JEE (ADVANCED) 2025 PAPER-2

[PAPER ANSWER KEY WITH SOLUTION]

HELD ON SUNDAY 18TH MAY 2025

MATHEMATICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

[:Q.1] Let x_0 be the real number such that $e^{x_0} + x_0 = 0$. For a given real number α, define for all real numbers x . $g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$. Then which one of the following statements is TRUE ?

[:A] For α = 2, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

[:B] For α = 2, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 1$

[:C] For α = 3, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

[:D] For α = 3, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = \frac{2}{3}$

[:ANS] C

[:SOLN] $g(x)=\dfrac{3xe^x+3x-\alpha e^x-\alpha x}{3\left(e^x+1\right)}$

$=\dfrac{3x\left(e^x+1\right)}{3\left(e^x+1\right)}-\dfrac{\alpha\left(e^x+x\right)}{3\left(e^x+1\right)}$

$= x-\dfrac{\alpha\left(e^x+x\right)}{3\left(e^x+1\right)}$

$g(x) + e^{x_0} = \left(x+e^{x_0}\right)-\dfrac{\alpha}{3}\left(\dfrac{e^x+x}{e^x+1}\right)$

$\because \boxed{e^{x_0}=-x_0}$

So $g(x) + e^{x_0}=(x-x_0)-\dfrac{\alpha}{3}\left(\dfrac{e^x+x}{e^x+1}\right)$

$\lim\limits_{x\to x_0}\left|\dfrac{g(x)+e^{x_0}}{x-x_0}\right| \qquad \left\{\dfrac{0}{0} \text{ form}\right\}$

Using L-hospital

$\lim\limits_{x\to x_0}\left|\dfrac{g(x)+e^{x_0}}{x-x_0}\right|=\lim\limits_{x\to x_0}\left|g'(x)\right|$

$=\left|g'(x_0)\right|$

$\because \quad g'(x)=1-\dfrac{\alpha}{3}\left\{\dfrac{\left(e^x+1\right)\left(e^x+1\right)-\left(e^x+x\right)\left(e^x\right)}{\left(e^x+1\right)^2}\right\}$

$g'(x) = 1-\dfrac{\alpha}{3}\left\{\left(\dfrac{e^{x_0}+1}{e^{x_0}+1}\right)^2\right\}=1-\dfrac{\alpha}{3}$

By check option (C) correct

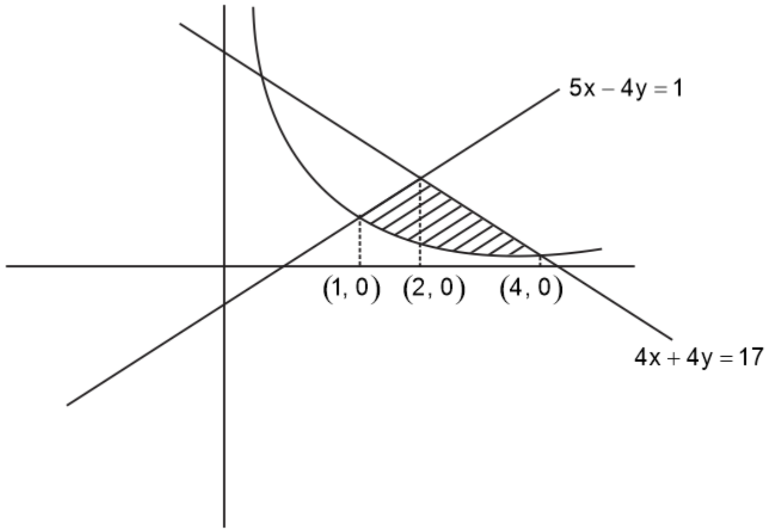
[:Q.2] Let R denote the set of all real numbers. Then the area of the region

$\left\{(x,\, y)\in R\times R : x>0, y>\dfrac{1}{x},\, 5x-4y-1>0,\, 4x+4y-17<0\right\}$ is

[:A] $\dfrac{17}{16}-\log_e 4$ [:B] $\dfrac{33}{8}-\log_e 4$ [:C] $\dfrac{57}{8}-\log_e 4$ [:D] $\dfrac{17}{2}-\log_e 4$

[:ANS] **B**

[:SOLN]



Area = $\int_1^2 \left(\frac{5x-1}{4} - \frac{1}{x}\right)dx + \int_2^4 \left(\frac{17-4x}{4} - \frac{1}{x}\right)dx$

= $\left(\frac{33}{8} - \ln 4\right)$

[:Q.3] The total number of real solutions of the equation $\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$ is

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\tan^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, respectively.)

[:A] 1 [:B] 2 [:C] 3 [:D] 5

[:ANS] **C**

[:SOLN] $\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$

Let $\tan\theta = x$

$\theta = \tan^{-1}x$

$\tan^{-1}x = \tan^{-1}(2x) - \frac{1}{2}\sin^{-1}\left(\frac{6x}{9+x^2}\right)$

$$\frac{1}{2}\sin^{-1}\left(\frac{6x}{9+x^2}\right)=\tan^{-1}2x-\tan^{-1}x$$

$$\frac{1}{2}\sin^{-1}\left(\frac{6x}{9+x^2}\right)=\tan^{-1}\left(\frac{x}{1+2x^2}\right)$$

$$\sin^{-1}\left(\frac{6x}{9+x^2}\right)=2\tan^{-1}\left(\frac{x}{1+2x^2}\right)$$

$$\sin^{-1}\left(\frac{2\left(\frac{x}{3}\right)}{1+\left(\frac{x}{3}\right)^2}\right)=2\tan^{-1}\left(\frac{x}{1+2x^2}\right)$$

$$2\tan^{-1}\left(\frac{x}{3}\right)=2\tan^{-1}\left(\frac{x}{1+2x^2}\right)$$

$$\frac{x}{3}=\frac{x}{1+2x^2}$$

$\Rightarrow$ $x = 0$ $\qquad$ $2x^2 + 1 = 3$

$x^2 = 1$

$x = \pm 1$

No. of solution = 3

[:Q.4] Let S denote the locus of the point of intersection of the pair of lines

$4x - 3y = 12\alpha$,

$4\alpha x + 3\alpha y = 12$,

Where α varies over the set of non-zero real numbers Let T be the tangent to S passing through the point (p, 0) and (0, q), q > 0, and parallel to the line $4x-\frac{3}{\sqrt{2}}y=0$. Then the value of pq is

[:A] $-6\sqrt{2}$ [:B] $-3\sqrt{2}$ [:C] $-9\sqrt{2}$ [:D] $-12\sqrt{2}$

[:ANS] **A**

[:SOLN] $4x - 3y = 12\alpha$ -------(1)

$4\alpha x + 3\alpha y = 12$ ---------(2)

$x = \frac{3}{2}\left(\alpha + \frac{1}{\alpha}\right)$; $y = 2\left(\frac{1}{\alpha} - \alpha\right)$

$\frac{2x}{3} = \alpha + \frac{1}{\alpha}$ ……………… (i)

$\frac{y}{2} = \frac{1}{\alpha} - \alpha$ ………………(ii)

On squaring and subtract

$\frac{4}{9}x^2 - \frac{y^2}{4} = 4$

$\boxed{\frac{x^2}{9} - \frac{y^2}{16} = 1}$ ……………..Hyperbola

Equ of tangent $y = mx \pm \sqrt{a^2m^2 - b^2}$

Given $m = \frac{4\sqrt{2}}{3}$

$y = \frac{4\sqrt{2}}{3}x \pm \sqrt{9m^2 - 16}$

$y = \frac{4\sqrt{2}}{3}x \pm 4$

at x = 0 $\therefore$ $y = \pm 4$ (0, q)

$\therefore$ q > 0

So x = 0 y = 4 & equ will be

$\boxed{y = \frac{4\sqrt{2}}{3}x + 4}$

So at y = 0 $x = -\frac{3}{\sqrt{2}}$

(P, 0) & (0, q) $\equiv \left(-\frac{3}{\sqrt{2}}, 0\right)$ & $(0, 4)$

$\boxed{pq = -6\sqrt{2}}$

SECTION 2 (Maximum Marks : 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

  Zero Marks : 0 If none of the options is chose (i.e. the question is unanswered);

  Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

  choosing ONLY (B) will get +1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option (i.e. the question is unanswered) will get 0 marks; and

  choosing any other combination of options will get –2 marks.

[:Q.5] Let $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ and $P=\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}$. Let Q = $\begin{pmatrix}x & y\\ z & 4\end{pmatrix}$ for some non-zero real numbers x, y and z for which there is a 2× 2 matrix R with all entries being non-zero real numbers, such that QR = RP. Then which of the following statements is (are) TRUE ?

[:A] The determinant Q – 2*I* is zero

[:B] The determinant Q – 6*I* is 12

[:C] The determinant Q – 3*I* is 15

[:D] yz = 2

[:ANS] **AB**

[:SOLN] ∵ QR = RP

$|QR| = |RP| \Rightarrow |Q| = |P|$

Also QR – 2R = RP – 2R

(Q – 2I)R = R(P – 2I)

$$|Q-2I| = |P-2I| = \begin{vmatrix}0 & 0\\ 0 & 1\end{vmatrix} = 0$$

& QR – 6R = RP – 6R

$$|Q-6I| = |P-6I| = \begin{vmatrix}-4 & 0\\ 0 & -3\end{vmatrix} = 12$$

& QR – 3R = RP – 3R

$$|Q-3I| = |P-3I| \neq 15$$

Let R = $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ a, b, c, d $\neq 0$

∵ QR = RP

$$\begin{pmatrix}xa+yc & xb+yd\\ az+4c & bz+4d\end{pmatrix} = \begin{pmatrix}2a & 3b\\ 2c & 3d\end{pmatrix}$$

On comparing

xa + yc = 2a ………….(i)

xb + yd = 3b ………….(ii)

az = –2c ………….(iii)

bz = –d ……………(iv)

(i) × d – (ii) × c

x(ad – bc) = 2ad – 3bc

$$x = \frac{2ad-3bc}{ad-bc}$$

$$z = \frac{-2c}{a} = \frac{-d}{b}$$

$x = \frac{b\cancel{c}}{\cancel{b}c} = 1$

$\boxed{ad = 2bc}$

$|Q| = |P|$; $4x - yz = 6$

$4 - 6 = yz$; $\boxed{yz = -2}$

[:Q.6] Let S denote the locus of the mid-points of those chords of the parabola $y^2 = x$, such that the area of the region enclosed between the parabola and the chord is $\frac{4}{3}$. Let R denote the region lying in the first quadrant, enclosed by the parabola $y^2 = x$, curve S, and the lines x = 1 and x = 4. The which of the following statements is (are) TRUE ?

[:A] $(4, \sqrt{3}) \in S$

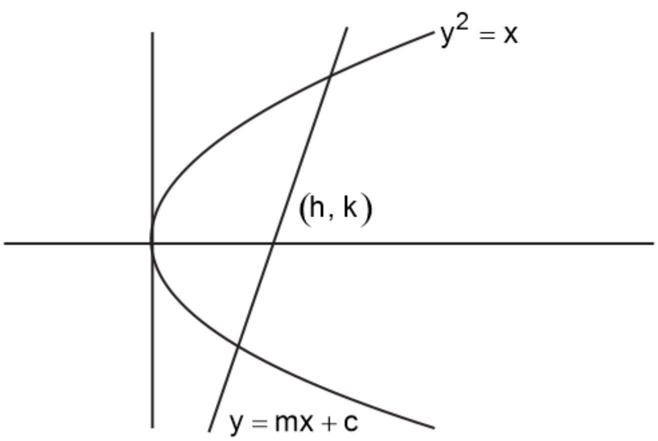
[:B] $(5, \sqrt{3}) \in S$

[:C] Area of R is $\frac{14}{3} - 2\sqrt{3}$

[:D] Area of R is $\frac{14}{3} - \sqrt{3}$

[:ANS] **AC**

[:SOLN]



$A = \frac{(1-4mc)^{\frac{3}{2}}}{6m^3}$ -----------------(i)

Given that $\frac{(1-4mc)^{\frac{3}{2}}}{6m^3} = \frac{4}{3}$

$\frac{(1-4mc)^3}{\cancel{36}m^6} = \frac{16}{\cancel{9}}$

$(1 - 4mc)^3 = 64\ m^6$

$1 - 4mc = 4m^2$

$\boxed{4m^2 + 4mc - 1 = 0}$ ------------(ii)

Equ of chord if (h, k) mid point

$T = S_1$; $y = \frac{x}{2k} + k - \frac{h}{2k}$ ---------(iii)

$m = \left(\frac{1}{2k}\right) \& \ c = \left(k - \frac{h}{2k}\right)$

Using in (ii)

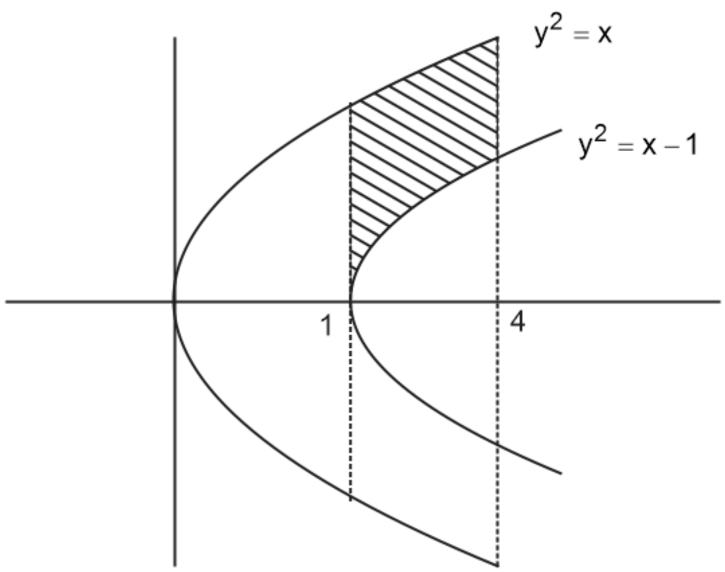
$\frac{4}{4k^2} + 4 \cdot \frac{1}{2k}\left(k - \frac{h}{2k}\right) - 1 = 0$

$\frac{1}{k^2} + 2 - \frac{h}{k^2} - 1 = 0$

$\frac{(1-h)}{k^2} + 1 = 0$

$k^2 = h - 1$

$\boxed{y^2 = x - 1}$



$S \equiv \boxed{y^2 = x - 1}$

So $\left(4, \sqrt{3}\right) \in S$

$\left(5, \sqrt{2}\right) \notin S$

Area = $\int_1^4 \left(\sqrt{x} - \sqrt{x-1}\right) dx$

$= \frac{2}{3}x^{3/2} - \frac{2}{3}(x-1)^{3/2}\Big]_1^4$

$= \left(\frac{2}{3}(4)^{3/2} - \frac{2}{3}\cdot 3^{3/2}\right) - \left(\frac{2}{3}\right)$

$= \frac{16}{3} - \frac{2}{3} - \frac{2}{3}\cdot 3^{3/2}$

$= \frac{14}{3} - \frac{2}{\cancel{3}}\cdot\cancel{3}\cdot\sqrt{3}$

$= \left(\frac{14}{3} - 2\sqrt{3}\right)$ area

[:Q.7] Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ such that $y_1 > 0$, and $y_2 > 0$. Let C denote the circle $x^2 + y^2 = 9$, and M be the point (3, 0). Suppose the line $x = x_1$ intersects C at R, and the line $x = x_2$ intersects C at S, such that the y-coordinates of R and S are positive. Let $\angle ROM = \frac{\pi}{6}$ and $\angle SOM = \frac{\pi}{3}$, where 0 denotes the origin (0, 0) Let $|XY|$ denote the length of the line segment XY. Then which of the following statements is (are) TRUE ?

[:A] The equation of the line joining P and Q is $2x + 3y = 3\left(1+\sqrt{3}\right)$

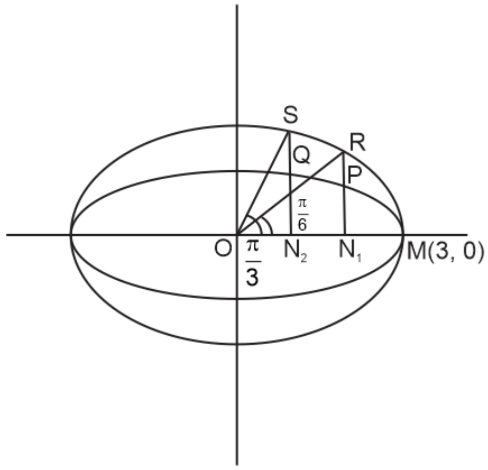
[:B] The equation of the line joining P and Q is $2x + y = 3\left(1+\sqrt{3}\right)$

[:C] If $N_2 = (x_2, 0)$, the $3|N_2Q| = 2|N_2S|$

[:D] If $N_1 = (x_1, 0)$, the $9|N_1P| = 4|N_1R|$

[:ANS] **AC**

[:SOLN]



$$N_1 \equiv \left(\frac{3\sqrt{3}}{2}, 0\right)$$

$$N_2 \equiv \left(\frac{3}{2}, 0\right)$$

$$P \equiv \left(3\cos\frac{\pi}{6}, 2\sin\frac{\pi}{6}\right) = \left(\frac{3\sqrt{3}}{2}, 1\right)$$

$$Q \equiv \left(3\cos\frac{\pi}{3}, 2\sin\frac{\pi}{3}\right) \equiv \left(\frac{3}{2}, \sqrt{3}\right)$$

Equation of PQ :

$$(y-1) = \frac{2\left(\sqrt{3}-1\right)}{\left(3-3\sqrt{3}\right)}\left(x-\frac{3\sqrt{3}}{2}\right)$$

$$(y-1) = -\frac{2}{3}\left(x-\frac{3\sqrt{3}}{2}\right)$$

$$3y - 3 = -2x + 3\sqrt{3}$$

$\boxed{2x+3y=3+3\sqrt{3}}$ [A]

$N_1P = 1$; $N_1R = \frac{3}{2}$

$N_2Q = \sqrt{3}; N_2S = \frac{3\sqrt{3}}{2}$

$3N_2Q = 2N_2S$ ---------[C]

$9N_1P \neq 4N_1R$ ----------[D] incorrect

[:Q.8] Let R denote the set of all real numbers. Let f : R → R be defined by

$$f(x)=\begin{cases}\dfrac{6x+\sin x}{2x+\sin x} & \text{if } x\neq 0\\[2mm] \dfrac{7}{3} & \text{if } x=0\end{cases}$$

Then which of the following statements is (are) TRUE ?

[:A] The point x = 0 is a point of local maxima of f

[:B] The point x = 0 is a point of local minima of f

[:C] Number of points of local maxima of f in the interval $[\pi, 6\pi]$ is 3

[:D] Number of points of local minima of f in the interval $[2\pi, 4\pi]$ is 1

[:ANS] **BCD**

[:SOLN] $\lim\limits_{x\to 0}\dfrac{6x+\sin x}{2x+\sin x}=\dfrac{7}{3}$ $\boxed{\text{cont. at } x=0}$

$f(x) = 1+\dfrac{4x}{2x+\sin x}$

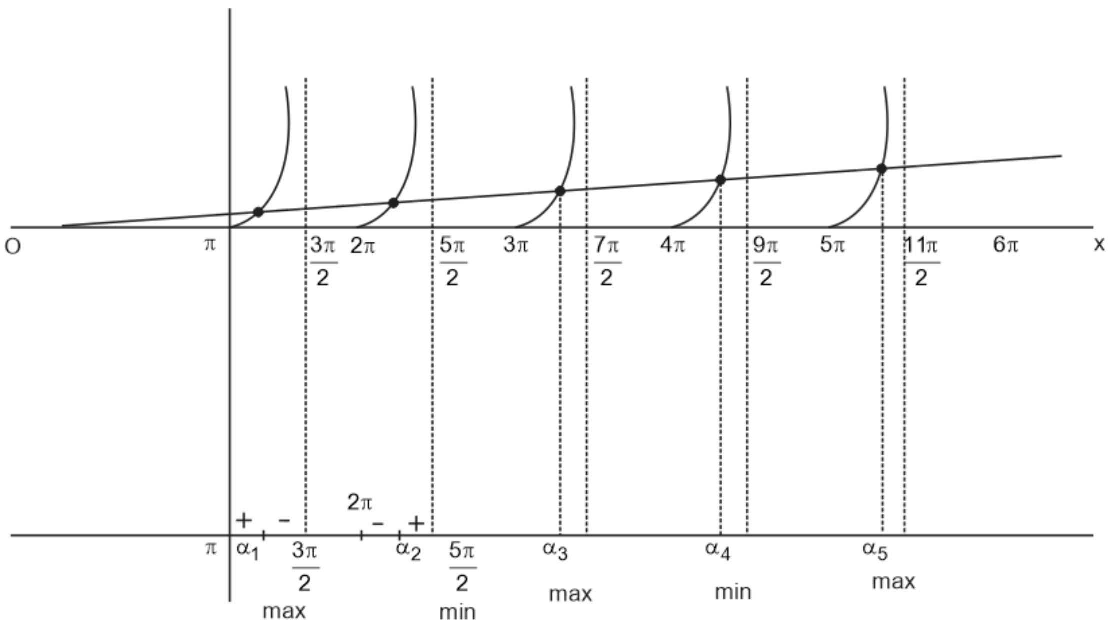
$f'(x) = \dfrac{4(\sin x - x\cos x)}{\oplus ve}=\dfrac{4\cos x(\tan x - x)}{\oplus ve}$

at $x \to 0$; cosx > 0

& $f'(x) = 0$ at x = 0

$f'(x)<0$ 0 $f'(x)>0$

So at x = 0; point of minima (B)



(3) point of max in $[\pi, 6\pi]$

(1) point of min in $[2\pi, 4\pi]$

SECTION 3 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value of to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : **+4** If ONLY the correct numerical value is entered in the designated place;
  Zero Marks **: 0** In all other cases.

[:Q.9] Let y(x) be the solution of the differential equation $x^2\frac{dy}{dx}+xy=x^2+y^2,\ x>\frac{1}{e}$, satisfying y(1) = 0. Then the value of $2\frac{(y(e))^2}{y(e^2)}$ is ________.

[:ANS] **0.75**

[:SOLN] $\frac{dy}{dx}=\frac{x^2+y^2-xy}{x^2}$, Put $\frac{y}{x}=V$

$V+x\frac{dV}{dx}=1+V^2-V$

$\frac{xdV}{dx}=1-2V+V^2 \qquad \Rightarrow \frac{dV}{(1-V)^2}=\frac{dx}{x}$

$\frac{-1}{V-1}=\ln(cx)$

$\therefore \quad \ln(cx) + \frac{x}{y-x}=0$

at x = 1, y = 0

$\therefore$ c = e

$\ln(cx) + \frac{x}{y-x}=0$

$\therefore \quad y(e) = \frac{e}{2}$

$y(e^2) = \frac{2e^2}{3}$

$\therefore \quad \frac{2\times\frac{e^2}{4}}{2\frac{e^2}{3}}=\frac{3}{4}$

[:Q.10] Let $a_0, a_1, \ldots\ldots, a_{23}$ be real numbers such that $\left(1+\frac{2}{5}x\right)^{23} = \sum_{i=0}^{23} a_i x^i$ for every real number x.

Let a_r be the largest among the numbers a_j for $0 \le j \le 23$. Then the value of r is _______.

[:ANS] **6**

[:SOLN] $\left(1+\frac{2x}{5}\right)^{23}$ NGT $\Rightarrow r \le \frac{(n+1)\cdot|X|}{1+|X|}$ (as NGT is for all 'x' So x $= \frac{2x}{5} @ x=1 \Rightarrow X = \frac{2}{5}$)

$$r \le \frac{24\cdot\left(\frac{2}{5}\right)}{1+\left(\frac{2}{5}\right)}$$

$r \le \frac{48}{7}$ $\qquad\qquad \therefore \quad r \le 6.8$

$\therefore \quad r = 6$

[:Q.11] A factory has a total of three manufacturing units, M_1, M_2, and M_3, which produce bulbs independent of each other. The units M_1, M_2 and M_3 produce bulbs in the proportions of 2 : 2 : 1, respectively. It is known that 20% of the bulbs produced in the factory are defective. It is also known that, of all the bulbs produced by M_1, 15% are defective. Suppose that, if a randomly chosen bulb produced in the factory is found to be defective, the probability that it was produced by M_2 is $\frac{2}{5}$. If a bulb is chosen randomly from the bulbs produced by M_3, then the probability that it is defective is ___________.

[:ANS] **0.3**

[:SOLN] Let No. of bulbs in M_1, M_2 & M_3 are 2x, 2x & x. respectively

Let p% are defective in M_2 & q% are in M_3

From questions

$2xp = \frac{2}{5} = \frac{40}{100}$

$2x \times 15 + 2xp + xq = x$

$\therefore \quad 30 + 2p + q = 100$

& $2p = 40$ $\qquad \therefore \qquad q = 30\% = 0.3$

[:Q.12] Consider the vectors $\vec{x} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{y} = 2\hat{i} + 3\hat{j} + \hat{k}$, and $\vec{z} = 3\hat{i} + \hat{j} + 2\hat{k}$. For two distinct positive real numbers α and β, define $\vec{X} = \alpha\vec{x} + \beta\vec{y} - \vec{z}$, $\vec{Y} = \alpha\vec{y} + \beta\vec{z} - \vec{x}$, and $\vec{Z} = \alpha\vec{z} + \beta\vec{x} - \vec{y}$. If the vectors $\vec{X}$, $\vec{Y}$ and $\vec{Z}$ lie in a plane, then the value of $\alpha + \beta - 3$ is ________.

[:ANS] **–2**

[:SOLN]
$$\begin{vmatrix} \alpha & \beta & -1 \\ -1 & \alpha & \beta \\ \beta & -1 & \alpha \end{vmatrix}\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \quad (\alpha^3 + \beta^3 - 1) - (\alpha\beta - \alpha\beta - \alpha\beta) = 0$

$\alpha^3 + \beta^3 + 3\alpha\beta = 1$

$\Rightarrow \quad \alpha^3 + \beta^3 + (-1)^3 = 3\alpha\beta(-1)$

$\Rightarrow \quad \alpha + \beta - 1 = 0 \qquad \therefore \quad \alpha + \beta - 3 = -2$

[:Q.13] For a non-zero complex number z, let arg(z) denote the principal argument of z, with $-\pi <$ arg(z) $\le \pi$. Let ω be the cube root of unity for which $0 < \arg(\omega) < \pi$. Let $\alpha = \arg\left(\sum_{n=1}^{2025} (-\omega)^n\right)$.

Then the value of $\frac{3\alpha}{\pi}$ is __________.

[:ANS] **–2**

[:SOLN] $\sum_{n=1}^{2025} (-\omega)^n$

Sum of GP = $\frac{a - lr}{1 - r}$

$$= \frac{-\omega - (-\omega)^{2025} \times (-\omega)}{1 - (-\omega)} = \frac{-\omega - (-\omega)^{2026}}{1 + \omega} = \frac{-\omega - \omega}{1 + \omega}$$

$$= \frac{-2\omega}{-\omega^2} = \frac{2}{\omega} = 2\omega^2$$

$\alpha = \arg(2\omega^2)$

$$= \arg\left(2 \times e^{\frac{i4\pi}{3}}\right) = \frac{4\pi}{3} \to \frac{4\pi}{3} - 2\pi = \frac{-2\pi}{3}$$

$$\therefore \quad \frac{3\alpha}{\pi} = -2$$

[:Q.14] Let R denote the set of all real numbers. Let f : R → R and g : R → (0, 4) be functions defined by $f(x) = \log_e(x^2 + 2x + 4)$, and $g(x) = \frac{4}{1+e^{-2x}}$. Define the composite function $f \circ g^{-1}$ by $(f \circ g^{-1})(x) = f(g^{-1}(x))$, where g^{-1} is the inverse of the function g. Then the value of the derivative of the composite function $f \circ g^{-1}$ at x = 2 is ____________.

[:ANS] **0.25**

[:SOLN] $f(g^{-1}(x))' = f'(g^{-1}(x)) \cdot (g^{-1}(x))'$, $g^{-1}(2) = 0$

$$= \frac{2x+2}{x^2+2x+4} \times \frac{1}{\frac{-4}{(1+e^{-2x})^2} \times -2e^{-2x}}$$

at x = 0

$$= \frac{1}{2} \times \left(\frac{1}{\left(\frac{8}{4}\right)} \right) = \frac{1}{4} = 0.25$$

[:Q.15] Let $\alpha = \frac{1}{\sin 60^o \sin 61^o} + \frac{1}{\sin 62^o \sin 63^o} + ... + \frac{1}{\sin 118^o \sin 119^o}$. Then the value of $\left(\frac{\operatorname{cosec} 1^o}{\alpha}\right)^2$ is _________.

[:ANS] **3**

[:SOLN] $\frac{\alpha}{\operatorname{cosec} 1^o} = \frac{\sin(61^o - 60^o)}{\sin 60^o \cdot \sin 61^o}$ ....

$\cot 60^o - \cot 61^o + \cot 62^o - \cot 63^o + \cot 64^o - \cot 65^o + ... +$

$\cot 116^o - \cot 117^o + \cot 118^o - \cot 119^o$

$= \cot 60^o = \frac{1}{\sqrt{3}}$

$\therefore \quad \frac{\operatorname{cosec} 1^o}{\alpha} = \sqrt{3}$

= 3 Ans

[:Q.16] If $\alpha = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1}x}{2x^2 - 3x + 2}dx$, then the value of $\sqrt{7}\ \tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right)$ is ________. (Here, the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)

[:ANS] **21**

[:SOLN] Putting x as $\frac{1}{x}$

$\therefore$ dx becomes $-\frac{1}{x^2}dx$

$$\alpha = \int_{2}^{\frac{1}{2}} \frac{\tan^{-1}\left(\frac{1}{x}\right)}{\left(\frac{2}{x^2} - \frac{3}{x} + 2\right)} \times \frac{-1}{x^2}dx$$

$$= -\int_{2}^{\frac{1}{2}} \frac{\tan^{-1}\left(\frac{1}{x}\right)}{2x^2 - 3x + 2}dx = \int_{\frac{1}{2}}^{2} \frac{\cot^{-1}x\,dx}{2x^2 - 3x + 2}$$

Adding, 2 or = $\frac{\pi}{2} \cdot \int_{\frac{1}{2}}^{2} \frac{dx}{2x^2 - 3x + 2} = \frac{\pi}{4}\int_{\frac{1}{2}}^{2} \frac{dx}{x^2 - \frac{3}{2}x + 1}$

$$\frac{8\alpha}{\pi} = \int_{\frac{1}{2}}^{2} \frac{dx}{\left(x - \frac{3}{4}\right)^2 + \frac{7}{16}} = \left[\frac{4}{\sqrt{7}} \cdot \tan^{-1}\frac{x - \frac{3}{4}}{\left(\frac{\sqrt{7}}{4}\right)}\right]_{\frac{1}{2}}^{2}$$

$$= \frac{4}{\sqrt{7}} \times \left(\tan^{-1}\frac{5}{\sqrt{7}} + \tan^{-1}\frac{1}{\sqrt{7}}\right) = \frac{4}{\sqrt{7}} \cdot \tan^{-1}\left(\frac{\frac{6}{\sqrt{7}}}{1 - \frac{5}{7}}\right) = \frac{4}{\sqrt{7}} \cdot \tan^{-1}\left(3\sqrt{7}\right)$$

$$\therefore \quad \frac{2\alpha}{\pi} = \tan^{-1}\left(3\sqrt{7}\right)$$

$$\sqrt{7} \times \tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right) = \sqrt{7} \times 3\sqrt{7} = 21$$