



INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN ASTRONOMY (NSEA)2025

(QUESTION PAPER CODE 41)

Date : 22/11/2025

Time : 120 Minute

Maximum Marks: 216

Write the question paper code (mentioned above) on YOUR OMR Answer Sheet (in the space provided), otherwise your Answer Sheet will NOT be evaluated. Note that the same Question paper code appears on each page of the question paper.

INSTRUCTIONS

1. Use of mobile phone, smart watches, and iPad during examination is **STRICTLY PROHIBITED**.
2. In addition to this question paper, you are given OMR Answer Sheet along with candidate's copy.
3. On the OMR sheet, make all the entries carefully in the space provided **ONLY** in BLOCK CAPITALS as well as by properly darkening the appropriate bubbles.
Incomplete/ incorrect/ carelessly filled information may disqualify your candidature.
4. On the OMR Answer sheet, use only **BLUE or BLACK BALL POINT PEN** for making entries and filling bubbles.
5. Your **Ten-digit roll number and date of birth** entered in the OMR Answer sheet shall remain your login credentials (means login id and password respectively) for accessing your performance/result in National Standard Examination in Astronomy – 2025.
6. Question paper has two parts. In part A-1 (**Q. No.1 to 48**) each question has four alternatives, out of which only one is correct. Choose the correct alternative (s) and fill the appropriate bubbles(s), as shown.

Q.No.22



In part A-2 (**Q. No. 49 to 60**) each question has four alternative out of which any number of alternative (s) (1, 2, 3, or 4) may be correct. You have to choose all correct alternative(s) and fill the appropriate bubbles(s), as shown

Q.No.54



7. For **Part A-1**, each correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer. In **Part A-2**, you get 6 marks. If all the correct alternative are marked. No negative marks in this part.
8. Rough work should be done only in the space provided. There are printed pages in this paper.
9. Use of **non-programmable scientific** calculator is allowed
10. No candidate should leave the examination hall before the completion of the examination.
11. After submitting answer paper, take away the question paper & candidate's copy of OMR for your reference
Please DO NOT make any mark other than filling the appropriate bubbles properly in the space provided on the OMR answer sheet.
OMR answer sheets are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED, Scratching or overwriting may result in wrong score.
DO NOT WRITE ON THE BACK SIDE OF THE OMR ANSWER SHEET.

Name of Student :

Batch :

Enrolment No.

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INDIAN ASSOCIATION OF PHYSICS TEACHERS

**NATIONAL STANDARD EXAMINATION IN
ASTRONOMY (NSEA) 2025**

PAPER CODE - 41

Date of Examination – 22th November, 2025

SOLUTIONS

Attempt All Sixty Questions

(NSEA) PART : A-1

ONLY ONE OUT OF FOUR OPTIONS IS CORRECT, BUBBLE THE CORRECT OPTION.

- [Q.1]** In a hypothetical scenario the force experienced by a particle is given by $F = Ae^{\frac{px}{t^2}} + BV_t^2$, where $V_t = \sqrt{\frac{2mg}{\rho\eta S}}$ is the expression for terminal velocity. Where m is the mass, g is the acceleration due to gravity, ρ is the density of the medium, S is the surface area of the particle, and η is the coefficient of viscosity of the medium. The possible units of A , B and p are
- [A] A : newton, B : newton kg m^{-1} , p : ms^2 [B] 4: no units, B : newton kg m^{-1} , p : m^{-1}s^2
- [C] 4: newton, B : kg m^{-1} , p : no units: [D] A : newton, B : newton $\text{s}^2 \text{m}^{-2}$, p : m^{-1}s^2

[ANS] D**[SOLN]** $[F] = [A]$

$$[A] = \text{Newton},$$

$$[B] = \frac{[F]}{[V]^2} = \frac{\text{N}}{\text{m}^2 \text{s}^{-2}} = \text{Nm}^{-2} \text{s}^2$$

$$\frac{[P][x]}{[t]^2} = 1 \Rightarrow [P] = \frac{[t]^2}{[x]} = \text{s}^2 \text{m}^{-1}$$

- [Q.2]** Gravity on the surface of Ganymede, a satellite of Jupiter, is $\left(\frac{1}{7^{\text{th}}}\right)$ of that on the Earth, while the gravity on the surface of the Moon is $\left(\frac{1}{6^{\text{th}}}\right)$ of that on the Earth. Two identical pendulums are taken one on Ganymede and the other on the Moon. The two pendulums start oscillating together, after how many oscillations on Moon will they come again in the same phase with approximately 1% uncertainty?

[A] 24

[B] 25

[C] 26

[D] 27

[ANS] D

[SOLN] $\frac{T_g}{T_m} = \sqrt{\frac{g_m}{g_{mg}}} = \sqrt{\frac{g/6}{g/7}} = \sqrt{\frac{7}{6}} = 1.68$

$$(\omega_m - \omega_g)t = 2\pi \times \frac{99}{100}$$

$$t = \frac{T_g T_m}{(T_g - T_m)} \times \frac{99}{100}$$

$$\therefore n = \frac{t}{T_m} = \frac{T_g}{(T_g - T_m)} \times \frac{99}{100} = \frac{T_g / T_m}{\left(\frac{T_g}{T_m} - 1\right)} \times \frac{99}{100} = \frac{\sqrt{\frac{7}{6}}}{\sqrt{\frac{7}{6}} - 1} \times \frac{99}{100} = 13.5$$

$$= \frac{27}{2}$$

[Q.3] Consider two telescopes having equal apertures of 400 mm. One has a focal ratio of $\frac{f}{5}$, the other has the focal ratio of $\frac{f}{10}$. What is the relation between their focal lengths?

[A] Focal length of the telescope with $\frac{f}{5}$ ratio is twice as compared to the focal length of telescope with $f/10$ ratio.

[B] Focal length of the telescope with $f/10$, ratio is twice as compared to the focal length of telescope with $f/5$ ratio.

[C] Focal length of the telescope with $f/10$ ratio is four times smaller as compared to the focal length of telescope with $f/5$ ratio.

[D] Focal lengths of both telescopes are the same

[ANS] A

[SOLN] f.no. = $\frac{f}{a} \rightarrow 5 \text{ nm}$

f.no. $\propto f$

Focal length of $f/5 = 2 \times$ focal length of $f/10$

[Q.4] **Statement I :** The radius vector of a planet sweeps equal area in equal time while revolving around the Sun .

Statement II : Gravitational force between the Sun and the planet is along the line joining the two.

[A] Both the Statements are true, and Statement II is correct reason of Statement I.

[B] Both the Statements are true, but Statement II is not the correct reason of Statement I

[C] The Statement I is true and the Statement II is false

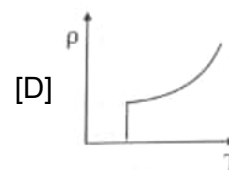
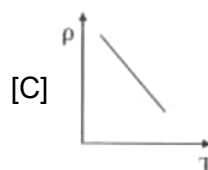
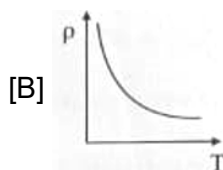
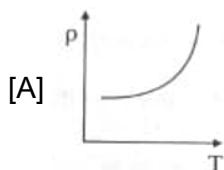
[D] The Statement I is false and the Statement II is true.

[ANS] A

[SOLN] $\frac{dA}{dt} = \frac{L}{2m}$

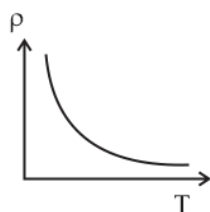
As $\vec{r} \parallel \vec{F}$, $\vec{\tau} = 0 \rightarrow L = \text{constant}$

[Q.5] Which of the following graphs qualitatively depicts the variation in resistivity of a semiconductor with respect to temperature correctly?

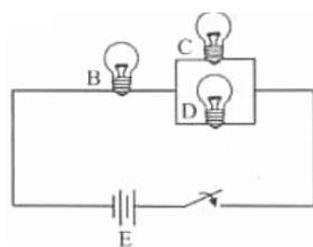


[ANS] B

[SOLN] fact based



[Q.6] Three identical bulbs, B, C and D are connected in a circuit as shown below. By connecting one more identical bulb



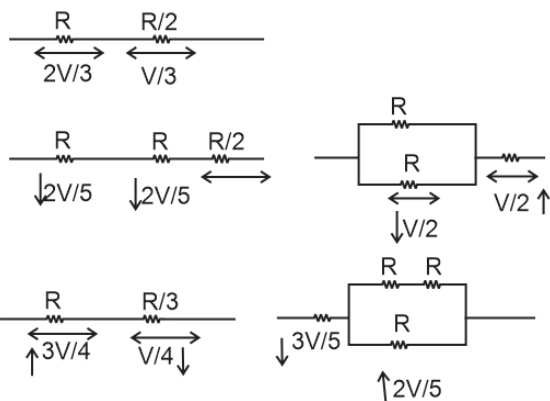
[A] in series with bulb B, the intensity of bulb B will decrease but that of bulb C and D will increase

[B] in parallel with bulb B, the intensities of all the three bulbs B, C and D will decrease.

[C] in parallel with bulbs C and D, the intensities of all the three bulbs B, C and D will increase

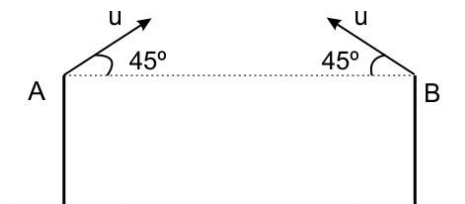
[D] in series with bulb C, the intensity of bulb B will decrease and that of bulb D will increase.

[ANS] D



[SOLN]

- [Q.7]** Two cannons are placed on 1000 m high towers at a horizontal distance of 400 m between them along x-axis. Ball A is fired at an angle of 45° to the +ve x-axis whereas Ball B is fired at an angle of 45° to the -ve x-axis. Initial velocity of projection given to each ball is $u = 40$ m/s in magnitude. The point $P(x, y)$ at which the two balls collide is

[A] $x = 200$ m, $y = -45$ m[B] $x = 200$ m, $y = 0$ [C] $x = 100$ m, $y = -200$ m

[D] the two balls will not collide

[ANS] **A**

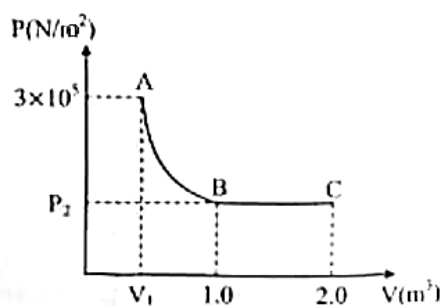
[SOLN] Time after which balls collide

$$t = \frac{(s_{\text{rel}})}{v_{\text{rel}}} = \frac{x_{\text{rel}}}{v_{x\text{rel}}} = \frac{400}{40\sqrt{2}} = 10 / \sqrt{2}$$

$$y = u_y t + \frac{1}{2} a_y t^2 = 20\sqrt{2} \times 10 / \sqrt{2} - \frac{1}{2} \times 9.8 \times \frac{100}{2}$$

$$= 200 - 245 = -45$$

- [Q.8]** Sixty moles of Helium gas are initially at 28°C (at A). It undergoes an isothermal process from A to B and then an isobaric process from B to C. Total change in the internal energy of the gas in the complete process from A to C would be approximately,



[A] 206 kJ

[B] 450 kJ

[C] 431 kJ

[D] 225 kJ

[ANS] D**[SOLN]** $\Delta u_{AB} = 0 \rightarrow$ Isothermal process

$$\Delta u_{BC} = nc_v \Delta T = n \frac{3}{2} R (T_3 - T_2)$$

↓

301K

Now, $B \rightarrow C$ ($V \propto T$)

$$T_3 = \frac{V_3}{V_2} T_2 = \frac{2}{1} \times 301 = 602$$

$$\therefore \Delta u_{BC} = \frac{3nR}{2} (602 - 301) = \frac{3 \times 60 \times 8.3}{2} \times 301$$

$$= 90 \times 8.3 \times 301 = 224.847 \text{ J} = 224.847 \text{ KJ} \approx 225 \text{ KJ}$$

[Q.9] In an atom, the nucleus consists of 2 protons and 2 neutrons and one electron is revolving around the nucleus. According to Bohr's atomic model, the only visible wavelength corresponds to the transition

(a) 2nd to 1st orbit[B] 4th to 2nd orbit[C] 4th to 3rd orbit[D] 5th to 3rd orbit**[ANS] C**

$$\frac{1}{\lambda} = Rz^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

↓

He = 2

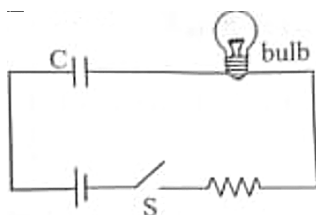
$$2 \rightarrow 1 \quad \lambda = \frac{1}{3R} = 303 \text{ Å (uv)} \quad \left[\frac{1}{2} = 911 \text{ Å} \right]$$

$$4 \rightarrow 2 \quad \lambda = \frac{4}{3R} = 1212 \text{Å} (\text{uv})$$

$$4 \rightarrow 3 \quad \lambda = \frac{36}{7R} = \frac{36}{7} \times 911 = 4685 \text{Å} (\text{visible})$$

$$5 \rightarrow 3 \quad \lambda = \frac{225}{64R} = \frac{225}{64} \times 911 = 3202 \text{ Å} (\text{uv})$$

[Q.10] Select the statement describing correctly the functioning of the circuit, shown below.



- [A] From the time the switch S is closed, the bulb will start glowing with intensity increasing uniformly to a maximum value and will turn off after some time.
- [B] As soon as the switch S is closed, the bulb will glow with maximum intensity which will then slowly decrease to zero.
- [C] From the time the switch S is closed, the bulb will start glowing with intensity increasing uniformly to a maximum value and will continue glowing thereafter.
- [D] If distance between the parallel plates of the capacitor is increased, bulb intensity will attain maximum value in shorter time after the switch is closed.

[ANS] B

[SOLN] The circuit is RC transient circuit

$$i = i_0 e^{-t/RC}$$

$$\frac{di}{dt} = \frac{i_0}{RC} e^{-t/RC}$$

Thus, intensity will be maximum just after closing the switch and it decreases exponentially with time.

[Q.11] A mechanical spring deviates from Hooke's law as $F \propto -k(e^x - 1)$, where x is the strain. At what value of the strain x , the force deviates from the one obeying Hooke's law with same k and unit length, just by 1% ?

- [A] $x = 1\%$ [B] $x = 2\%$ [C] $x = 5\%$ [D] $x = 20\%$

[ANS] B

[SOLN] $\frac{F' - F}{F} \times 100 = 1$

$$\frac{k(e^x - 1) - kx}{kx} \times 100 = 1 \quad \left[\text{use } e^x = 1 + x + \frac{x^2}{2!} \right]$$

$$x = 2\%$$

[Q.12] Three lenses with focal lengths f , f' and f respectively are kept with common principal axis in that order with successive separation of $\frac{f}{2}$ between each pair. If the point object is placed on the common principal axis to the left of 1st lens at $2f$ distance from it. What should be the value of focal length f' so as to form the final image at distance of $2f$ from 3rd lens on the right of it ?

[A] $-\frac{4f}{3}$

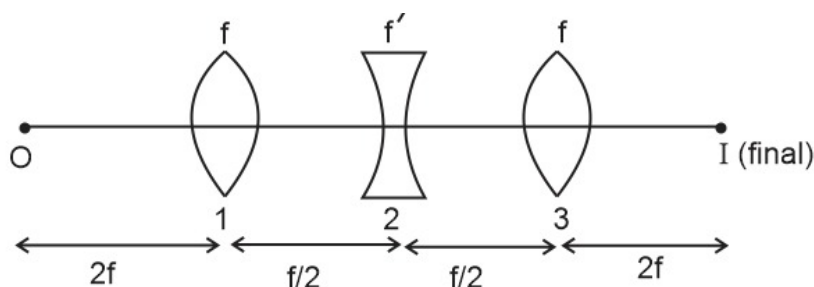
[B] $+\frac{4f}{3}$

[C] $-\frac{3f}{4}$

[D] $-\frac{3f}{5}$

[ANS] C

[SOLN]



for Lens 1:

$$u = -2f$$

$$\Rightarrow v = +2f$$

for Lens 2:

$$u = +\frac{3f}{2}$$

$$f = f'$$

$$v = \frac{uf}{u + f} = \frac{\left(\frac{3f}{2}\right)(f')}{\frac{3f}{2} + f'} = \frac{3ff'}{3f + 2f'}$$

for Lens 3:

$$u = \frac{3ff'}{3f + 2f'} - \frac{f}{2}$$

$$f = f$$

$$v = +2f$$

$$\text{using } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

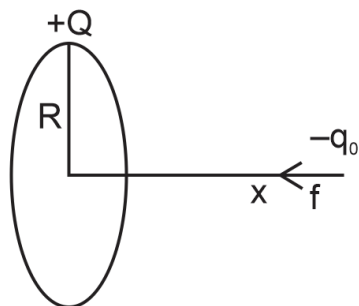
$$\text{we get } f' = -\frac{3f}{4}$$

[Q.13] A negatively charged small ball with charge -20pC and mass 1.00 mg is placed at the centre of a uniformly charged ring of radius 5.00 cm . The negatively charged ball is allowed to move only along the axis of the ring. The ball executes SHM with frequency of 1.00 kHz . The charge on the ring is

- [A] 27.4 mC [B] 0.55 mC [C] $13.9\text{ }\mu\text{C}$ [D] $0.69\text{ }\mu\text{C}$

[ANS] A

[SOLN]



Force

$$f = \frac{-KQq_0x}{(R^2 + x^2)^{\frac{3}{2}}} \approx -\left[\frac{KQq_0}{R^3}\right]x, K_0 = \frac{KQq_0}{R^3}$$

Time period

$$T = 2\pi\sqrt{\frac{m}{K_0}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K_0}{m}} = \frac{1}{2\pi}\sqrt{\frac{KQq_0}{R^3}}$$

$$\frac{1}{2\pi}\sqrt{\frac{9 \times 10^9 \times Q \times 20 \times 10^{-12}}{125 \times 10^{-6}}} = 1 \times 10^3$$

Solving

$$Q = 27.4\text{ mC}$$

[Q.14] Two hunters approach a prey from opposite directions. One hunter has a probability of $\frac{1}{3}$ for hitting the prey, the other has a probability of $\frac{2}{3}$ for hitting the prey. Both fires burst of 3 shots

each simultaneously. Assume that both the hunters fire one shot each at the same time simultaneously, followed by another shot each at the same time, followed by the third shot each at the same time, each round of shots have negligible time gap between them. The probability that the prey, in question, is hit

[A] after the three rounds is 1

[B] in the first round of shots is 1

[C] after the three rounds is $\frac{7}{9}$

[D] after the three rounds is $\frac{721}{729}$

[ANS] D

[SOLN] Let A_i : First hunter hits the prey in i th shot

B_i : 2nd hunter hits the prey in i th shot

E_i : The prey is hit in the i th shot

for $i = 1, 2, 3$

Then $P(E_i) = P(A_i \cup B_i)$

$$= 1 - P(\overline{A_i})P(\overline{B_i})$$

$$= 1 - \frac{2}{3} \times \frac{1}{3} = \frac{7}{9}$$

Probability that the prey is hit after the 3 rounds

$$= P(E_1 \cup E_2 \cup E_3)$$

$$= 1 - P(\overline{E_1})P(\overline{E_2})P(\overline{E_3})$$

$$= 1 - \frac{2}{9} \times \frac{2}{9} \times \frac{2}{9} = \frac{721}{729}$$

[Q.15] A particle moving with constant velocity along X-axis has angular momentum $p = 120$ units about a reference point in XY plane. Which of the following statements is correct ?

[A] If the reference point is shifted parallel to Y-axis, angular momentum need not change necessarily.

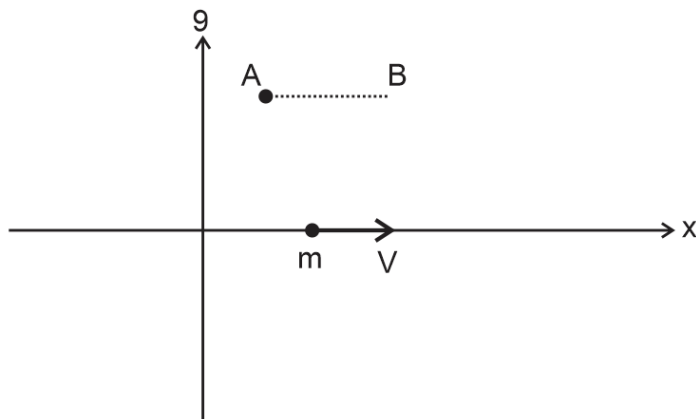
[B] If the reference point is taken to double the present distance from origin, instantaneous angular velocity become half the present value.

[C] It is always possible to increase instantaneous angular velocity to double the present value by shifting reference point parallel to X-axis

[D] If the reference point is shifted parallel to X-axis, angular momentum will surely not change about it.

[ANS] D

[SOLN]



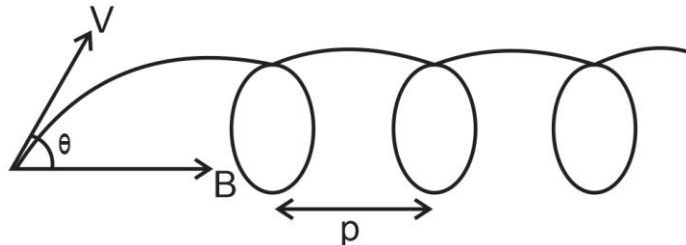
Angular momentum about A & B will be same.

[Q.16] An electron moving with speed of 0.1% speed of light, enters a uniform magnetic field of strength 2.5 gauss at 60° angle. Calculate the number of helical turns it takes in moving 1 km distance.

[A] 20000 to 22000 [B] 22000 to 25000 [C] 30000 to 32000 [D] 45000 to 47000

[ANS] D

[SOLN]



$$N \times p = 1 \text{ km} \quad p = V \cos \theta \times \frac{2\pi m}{qB}$$

$$N \times V \cos \theta \times T = 1 \text{ km}$$

$$N = \frac{1 \times 10^3 \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4}}{3 \times 10^5 \times 2\pi \times 9.1 \times 10^{-31}} \approx 46600$$

[Q.17] If $a + \frac{1}{a} = \sqrt{3}$ then $a^6 + \frac{1}{a^6}$ is

[A] 27

[B] 0

[C] -2

[D] 81

[ANS] C

[SOLN] $a + \frac{1}{a} = \sqrt{3}$

$$\Rightarrow a^2 + \frac{1}{a^2} + 2 = 3$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 1$$

Using both sides,

$$a^6 + \frac{1}{a^6} + 3\left(a^2 + \frac{1}{a^2}\right) = 1$$

$$\Rightarrow a^6 + \frac{1}{a^6} + 3 = 1$$

$$\Rightarrow a^6 + \frac{1}{a^6} = -2$$

[Q.18] If $\sin x + \cos x = \sqrt{2}$ then $\sin^4 x + \cos^4 x$ is

[A] 4

[B] -1

[C] 1

[D] $\frac{1}{2}$

[ANS] D

[SOLN] $\sin x + \cos x = \sqrt{2} \Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x = 2$

$$\Rightarrow 2\sin x \cos x = 1$$

$$\therefore \sin^4 x + \cos^4 x = 1 - 2\sin^2 x \cos^2 x$$

$$= 1 - 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

[Q.19] If a, b, c are in arithmetic progression and a^2, b^2, c^2 are in geometric progression, then the common ratio is

[A] 0

[B] 1

[C] 2

[D] 3

[ANS] B

[SOLN] a, b, c are in A.P.

$$\Rightarrow b = \frac{a+c}{2}$$

a^2, b^2, c^2 in G.P.

$$\Rightarrow b^4 = a^2 c^2 \Rightarrow b^2 = \pm ac$$

$$\Rightarrow \left(\frac{a+c}{2} \right)^2 = \pm ac$$

$$(a+c)^2 \mp 4ac = 0$$

Case I

$$(a+c)^2 - 4ac = 0$$

$$\Rightarrow (a-c)^2 = 0 \quad \Rightarrow a = c$$

$$\therefore a = b = c$$

$$\therefore \text{c.r.} = \frac{b^2}{a^2} = 1$$

Case II

$$(a+c)^2 + 4ac = 0$$

$$\Rightarrow a^2 + 6ac + c^2 = 0$$

$$\Rightarrow \left(\frac{c}{a} \right)^2 + 6 \frac{c}{a} + 1 = 0$$

$$\Rightarrow \frac{c}{a} = \frac{-6 \pm \sqrt{32}}{2} = -3 \pm 2\sqrt{2}$$

$$\therefore \text{c.r.} = \left| \frac{c^2}{a^2} \right| = \left| \frac{c}{a} \right| = (3 \pm 2\sqrt{2})$$

[Q.20] $\sum_{n=1}^{\infty} \frac{1}{n(n+3)} =$

[A] $\frac{1}{3}$

[B] $\frac{11}{18}$

[C] $\frac{1}{2}$

[D] $\frac{4}{17}$

[ANS] B

[SOLN] $\sum_{n=1}^{\infty} \frac{1}{n(n+3)} = \frac{1}{3} \sum_{n=1}^{\infty} \frac{n+3-n}{n(n+3)}$

$$= \frac{1}{3} \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+3} \right)$$

$$= \frac{1}{3} \left[\left(\frac{1}{1} - \frac{1}{4} \right) + \left(\frac{1}{2} - \frac{1}{5} \right) + \left(\frac{1}{3} - \frac{1}{6} \right) + \left(\frac{1}{4} - \frac{1}{7} \right) + \dots \text{to } \infty \right]$$

$$= \frac{1}{3} \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} \right)$$

$$= \frac{11}{18}$$

[Q.21] Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \max\{x^2, 1\}$. Then

[A] f is differentiable everywhere

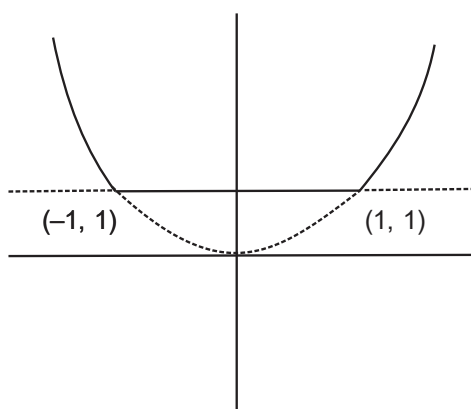
[B] f is continuous everywhere but not differentiable at ± 1 .

[C] f is not continuous at $x = \pm 1$

[D] f is neither continuous nor differentiable.

[ANS] B

[SOLN]



$$f : \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = \max\{x^2, 1\}$$

From the graph it is clear that f is continuous everywhere but not differentiable at $x = \pm 1$.

[Q.22] Let T_1 denote an equilateral triangle of side length a , T_2 be the triangle whose vertices are midpoints of sides of T_1 , T_3 be the triangle whose vertices are midpoints of sides of T_2 . We continue in similar number to obtain triangles $T_4, T_5, \dots$. For each natural number n , let P_n denote the perimeter of the triangle T_n . Then $\sum_{n=1}^{\infty} P_n =$

[A] $12a$

[B] $8a$

[C] $6a$

[D] $3a$

[ANS] C

[SOLN] T_1 has side length a , therefore, T_2 has side length $\frac{a}{2}$ and this trend continues, hence

$$\sum_{n=1}^{\infty} P_n = \sum_{n=1}^{\infty} 3 \frac{a}{2^{n-1}} = 6a$$

[Q.23] $\sqrt{i} + \sqrt[3]{i^2} = x + iy$, where $i = \sqrt{-1}$, $(x, y) = ?$

- [A] $(-0.293, 1.573)$ [B] $(-0.293, 0.707)$ [C] $(-0.207, 1.573)$ [D] $(1.207, 1.573)$

[ANS] B

[SOLN] $\sqrt{i} = \pm \frac{1+i}{\sqrt{2}}$ $\sqrt[3]{i^2} = \sqrt[3]{-1} = -1, -\frac{-1 \pm i\sqrt{3}}{2}$

So we have six possibilities

On checking, $\frac{1+i}{\sqrt{2}} - 1 = \frac{\sqrt{2}}{2} - 1 + \frac{\sqrt{2}}{2}i = (-0.293, 0.707)$

[Q.24] A standard parabola $x^2 = 36y$ is approximated as an arc of a circle for small values of x . What will be the radius of that circle ?

- [A] 72 [B] 36 [C] 18 [D] 9

[ANS] C

[SOLN] $y = \frac{x^2}{36}$

$$\frac{dy}{dx} = \frac{x}{18}$$

$$\frac{d^2y}{dx^2} = \frac{1}{18}$$

$$\text{ROC, } r = \left| \frac{\left[1 + \left(\frac{x}{18} \right)^2 \right]^{3/2}}{1/18} \right|$$

for $x \rightarrow 0$

$r = 18$ unit

[Q.25] $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx =$

- [A] $\frac{\pi}{2}$ [B] $\frac{\pi}{4}$ [C] 0 [D] π

[ANS] B

[SOLN] $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$$

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$2I = \int_0^{\pi/2} dx = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

[Q.26] Let $f(x)$ be continuous on $[0, \pi]$ and $f(x) + f(\pi - x) = \pi$. Then $\int_0^\pi f(x) dx =$

[A] π^2

[B] $\frac{\pi}{2}$

[C] $\frac{\pi^2}{2}$

[D] $\frac{\pi^2}{4}$

[ANS] C

[SOLN] Let $I = \int_0^\pi f(x) dx$... (1)

Then Let $I = \int_0^\pi f(\pi - x) dx$... (2)

Adding (1) & (2)

$$2I = \int_0^\pi f(x) + f(\pi - x) dx$$

$$\int_0^\pi \pi dx = \pi^2$$

$$\therefore I = \frac{\pi^2}{2}$$

[Q.27] $\lim_{x \rightarrow 3} \frac{\sqrt{1 - \cos 2(x-3)}}{x-3}$

[A] Exists and it equals $\sqrt{2}$

[B] Exists and it equal $-\sqrt{2}$

[C] Does not exist because $(x-1) \rightarrow 0$

[D] Does not exist because left hand limit is not equal to right hand limit.

[ANS] D

[SOLN] $\lim_{x \rightarrow 3} \frac{\sqrt{1 - \cos 2(x-3)}}{x-3}$

$$= \lim_{x \rightarrow 3} \frac{\sqrt{1 - \cos 2\theta}}{\theta} \quad (\theta = x - 3)$$

$$= \lim_{\theta \rightarrow 0} \frac{\sqrt{2} |\sin \theta|}{\theta}$$

$$\text{But } \lim_{\theta \rightarrow 0} \frac{\sqrt{2} |\sin \theta|}{\theta} = \lim_{\theta \rightarrow 0^+} \frac{\sqrt{2} |\sin \theta|}{\theta} = \sqrt{2} \text{ and } \lim_{\theta \rightarrow 0} \frac{\sqrt{2} |\sin \theta|}{\theta} = \lim_{\theta \rightarrow 0^-} \frac{-\sqrt{2} \sin \theta}{\theta} = -\sqrt{2}$$

$\therefore$ limit does not exist.

[Q.28] A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{3}{4}\right)$. What is poured into it at a constant rate of $5 \text{ m}^3/\text{min}$. Then, the rate (in m/min) at which the water surface moves along the slant surface at the instant when the depth of water in the tank is 10 m is

[A] $\frac{1}{9\pi}$

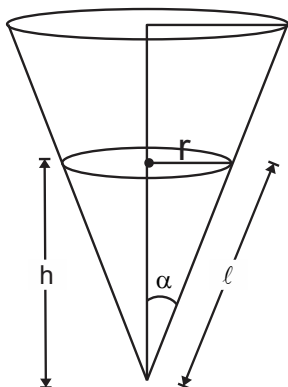
[B] $\frac{9}{192\pi}$

[C] $\frac{1875}{4\pi}$

[D] $\frac{4}{27\pi}$

[ANS] A

[SOLN]



$$\alpha = \tan^{-1} \frac{3}{4} \Rightarrow \tan \alpha = \frac{3}{4}$$

$$\therefore \cos \alpha = \frac{4}{5} - \frac{h}{\ell} \Rightarrow h = \frac{4}{5} \ell$$

Now volume at any instant t ,

$$V = \frac{1}{3} \pi r^2 h = \frac{\pi}{3} \left(\frac{3}{5} \ell \right)^2 \left(\frac{4}{5} \ell \right)$$

$$V = \frac{1}{3} \pi r^2 h = \frac{\pi}{3} \left(\frac{3}{5} \ell \right)^2 \left(\frac{4}{5} \ell \right) = \frac{12x}{125} \ell^3$$

$$\therefore \frac{dV}{dt} = \frac{36\pi}{125} \ell^2 \frac{d\ell}{dt}$$

$$\Rightarrow \frac{d\ell}{dt} = \frac{125}{36\pi\ell^2} \ell^2 \frac{dv}{dt}$$

When $h = 10 \text{ cm}$

$$\ell = \frac{5h}{4} = \frac{25}{2} \text{ m}, \quad \frac{dv}{dt} = 5 \text{ m}^2 / \text{min}$$

$$\therefore \frac{d\ell}{dt} = \frac{125}{3 \times \frac{25}{2} \times \frac{25}{2}} \times 5 \text{ m} / \text{min} = \frac{1}{9\pi} \text{ m} / \text{min}$$

[Q.29] How many people should there be in a group such that there is more than half the probability, that two people from the group have their birthdays with the same date, irrespective of in which months they were born.

[A] 3 or more [B] 4 or more [C] 7 or more [D] 23 or more

[ANS] C

[SOLN] Let there are n people in the group.

If $n > 31$, then obviously two people will have same birth date.

So we assume that $n < 31$.

$P(\text{the people have same birth date})$

$= 1 - P(\text{all have diff. birth date-})$

$$= 1 - \frac{{}^{31}P_n}{31^n} > \frac{1}{2} \quad \Rightarrow \quad 1 - \frac{{}^{31}P_n}{31^n} < \frac{1}{2}$$

$$\Rightarrow \frac{31 \times 30 \times 29 \times \dots \times (32 - n)}{31 \times 31 \times 31 \times 31 \times \dots \text{times}} = \frac{1}{2}$$

On checking, $n \geq 7$

[Q.30] A and B alternatively toss a coin. The one who gets a head first wins. If A starts the game, then what is the probability that A wins ?

[A] $\frac{1}{2}$ [B] $\frac{2}{3}$ [C] $\frac{3}{4}$ [D] $\frac{3}{5}$

[ANS] B

[SOLN] $P(A_{\text{wins}}) = A + A'B'A + A'B'A'B'A + \dots$

$$= \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \dots$$

$$\frac{1/2}{1-\frac{1}{4}} = \frac{2}{3}$$

[Q.31] Consider the expression $Z = 1 + 2^2 + (3^3)^3 + ((4^4)^4)^4 + \dots$. From which term onwards, the total value of the expression Z exceeds 10^{1000} ?

[A] 4

[B] 6

[C] 8

[D] 10

[ANS] B

[SOLN] $z = 1 + 2^2 + (3)^3 + \left((4^4)^4\right) + \left(\left((5^5)^5\right)^5\right) + \left(\left(\left((6^6)^6\right)^6\right)^6\right) + \dots$

With the help of logarithm, we see that $\left(\left((5^5)^5\right)^5\right)^5$ has 437 digits and $\left(\left(\left((6^6)^6\right)^6\right)^6\right)^6$ has

6051 digits & hence from this onwards z exceeds 10^{1000}

[Q.32] Let $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$. Which of the following is not true?

[A] A is symmetric[B] If $a + b + c = 0$ then $\det A = 0$ [C] $\det A = a^3 + b^3 + c^3 - 3abc$ [D] $\det A = \det A^T$

[ANS] C

[SOLN] $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ is symmetric

with $\det A = 3abc - a^3 - b^3 - c^3$

$\therefore$ option C is not correct

[Q.33] It has been four hours since Regulus (10h08m, +11°58') has crossed the local meridian at Mumbai (19°2'11.11"N, 72°51'34.09"E). Which of the following stars will be closest to the meridian now?

[A] Arcturus (14h16m50.74s, 19°02'8")

[B] Sirius (6h46m15.1s, -16°44'.6")

[C] Betelgeuse (5h56m31.86s, 19°02', 8")

[D] Spica (13h26m32.96s, -11°17'44.7")

[ANS] A

[SOLN] LST = 10h 08 m + 4h 00m = 14 H 08 M

Difference (Δ) [LST to star right Ascensions (RA)]

$$\Delta_a = 14h16m51s - 14h08m \approx 8m51s$$

$$\Delta_b = 14h08m - 5h56m32s \approx 8m11m$$

$$14 \text{ h } 08 \text{ m} - 6 \text{ h } 46 \text{ m } 15 \text{ sec} \approx 7 \text{ h } 22 \text{ m}$$

$$14 \text{ h } 08 \text{ m} - 13 \text{ h } 26 \text{ m } 33 \text{ s} \approx 41 \text{ m } 27 \text{ s}$$

[Q.34] A star has an apparent magnitude of 10, and an Absolute magnitude of 10. How many parsecs away from the Earth is it?

[A] 100

[B] 10

[C] 1

[D] 0.1

[ANS] B

[SOLN] Use the distance modulus formula

$$m - M = 5 \log_{10} \left(\frac{d}{10 \text{ pc}} \right)$$

Given that $m = M = 10$

$$\log_{10} \left(\frac{d}{10} \right) = 0$$

$$\Rightarrow \frac{d}{10} = 1 \Rightarrow d = 10 \text{ pc}$$

[Q.35] Which of the following statement or statements about stars are true?

[A] Among all one solar mass stars, the one with the largest radius is also the hottest.

[B] Type I supernovae are characterized by the absence of Hydrogen in the spectrum

[C] Cooler stars show less absorption lines in their spectra.

[D] The stars are spectrally classified as O, B, A, F, G, K, M; with O type stars showing dense hydrogen line while M type stars having very less hydrogen lines.

[ANS] B

[SOLN] CONCEPTUAL

[Q.36] As seen from Earth, angular separation between Proxima Centauri and Alpha Centauri is 2.2° . What is the physical separation between the two stars?

[A] 4.25 light years

[B] 0.16 light years

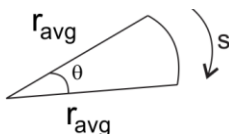
[C] 2.2 light years

[D] 0.33 light years

[ANS] B

[SOLN] Distance Proxima Centauri $\approx 4.25 \text{ Ly}$

Distance of Alpha Centauri $\approx 4.34 \text{ Ly}$



$$s = r\theta = r_{\text{avg}} = \frac{4.25\text{Ly} + 4.34\text{Ly}}{2} = 4.295\text{Ly}$$

$$\theta = 2.2^\circ \frac{\pi}{180} = 0.0384 \text{ radian}$$

$$s \approx 4.295\text{Ly} \times 0.0384 \text{ radian} \approx 0.165\text{Ly}.$$

[Q.37] Two stars in a binary system are separated by 3.0 AU and have mass ratio of 2: 1. Their orbital period is 6.0 years. What are the masses of the stars in terms of solar mass?

[A] 2.0, 1.0

[B] 0.50, 0.25

[C] 2.25, 1.125

[D] 1.78, 0.88

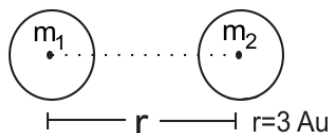
[ANS] B

[SOLN] $T = 6 \text{ yrs}$

$$T^2 = \frac{4\pi^2 r^3}{G(m_1 + m_2)}$$

If r is in Au

$$T^2 = \frac{r^3}{(m_1 + m_2)}$$



Given that

$$\frac{m_1}{m_2} = \frac{2}{1} \dots\dots\dots (1)$$

$$\Rightarrow m_1 + m_2 = \frac{27}{26} = 0.75 \text{ solar mass} - (ii)$$

From (i) and (ii)

$$\Rightarrow m_1 = 0.50 \text{ \& } m_2 = 0.25$$

[Q.38] A comet's closest approach to Sun is at 1 AU. What is the radial component of its velocity at this position?

[A] 0 km/s

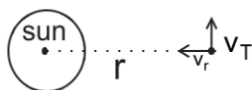
[B] 21.2 km/s

[C] 30 km/s

[D] 42.4 km/s

[ANS] B

[SOLN] At the moment of closest approach $\left(\frac{dr}{dt} = v_r = 0\right)$



[Q.39] A new space station orbits the sun every four and a half years. In a particular year, it is seen on the local meridian at 1: 00 am of the 21st of June, 2020. It will be again seen on the local meridian from the Earth, approximately, on

[A] 21 June, 2024, at 1: 00 am

[B] 20 December, 2024, at 4: 00 pm

[C] 3 October, 2021, 4: 00 pm

[D] 21 March, 2021, at 9: 00 pm

[ANS] C

[SOLN] The interval between successive meridian passages as seen from earth is the synodic period

$$\frac{1}{S} = \left| \frac{1}{P} - \frac{1}{1\text{yr}} \right| = \left| \frac{1}{4.5} - 1 \right| = 0.77778$$

To calculate the time delay

$$T = \frac{1}{S} = 1.2857 \text{ yrs} \approx 1 \text{ yrs} + 104.36 \text{ days}$$

Approx. [June 21, 2020 + 1 Year + 104.36 days] = 3 oct. 2021

1 am

4 pm

[Q.40] What is the advantage of an equatorial telescope mount over an alt-azimuth mount?

[A] Reduced vibrations and provides a more stable viewing platform.

[B] It allows tracking celestial objects using only one axis of motion.

[C] It eliminates the need for polar alignment before observing.

[D] It is easier to carry and transport due to its lightweight design.

[ANS] B

[SOLN] It allows tracking celestial objects using only one axis of rotation.

[Q.41] Which of the following statements is correct about constellations?

[A] Any star cannot belong to two constellations simultaneously.

[B] Only the bright stars which are imagined as some figure in the sky make constellations.

[C] Brightest star in any constellation has magnitude 1.

[D] There are only 12 constellations along the Ecliptic belt.

[ANS] A

[SOLN] Any star cannot belong to two constellations simultaneously.

[Q.42] What would be the speed of a comet, on a parabolic orbit around the sun, whose point of closest approach is 1AU, when at a distance of 4.0 AU?

[A] 42.1 km/s

[B] 29.8 km/s

[C] 21.1 km/s

[D] 84.4 km/s

[ANS] C

[SOLN] $V_{\text{esc}} = \sqrt{\frac{2GM}{r}}$

$$\text{In } \frac{\text{AU}}{V_{\text{esc}}} = V_E \sqrt{\frac{2\text{AU}}{r}} = (29.78 \text{ km/s}) \sqrt{\frac{2 \times 1\text{AU}}{4\text{AU}}} = 21.1 \text{ km/s}$$

[Q.43] Globular clusters are typically found in the halo of Milky Way. In which of the following constellation, there are higher chances of seeing globular cluster?

[A] Orion

[B] Sagittarius

[C] Ursa Minor

[D] Virgo

[ANS] B

[SOLN] Globular cluster are highly concentrated around the Galactic centre.

The centre of our milky way galaxy lies in the direction of the constellation Sagittarius

[Q.44] Ecliptic plane makes approximately 60° with the Milky Way plane. One point of intersection lies in the constellation of Sagittarius, in which constellation does the other intersection point lie?

[A] Aquarius

[B] Libra

[C] Pieces

[D] Gemini

[ANS] D

[SOLN] The opposite direction from Sagittarius along the ecliptic lies in the constellation gemini.

[Q.45] Which of the following places will have minimum duration between the two zero shadow days in a given calendar year.

[A] Manila (14° 36' N, 120° 59' E)

[B] Monteiro (7° 53' S, 37° 7' W)

[C] Kansanshi (12° 6' S, 26° 26' W)

[D] Barah (13° 42' N, 30° 22' E)

[ANS] A

[SOLN] comparing the latitudes to the nearest tropic (around 23.5°)

(i) Manila $|23.5^\circ - 14^\circ 36'| \approx 8^\circ 54'$ (smallest)

(ii) Monteiro $|23.5^\circ - 7^\circ 53'| \approx 15^\circ 37'$

(iii) Kansanshi $|23.5^\circ - 12^\circ 6'| \approx 11^\circ 24'$

(iv) Barah $|23.5^\circ - 13^\circ 42'| \approx 9^\circ 48'$

[Q.46] An observer from Delhi, will see the Sun on the local meridian 365 times in the year 2025. A star, located on the celestial equator, will be seen how many times on the local meridian by the same observer in 2025?

[A] 364

[B] 365

[C] 366

[D] 367

[ANS] C

[SOLN] The earth makes one extra rotation relative to the fixed stars during one solar year.

[Q.47] Considering the nuclear reactions that power the energy output of the stars, the correct statements is :

[A] the p-p chain is the dominant process that creates He in very massive stars.

[B] the fusion of H into He is an exothermic process.

[C] the CNO (Carbon-Nitrogen-Oxygen) cycle results in the creation of the elements like Si, S and P following the fusion of the lighter elements.

[D] the fusion process is replaced by the fission process in heavier stars.

[ANS] B

[SOLN] fusion of H into He is an exothermic

[Q.48] Which of the following constellations is broken in two disjoint parts in the sky?

[A] Draco

[B] Ursa

[C] Serpens

[D] Eridanus

[ANS] C

[SOLN] The constellation that is broken in two disjoint parts in the sky is serpens (The serpent)

(NSEA) PART : A-2

ANY NUMBER OF OPTIONS (4, 3, 2 OR 1) MAY BE CORRECT MARKS WILL BE AWARDED ONLY IF ALL THE CORRECT OPTIONS ARE BUBBLED AND INCORRECT.

[Q.49] Inside a cylindrical well, at the bottom and touching the wall, a red ball is thrown at an angle of 45° to the horizontal towards the diametrically opposite end of the wall, and it hits the wall after a time interval of $\frac{v\sqrt{2}}{g}$, where v is the magnitude of velocity of the red ball and g is acceleration due to gravity. A black ball identical in shape and mass to the red ball is thrown vertically upwards from the bottom of the well with a kinetic energy half of that of the red ball. A green ball having a mass half of the red ball is thrown from the bottom of the well but diametrically opposite to the red ball with a kinetic energy half of that of the red ball. All the balls, if and when they hit the wall, undergo completely elastic collision.

[A] The red ball bounces off the bottom of the well for the first time when it hits the wall the third time.

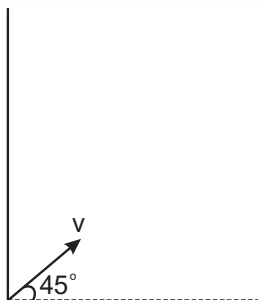
[B] The black ball will take double the time to hit the bottom of the well to that of the red ball.

[C] The green ball hits the bottom of the well at the same spot as the red ball.

[D] The green ball will take the same time as the red ball to hit the bottom of the well the first time.

[ANS] C,D

[SOLN]



$$T_{RB} = \frac{2v \sin 45^\circ}{g} = \sqrt{2} \frac{v}{g}$$

$$T_{BB} = \frac{2}{g} \frac{v}{\sqrt{2}} = \frac{\sqrt{2}v}{g}$$

$$T_{GB} = \frac{\sqrt{2}v}{g}$$

$$\left(\begin{array}{l} K_{BB} = \frac{1}{2} K_{RB} \\ \frac{1}{2} m v'^2 = \frac{1}{2} \frac{1}{2} m v^2 \\ v' = v / \sqrt{2} \end{array} \right)$$

$$\left(\begin{array}{l} K_{GB} = \frac{1}{2} K_{RB} \\ \frac{1}{2} m v''^2 = \frac{1}{2} \frac{1}{2} m v^2 \\ v'' = v \end{array} \right)$$

[Q.50] Read the two statements, I and II, about a RLC series circuit driven by an AC voltage source using an inductor having internal resistance r . Assume that the maximum amplitude of ac signal is v_m and frequency is F :

I : when frequency F is equal to the resonance frequency of the circuit, potential difference across the series combination of L and C has a non-zero amplitude.

II : when frequency F is equal to the resonance frequency of the circuit, current in the circuit has amplitude less than v_m/R .

Select correct statement(s) from the following:

[A] If statement I is correct, statement II has to be correct.

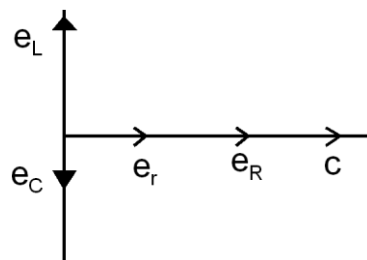
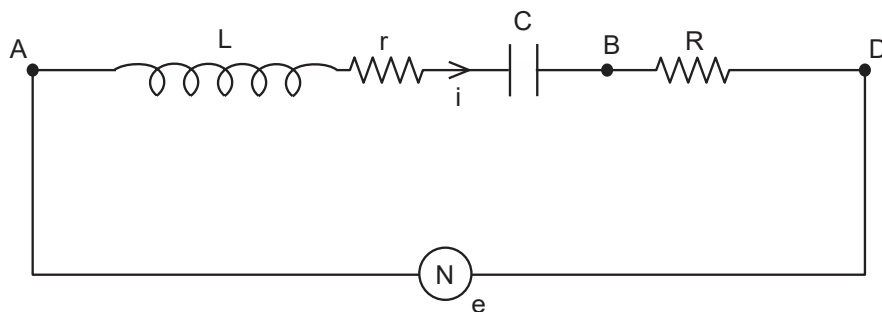
[B] If statement II is false, statement I cannot be false

[C] Statement II is false for most of the RLC series circuits with ac voltage source

[D] Statement I is correct for most of the RLC series circuits with ac voltage source.

[ANS] A

[SOLN]



$$e_{AB} = \sqrt{(e_L - e_C)^2 + e_r^2}$$

for resonance $e_L = e_C$

$$\therefore e_{AB} = e_r \neq 0$$

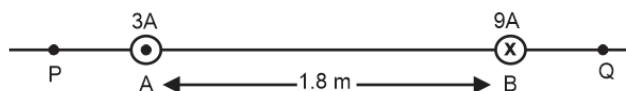
$$e_{AD} = (e_r + e_R)$$

$$iZ = i r + i R$$

$$Z = (r + R)$$

$$\text{Current amplitude, } i_m = \frac{V_m}{r + R} < \frac{V_m}{R}$$

[Q.51] Two current carrying wires A and B are held fixed, parallel to each other, at a distance of 1.8 m. A current of 3A flows through wire A in a direction, coming out of the plane of paper and that through the wire B is 9A going in to the plane of paper as shown.



Which of the following statement(s) is/are true with reference to the given situation?

- [A] There can be a point on the left of wire A (PA side) where net magnetic field is zero.
 [B] There can be a point on the right of wire B (BQ side) where net magnetic field is zero.
 [C] There can be a point between the wires A and B, closer to A, where magnetic field produced by the two wires will be in the same direction having equal magnitude.
 [D] There can be a point Q on the right of wire B (BQ side) where net magnetic field is in the upward direction parallel to the plane of paper.

[ANS] **A, C**

[SOLN] $B_p = 0$

$$(A) \frac{\mu_0 \times 3}{2\pi x} = \frac{\mu_0 9}{2\pi(1.8 + x)}$$

$$3x = 1.8x$$

$$x = 0.9\text{m}$$

(B) b/w A & B field can't be zero as field of both wires are along same direction.

$$(C) \frac{\mu_0 \times 3}{2\pi x} = \frac{\mu_0 \times 9}{2\pi(1.8 - x)}$$

$$4x = 1.8$$

$$x = 1.8 = 0.45\text{m} \quad (\text{from A})$$

(D) Not possible as field of wire B will be dominating which is in downside.

[Q.52] Two black bodies A and B are emitting in the approximate ratio 2:5. Which of the following statements may be correct from the given information?

- [A] Body B is 20% hotter and 10% larger in diameter than body A.
 [B] Body B is 50% hotter but half in diameter than body A.
 [C] Body B is 10% hotter and 30% larger in diameter than body A.
 [D] Body B has double the temperature of A and only 40% diameter of body A.

[ANS] **A, C**

[SOLN] $\frac{d_A^2 T_A^4}{d_B^2 T_B^4} = \frac{2}{5} \quad (\text{Given condition})$

$$(A) T_B = 1.2T_A \quad d_B = 1.1d_A$$

This data will satisfy the given condition.

$$(B) T_B = 1.5T_A \quad d_B = \frac{1}{2}d_A$$

Will not satisfy given condition

$$(C) T_B = 1.1T_A \quad d_B = 1.3d_A$$

Nearly satisfy the given condition

$$(D) T_B = 2 T_A \quad d_B = 0.4 d_A$$

Will not satisfy the given condition

[Q.53] Let $\delta = \begin{vmatrix} x & x^2 & 1 \\ 1 & x & x^2 \\ x^2 & 1 & x \end{vmatrix}$. Which of the following statements is not true?

[A] δ is an even function

[B] $\delta = 0$ for all real x

[C] δ is a polynomial of degree 6

[D] If $x = \sqrt[3]{1}$, $\delta = 0$ for all roots

[ANS] C, D

[SOLN]
$$\delta = \begin{vmatrix} x & x^2 & 1 \\ 1 & x & x^2 \\ x^2 & 1 & x \end{vmatrix}$$

$$= \begin{vmatrix} 1+x+x^2 & x^2 & 1 \\ 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x^2 \end{vmatrix} \quad (C_1 + C_1 + C_2 + C_3)$$

$$= (1+x+x^2) \begin{vmatrix} 1 & x^2 & 1 \\ 1 & x & x^2 \\ 1 & 1 & x \end{vmatrix}$$

$$= (1+x+x^2) \{1(x^2 - x^2) - 1(x^3 - 1) + 1(x^2 - x)\}$$

$$= (1+x+x^2)(x^4 - x^3 - x + 1)$$

$$= (1+x+x^2)(x^3(x-1) - 1(x-1))$$

$$= (x-1)(x^2+x+1)(x^3-1)$$

$$= (x^3-1)^2$$

$$\delta = 0 \Rightarrow$$

$$x^3 = 1$$

$$\Rightarrow x = \sqrt[3]{1}$$

[Q.54] Which of the following functions are periodic?

[A] $\sin x + \cos x$

[B] $\tan x$

[C] $x \sin x$

[D] $\sin x + \sin \sqrt{2}x$

[ANS] A, B

[SOLN] (a), (b) are clearly periodic and (c) is non-periodic.

For (d),

Sin x has period 2π and $\sin\sqrt{2}x$ has period $\frac{2\pi}{\sqrt{2}}$

But l.c.m of 2π and $\sqrt{2}\pi$ does not exist,

So it is non-periodic.

[Q.55] If $x + \frac{1}{x} = 2$ then which of the following are true?

[A] $x^2 + \frac{1}{x^2} = 2$

[B] $x^3 + \frac{1}{x^3} = 2$

[C] $x = 1$

[D] $x^4 + \frac{1}{x^4} = 2$

[ANS] A, B, C, D

[SOLN] $x + \frac{1}{x} = 2$

$$\Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow (x-1)^2 = 0$$

[Q.56] Which of the following series are not convergent?

[A] $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)$

[B] $\sum_{n=1}^{\infty} \left(3 + \left(\frac{2}{3}\right)^n\right)$

[C] $\sum_{n=1}^{\infty} n^{\frac{1}{n}}$

[D] $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

[ANS] A, B, C

[SOLN] (a) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) = 1$

$\therefore$ divergent

(b) $\lim_{n \rightarrow \infty} \left(3 + \left(\frac{2}{3}\right)^n\right) = 3$

$\therefore$ divergent

(c) $\lim_{n \rightarrow \infty} n^{1/n} = 1$

$\therefore$ The series is divergent

(d) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1$

$\therefore$ convergent

[Q.57] The pole star

[A] Can be seen at night from all locations on the Earth.

[B] Will be visible during a solar eclipse from nearly the whole of the northern hemisphere.

[C] Will be visible during a solar eclipse from the equator.

[D] Can, in principle, be observed from India anytime during the day and night.

[ANS] B, D

[SOLN] Conceptual

[Q.58] An observer measures the location of the Sun, moon, planets, and bright stars like Sirius, very diligently. The correct conclusion that may reach is/are:

[A] Planets rise and set at the same time as per the sidereal clock but not as per the solar clock for all days of the year.

[B] The Sun rises and sets at the same time as per the solar clock and not as per the sidereal clock for all days of the year.

[C] The stars rise and set at the same time as per the sidereal clock but not as per the solar clock for all days of the year.

[D] The moon rises and sets at different times as per the sidereal clock and also as per the solar clock for all days of the month.

[ANS] C, D

[SOLN] Conceptual

[Q.59] Which of the following stars is/are circumpolar in Warsaw ($52^{\circ}14'N$ $21^{\circ}01'E$)

[A] α Cygni (16h 41m, $+31^{\circ}36'$)

[B] β Böotis (15h 01m, $+40^{\circ}23'$)

[C] θ Aurigae (5h 59m, $+37^{\circ}12'$)

[D] γ Draconis (17h 56m, $+51^{\circ}26'$)

[ANS] B, D

[SOLN] Observer's Latitude (ϕ) :-

$$\phi = 52^{\circ}14' = 53.23^{\circ}N$$

In the NH, $\delta > 90^{\circ} - \phi$ (Condition of Circumpolar)

$$\delta > 37.77^{\circ}$$

$$\delta > 37^{\circ}46'$$

So, only

b & d (β Böotis α Draconis)

(will be circumpolar)

[Q.60] Star A rises half an hour before star B and it sets half an hour after star B from a particular location. Which of the following statement(s) is/are correct?

[A] The location is on equator.

[B] The location is in southern hemisphere and star A is more south than star B.

[C] The location is in northern hemisphere and star A is more north than star B.

[D] Both stars have the same right ascension.

[ANS] B, C, & D

[SOLN] Conceptual