n) = \begin{cases} 3+2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0, \end{cases}$$

Define $(gof)(n) = g(f(n))$ for all $n \in N$, and $(fog)(n) = f(g(n))$ for all $n \in Z$.

Then which of the following statements is (are) TRUE?

[:A] $g\,o\,f$ is NOT one-one and $g\,o\,f$ is NOT onto

[:B] $f\,o\,g$ is NOT one-one but $f\,o\,g$ is onto

[:C] g is one-one and g is onto

[:D] f is NOT one-one but f is onto

[:ANS] **AD**

[:SOLN] f(1) = f(2) = 1 $\Rightarrow$ f is not one – one

Let $m \in z$

If m > 0, f (2m -1) = $\frac{2m-1+1}{2} = m$

& if m < 2, $f(4-2m) = \frac{4-(4-2m)}{2} = m$

f(3) = 2 & $f(4) = 0$, thus for any integer m there exists a natural number n such that f(n) = m , therefore f is onto

when $n_1 \geq 0 \,\&\, n_2 \geq 0$, $g(n_1) = g(n_2) \Rightarrow 3 + 2n_1 = 3 + 2n_2$

$\Rightarrow n_1 = n_2$

If $n_1 < 0 \,\&\, n_2 < 0$, $g(n_1) = g(n_2) \Rightarrow -2n_1 = -2n_2$

$\Rightarrow n_1 = n_2$

& if $n_1 >, 0 \,\&\, n_2 < 0$ $g(n_1) = 3 + 2n_1 \to \text{odd}$

$\& g(n_2) = -2n_2 \to \text{even}$

$\Rightarrow g(n_1) \neq g(n_2)$

Therefore, g is one - one

g is not onto as for $n \geq 0$ g(n) is odd & ≥ 3 and for n < 0 g(n) is even & ≥ 2,

therefore g can never be equal to I.

Now $g\,o\,f\left(n\right) = g\left(f\left(n\right)\right)$

g o f (1) = g(f(1)) = g(1) = 5

g o f (2) = g(f(2)) = g(1) = 5

g o f (n) is not one – one

if n is odd natural number f(n) $= \frac{n+1}{2} > 0$

$\Rightarrow$ g o f (n) = $3 + 2\left(\frac{n+1}{2}\right) = n + 4 \geq 5$

& if n is even f(n) = $\frac{4-n}{2}$

If $\frac{4-n}{2} \geq 0$, g o f (n) = 3 + 2 $\left(\frac{4-n}{2}\right) = 7 - n$

& if $\frac{4-n}{2} < 0, g\,o\,f(n) = n - 4 \geq 0$

$\therefore$ g o f is not onto

If $n \geq 0, f\left(g\left(n\right)\right) = f\left(3 + 2n\right) = \frac{3 + 2n + 1}{2} = 2 + n$

If n < 0 $f\left(g\left(n\right)\right) = f\left(-2n\right) = \frac{4 - \left(-2n\right)}{2} = 2 + n$

f(g(n)) = n + 2 for all $n \in Z$

f o g (n) is one-one & onto

[:Q.7] Let R denote the set of all real numbers. Let $z_1 = 1 + 2i$ and z_2 = 3i be two complex numbers, where $i = \sqrt{-1}$. Let

$$S = \left\{\left(x, y\right) \in R \times R : \left|x + iy - z_1\right| = 2\left|x + iy - z_2\right|\right\}.$$

Then which of the following statements is (are) TRUE?

[:A] S is a circle with centre $\left(-\frac{1}{3}, \frac{10}{3}\right)$

[:B] S is a circle with centre $\left(\frac{1}{3}, \frac{8}{3}\right)$

[:C] S is a circle with radius $\frac{\sqrt{2}}{3}$

[:D] S is a circle with radius $\frac{2\sqrt{2}}{3}$

[:ANS] **A,D**

[:SOLN] $\left|x+iy-(1+2i)\right| = 2\left|x+iy-3i\right|$

$$\sqrt{(x-1)^2+(y-2)^2} = 2\sqrt{x^2+(y-3)^2}$$

$$\Rightarrow 3x^2+3y^2+2x-20y+31=0$$

$$\Rightarrow x^2+y^2+\frac{2}{3}x-\frac{20}{3}y+\frac{31}{3}=0$$

Which is a circle with centre $\left(-\frac{1}{3},\frac{10}{3}\right)$ and radius $\frac{2\sqrt{2}}{3}$

SECTION-3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

[:Q.8] Let the set of all relations R on the set {a, b, c, d, e, f }, such that R is reflexive and symmetric, and R contains exactly 10 elements, be denoted by S.
Then the number of elements in S is ________________.

[:ANS] **105**

[:SOLN] Since R has 10 elements & it has to be reflexive six elements will be (a,a) , (b,b), (c,c) (d,d), (e,e), (f,f) and since
R is symmetric remaining 4 elements must come in pairs of the form (x,y) and (y, x) therefore, there must be two such pairs, there are ${}^{6}C_2=15$ such possible pairs, therefore, required number of ways = ${}^{15}C_2=105$.

[:Q.9] For any two points M and N in the XY -plane, let $\overrightarrow{MN}$ denote the vector from M to N, and $\vec{0}$ denote the zero vector. Let P, Q and R be three distinct points in the XY -plane. Let S be a point inside the triangle Δ PQR such that

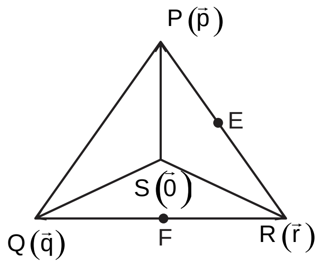
$$\overrightarrow{SP}+5\overrightarrow{SQ}+6\overrightarrow{SR}=\vec{0}.$$

Let E and F be the mid-points of the sides PR and QR, respectively. Then the value of

$\frac{\text{length of the line segment EF}}{\text{length of the line segment ES}}$ is __________.

[:ANS] **1.2**

[:SOLN] Fix origin at S and take position vectors of P, Q, R to be $\vec{P}, \vec{q}, \vec{r}$ respectively



$\overrightarrow{SP} + 5\overrightarrow{SQ} + 6\overrightarrow{SR} = \vec{0}$

$\Rightarrow \vec{P} + 5\vec{q} + 6\vec{r} = \vec{0}$

$\overrightarrow{SE} = \dfrac{\overrightarrow{SP} + \overrightarrow{SR}}{2} = \dfrac{\vec{p} + \vec{r}}{2}$

$\overrightarrow{SF} = \dfrac{\overrightarrow{SQ} + \overrightarrow{SR}}{2} = \dfrac{\vec{q} + \vec{r}}{2}$

$\therefore \overrightarrow{EF} = \overrightarrow{SF} - \overrightarrow{SE} = \dfrac{\vec{q} - \vec{p}}{2}$

$\therefore \dfrac{\left|\overrightarrow{EF}\right|}{\left|\overrightarrow{ES}\right|} = \dfrac{|\vec{q} - \vec{p}|}{|\vec{p} + \vec{r}|} = \dfrac{|\vec{q} + 5\vec{q} + 6\vec{r}|}{|-5\vec{q} - 6\vec{r} + \vec{r}|}$

$= \dfrac{6|\vec{q} + \vec{r}|}{5|\vec{q} + \vec{r}|} = \dfrac{6}{5} = 1.2$

[:Q.10] Let S be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in S, but 0210222 is NOT in S.
Then the number of elements x in S such that at least one of the digits 0 and 1 appears exactly twice in x, is equal to _____________.

[:ANS] **762**

[:SOLN] Let A be the set of the seven digit numbers containing exactly two zeros and B that of the numbers containing exactly two ones.

Then $n(A) = {}^6C_2 \times 2^5 = 480$

$n(B) = {}^7C_2 \times 2^5 - {}^6C_2 \times 2^4 = 432$

$n(A \cap B) =$ no. of 7 digit no. formed

Using 0,0,1,1,2,2,2

$= \dfrac{\lfloor 7}{\lfloor 2 \lfloor 2 \lfloor 3} - \dfrac{\lfloor 6}{\lfloor 2 \lfloor 3} = 150$

$\therefore$ required no. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 480 + 432 - 150 = 762$

[:Q.11] Let α and β be the real numbers such that

$$\lim_{x\to 0}\frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt+\beta x\cos x\right)=2.$$

Then the value of $\alpha+\beta$ is _____.

[:ANS] **2.4**

[:SOLN] $2 = \lim_{x\to 0}\dfrac{\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt+\beta x\cos x}{x^3}\left[\frac{0}{0}\text{ form}\right]$

$$\lim_{x\to 0}\frac{\frac{\alpha}{2}\cdot\frac{1}{1-x^2}+\beta(\cos x-x\sin x)}{3x^2}$$ L′ Hopital's rule

$\because D^r \to 0 \therefore N^r \to 0$

$\Rightarrow \frac{\alpha}{2}+\beta=0$

Again applying L′ Hopital's Rule

$$2=\lim_{x\to 0}\frac{-\beta\cdot\frac{-1(-2x)}{(1-x^2)^2}+\beta(-2\sin x-x\cos x)}{6x}$$

$$=\lim_{x\to 0}\frac{-\beta}{6}\left[\frac{2}{(1-x^2)^2}+\frac{2\sin x}{x}+\cos x\right]$$

$$=\frac{-\beta}{6}(2+2+1)$$

$$\Rightarrow \beta=\frac{-12}{5}=-2.4$$

$\alpha+\beta=-2\beta+\beta=-\beta=2.4$

[:Q.12] Let R denote the set of all real numbers. Let f : R $\to$R be a function such that f(x) > 0 for all x$\in$R, and f(x+y) = f(x) f(y) for all x,y $\in$R.

Let the real numbers $a_1, a_2, ...a_{50}$ be in an arithmetic progression. If $f(a_{31})=64f(a_{25})$, and

$\sum_{i=1}^{50} f(a_i)=3\left(2^{25}+1\right)$,

Then the value of

$\sum_{i=6}^{30} f(a_i)$ is _________.

[:ANS] **96**

[:SOLN] f(x + y) = f(x) f(y) – (i)

Putting x = y = 0,

f(0) = f(0) . f(0) = f(0) = 1

Putting x = d, y = (k-1)d ; $k \in N$

f(kd) = f(d) f(k-1)d)

$= (f(d))^2 \, f\left(\left(k-2\right)d\right)$

$= \ldots\ldots = \left(f\left(d\right)\right)^k f\left(0\right)$

$= \left(f\left(d\right)\right)^k$

Let $a_i = a + \left(i-1\right)d$

$\therefore f\left(a_i\right) = f\left(a\right) f\left(\left(i-1\right)d\right) = f\left(a\right)\left(f\left(d\right)\right)^{i-1}$

$\therefore f\left(a_1\right), f\left(a_2\right), f\left(a_3\right), \ldots.$ are in G.P. with first term f(a) and c.r. f(d)

$f\left(a_{31}\right) = 64 f\left(a_{25}\right)$

$\Rightarrow f\left(a\right)\cdot\left(f\left(d\right)\right)^{30} = 64 f\left(a\right)\left(f\left(a\right)\right)^{24}$

$\Rightarrow \left(f\left(d\right)\right)^6 = 2^6 \Rightarrow f\left(d\right) = 2$

$$\sum_{i=1}^{50} f\left(a_i\right) = f\left(a\right)\frac{\left(\left(f\left(d\right)\right)^{50}-1\right)}{f\left(d\right)-1}$$

$= f\left(a\right)\left(2^{50}-1\right) = 3\left(2^{25}+1\right)$

$$\Rightarrow f\left(a\right) = \frac{3}{2^{25}-1}$$

$$\therefore \sum_{i=6}^{30} f\left(a_i\right) = f\left(a\right)\left(f\left(d\right)\right)^5 \frac{\left[\left(f\left(d\right)\right)^{25}-1\right]}{f\left(d\right)-1}$$

$$= \frac{3}{2^{25}-1} \times 2^5 \times \frac{2^{25}-1}{2-1} = 96$$

[:Q.13] For all x > 0, let $y_1(x), y_2(x)$ and $y_3(x)$ be the functions satisfying

$$\frac{dy_1}{dx} - (\sin x)^2 y_1 = 0, y_1(1) = 5,$$

$$\frac{dy_2}{dx} - (\cos x)^2 y_2 = 0, y_2(1) = \frac{1}{3},$$

$$\frac{dy_3}{dx} - \left(\frac{2-x^3}{x^3}\right) y_3 = 0, y_3(1) = \frac{3}{5e},$$

respectively. Then

$$\lim_{x\to 0^+} \frac{y_1(x)y_2(x)y_3(x)+2x}{e^{3x}\sin x}$$

Is equal to ___________.

[:ANS] **2**

[:SOLN] $\frac{dy_1}{dx} = (\sin^2 x) y_1$

$$\int \frac{dy_1}{y_1} = \int \sin^2 x dx = \int \frac{1-\cos 2x}{2} dx$$

$$\Rightarrow \ln y_1 = \frac{x}{2} - \frac{\sin 2x}{4} + c_1$$

Similarly , $\ln y_2 = \frac{x}{2} + \frac{\sin 2x}{4} + c_2$

and , $\ln y_3 = -x^{-2} - x + c_3$

Adding, $\ln(y_1 y_2 y_3) = \frac{-1}{x^2} + c$ $\quad (c = c_1 + c_2 + c_3)$

Putting x = 1, $\ln\left(5 \times \frac{1}{3} \times \frac{3}{5e}\right) = -1 + c$

$\Rightarrow c = 0$

$\therefore y_1 y_2 y_3 = e^{-1/x^2}$

$$\therefore \lim_{x\to 0^+} \frac{y_1(x)y_2(x)y_3(x)+2x}{e^{3x}\sin x}$$

$$= \lim_{x\to 0^+} \frac{e^{-1/x^2}+2x}{e^{3x}\sin x}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{xe^{1/x^2}}+2}{e^{3x}\cdot\frac{\sin x}{x}}$$

$$= \frac{0+2}{1\times 1} = 2$$

SECTION-4 (Maximum Marks : 12)

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II.**
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

[:Q.14] Consider the following frequency distribution:

Value	4	5	8	9	6	12	11
Frequency	5	f_1	f_2	2	1	1	3

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6. For the given frequency distribution, let α denote the mean deviation about the mean, β denote the mean deviation about the median, and σ^2 denote the variance.

Match each entry in List – I to the correct entry in List – II and choose the correct option.

List – I	**List – II**
(P) $7f_1 + 9f_2$ is equal to	(1) 146
(Q) 19α is equal to	(2) 47
(R) 19β is equal to	(3) 48
(S) $19\sigma^2$ is equal to	(4) 145
	(5) 55

[:A] (P) ⟶ (5) (Q) ⟶ (3) (R) ⟶ (2) (S) ⟶ (4)

[:B] (P) ⟶ (5) (Q) ⟶ (2) (R) ⟶ (3) (S) ⟶ (1)

[:C] (P) ⟶ (5) (Q) ⟶ (3) (R) ⟶ (2) (S) ⟶ (1)

[:D] (P) ⟶ (3) (Q) ⟶ (2) (R) ⟶ (5) (S) ⟶ (4)

[:ANS] **C**

[:SOLN]

x_i	4	5	6	8	9	11	12
f_i	5	f_1	1	f_2	2	3	1

$$\sum_{i=1}^{7} f_i = f_1 + f_2 + 12 = 19 \Rightarrow f_1 + f_2 + 7$$

Median = 6 = 10th term

$$\Rightarrow 6 + f_1 = 10 \Rightarrow f_1 = 4$$

$$\therefore f_2 = 3$$

x_i	f_i	$x_i f_i$	$\lvert x_i - 7\rvert \cdot f_i$	$\lvert x_i - 6\rvert f_i$	$x_i^2 f_i$
4	5	20	15	10	80
5	4	20	8	4	100
6	1	6	1	0	36
8	3	24	3	6	192
9	2	18	4	6	162
11	3	33	12	15	363
12	1	12	5	6	144
	19	133	48	47	1077

Mean $\bar{x} = \dfrac{\sum x_i f_i}{\sum f_i} = \dfrac{133}{19} = 7$

$$\alpha = M \cdot D.\left(\bar{x}\right) = \frac{\sum \lvert x_i - \bar{x}\rvert \cdot f_i}{\sum f_i} = \frac{48}{19}$$

$$\beta = M.D.\left(Me\right) = \frac{\sum \lvert x_i - 6\rvert \cdot f_i}{\sum f_i} = \frac{47}{19}$$

$$\sigma^2 = \frac{\sum x_i^2 fi}{19} - \bar{x}^2$$

$$= \frac{1077}{19} - 7^2 = \frac{146}{19}$$

[:Q.15] Let R denote the set of all real numbers. For a real number x, let $[x]$ denote the greatest integer less than or equal to x. Let n denote a natural number.

Match each entry in List – I to the correct entry in List – II and choose the correct option.

List – I	**List – II**
(P) The minimum value of n for which the function Function $f(x)=\left[\frac{10x^3-45x^2+60x+35}{n}\right]$ is continuous on the interval [1,2], is	(1) 8
(Q) The minimum value of n for which g(x) = $(2n^2-13n-15)(x^3+3x)$, $x \in$R, is an increasing function on R, is	(2) 9
(R) The smallest natural number n which is greater than 5, such That x = 3 is a point of local minima of $h(x)=(x^2-9)^n(x^2+2x+3)$,	(3) 5
(S) Number of $x_0 \in R$ such that $I(x)=\sum_{k=0}^{4}\left(\sin\lvert x-k\rvert+\cos\left\lvert x-k+\frac{1}{2}\right\rvert\right)$, $x \in R$, is NOT differentiable at x_0, is	(4) 6
	(5) 10

[:A] (P) ⟶ (1) (Q) ⟶ (3) (R) ⟶ (2) (S) ⟶ (5)

[:B] (P) ⟶ (2) (Q) ⟶ (1) (R) ⟶ (4) (S) ⟶ (3)

[:C] (P) ⟶ (5) (Q) ⟶ (1) (R) ⟶ (4) (S) ⟶ (3)

[:D] (P) ⟶ (2) (Q) ⟶ (3) (R) ⟶ (1) (S) ⟶ (5)

[:ANS] **B**

[:SOLN] (P) $g(x)=10x^3-45x^2+60x+35, x\in[1,2]$

$g'(x)=30x^2-90x+60$

$=30(x-1)(x-2)<0\forall x\in(1,2)$

$\therefore$ range of g(x) = $[g(2),g(1)]=[55,60]$

Minimum value of n = 9

(Q) g'(x) = $(2n-15)(n+1)(3x^2+3)\geq,0\forall x\in R$

$\therefore$ minimum value of n= 8

(R) n must be even

$\therefore$ smallest n = 6

(S) $I(x)=\sum_{k=0}^{4}\sin|x-k|+\sum_{k=0}^{4}\cos\left(x-k+\frac{1}{2}\right)$ $(\because \cos|x|=\cos x)$

$\therefore I(x)$ is not different at x = 0, 1, 2, 3, 4

i. e. 5 values of x_o.

[:Q.16] Let $\vec{W}=\hat{i}+\hat{j}-2\hat{k}$, and $\vec{u}$ and $\vec{v}$ be two vectors, such that $\vec{u}\times\vec{v}=\vec{w}$ and $\vec{v}\times\vec{w}=\vec{u}$. Let α,β,γ and t be real numbers such that

$\vec{u}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}, -t\alpha+\beta+\gamma=0,\ \alpha-t\beta+\gamma=0,$ and $\alpha+\beta-t\gamma=0.$

Match each entry in List – I to the correct entry in List – II and choose the correct option.

List – I	**List – II**
(P) $\|\vec{v}\|^2$ is equal to	(1) 0
(Q) If $\alpha=\sqrt{3}$, then γ^2 is equal to	(2) 1
(R) If $\alpha=\sqrt{3}$, then $(\beta+\gamma)^2$ is equal to	(3) 2
(S) If $\alpha=\sqrt{2}$, then t + 3 is equal to	(4) 3
	(5) 5

[:A] (P) ⟶ (2) (Q) ⟶ (1) (R) ⟶ (4) (S) ⟶ (5)

[:B] (P) ⟶ (2) (Q) ⟶ (4) (R) ⟶ (3) (S) ⟶ (5)

[:C] (P) ⟶ (2) (Q) ⟶ (1) (R) ⟶ (4) (S) ⟶ (3)

[:D] (P) ⟶ (5) (Q) ⟶ (4) (R) ⟶ (1) (S) ⟶ (3)

[:ANS] **A**

[:SOLN] $\vec{u}\times\vec{v}=\vec{w}\neq\vec{0}$

$\Rightarrow \vec{u}\perp\vec{w}$ and $\vec{v}\perp\vec{w}$

$\vec{v}\times\vec{w}=\vec{u}$

$\Rightarrow \vec{v}\perp\vec{u}$ and $\vec{w}\perp\vec{u}$

$\therefore |\vec{u}\times\vec{v}|=|\vec{u}||\vec{v}|=|\vec{w}|=\sqrt{6}$ (i)

$|\vec{v}||\vec{w}|=|\vec{u}|\Rightarrow\sqrt{6}|\vec{v}|=|\vec{u}|$ (ii)

From (i) and (ii), $|\vec{v}|=1$ and $|\vec{u}|=\sqrt{6}$

$\therefore \alpha^2+\beta^2+\gamma^2=6$

Now, $\begin{vmatrix} -t & 1 & 1 \\ 1 & -t & 1 \\ 1 & 1 & -t \end{vmatrix}=0\Rightarrow t=2 \text{ or } -1$

Case I t = 2

Subtracting
$$\begin{array}{r} -2\alpha+\beta+\gamma=0 \\ \alpha-2\beta+\gamma=0 \\ \hline -3\alpha+3\beta=0 \end{array}$$

$$\Rightarrow \alpha=\beta$$

Similarly $\beta=\gamma \therefore \alpha=\beta=\gamma=\pm\sqrt{2}$

Case II t = –1

$\alpha+\beta+\gamma=0$ and $\alpha^2+\beta^2+\gamma^2=6$

$\alpha=\sqrt{3}\Rightarrow\beta+\gamma=-\sqrt{3}$ and $\beta^2+\gamma^2=3$

$\Rightarrow(\beta,\gamma)=\left(0,-\sqrt{3}\right)$

or $\left(-\sqrt{3},0\right)$

But, $\vec{u}\cdot\vec{w}=0\Rightarrow\alpha+\beta-2\gamma=0\Rightarrow\beta-2\gamma=-\sqrt{3}$

$\therefore \gamma=0,\beta=-\sqrt{3}$