JEE (ADVANCED) 2026 PAPER-2

[PAPER ANSWER KEY WITH SOLUTION]

HELD ON SUNDAY 17TH MAY 2026

MATHEMATICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +3 If **ONLY** the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.

[Q.1] Let $\vec{a}, \vec{b}$ be two vectors, and let P, Q and R be the points with position vectors $\vec{a}, \vec{b}$ and $\vec{a}+\vec{b}$, respectively, with respect to the origin O. If $\left|\vec{a}+\vec{b}\right| = \sqrt{21}$ $\left|\vec{a}-\vec{b}\right| = 3$, and $\vec{a}$ and $\left(\vec{a}-\vec{b}\right)$ are perpendicular to each other, then the area of the triangle OPR is

[A] $\sqrt{3}$ [B] $\frac{\sqrt{3}}{2}$ [C] $\frac{3\sqrt{3}}{2}$ [D] $\frac{3}{2}$

[ANS] **C**

[SOLN] $\left|\vec{a}+\vec{b}\right| = \sqrt{21} \Rightarrow a^2 + b^2 + 2\vec{a}.\vec{b} = 21$

$\left|\vec{a}+\vec{b}\right| = 3 \Rightarrow a^2 + b^2 - 2\vec{a}.\vec{b} = 9$

$\Rightarrow \vec{a}.\vec{b} = 3$

$\vec{a}.\left(\vec{a}-\vec{b}\right) = 0 \Rightarrow a^2 - \vec{a}.\vec{b} = 0$

$a^2 = 3$

$a^2 + b^2 - 2\,\vec{a}.\vec{b} = 9 \Rightarrow a^2 + b^2 = 15$

$\Rightarrow b^2 = 12$

$ar\left(\Delta QPR\right) = \frac{1}{2} \times ab\sin\theta$

$= \frac{1}{2}\sqrt{3} \times 2\sqrt{3} \times \frac{\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2}$

[Q.2] Let T be the tangent to the parabola $y^2 = 16x$ at the point (64, 32). Let L be the tangent to the same parabola at another point (x_1, y_1) on the parabola. If L and T are perpendicular to each other, then the distance between the point (x_1, y_1) and the focus of the parabola, is

[A] $\frac{15}{4}$ [B] 4 [C] $\frac{17}{4}$ [D] 5

[ANS] **C**

[SOLN] $\frac{2}{\frac{1}{L_1} + \frac{1}{L_2}} = 8$

Here, $L_1 = 4 + 64 = 68$

$\Rightarrow \frac{1}{L_1} + \frac{1}{L_2} = \frac{1}{4} \Rightarrow \frac{1}{68} + \frac{1}{L_2} = \frac{1}{4}$

$\Rightarrow \frac{1}{L_2} = \frac{16}{68} = \frac{4}{17} \Rightarrow L_2 = \frac{17}{4}$

[Q.3] Let $y:(-\infty,\infty)\to(0,\infty)$ be the solution of the differential equation

$$\frac{dy}{dx}=\frac{e^{5x}y^3+y^3}{e^x+e^xy^4},$$

satisfying $y(0)=\frac{1}{\sqrt{2}}$. Then the value of $y(\log_e 2)$ is

[A] $\sqrt{\frac{5+\sqrt{35}}{2}}$ [B] $\sqrt{\frac{7+\sqrt{53}}{2}}$

[C] $\frac{7+\sqrt{53}}{2}$ [D] $\frac{5+\sqrt{35}}{2}$

[ANS] **B**

[SOLN] $$\frac{dy}{dx}=\frac{(e^{5x}+1)y^3}{e^x(1+y^4)}$$

$$\Rightarrow \int\frac{1+y^4}{y^3}dy=\int\frac{e^{5x}+1}{e^x}dx$$

$$\Rightarrow -\frac{1}{2y^2}+\frac{y^2}{2}=\frac{e^{4x}}{4}-e^{-x}+c$$

$$x=0,\ y=\frac{1}{\sqrt{2}}\Rightarrow -1+\frac{1}{4}=\frac{1}{4}-1+c\Rightarrow c=0$$

Hence $y^2-\frac{1}{y^2}=\frac{e^{4x}}{2}-2e^{-x}$

Put $x=\ln(2)$ $\quad y^2-\frac{1}{y^2}=8-1=7$

$\Rightarrow y^4-7y^2-1=0,\quad y=\sqrt{\frac{7+\sqrt{53}}{2}}$

[Q.4] The value of the definite integral $\int_0^2\frac{1}{3^x+3}dx$ is

[A] $\frac{1}{2}$ [B] $\frac{1}{3}$ [C] $\frac{\log_3 3}{3}$ [D] $\frac{\log_3 3}{2}$

[ANS] **B**

[SOLN] $$I=\int_0^2\frac{dx}{3^{2-x}+3}=\int_0^2\frac{3^x dx}{9+3.3^x}=\int_0^2\frac{3^{x-1}}{3+3^x}dx$$

$$2I=\int_0^2\frac{1+3^{x-1}}{3+3^x}dx\Rightarrow\int_0^2\frac{dx}{3}\Rightarrow\frac{2}{3}$$

$$\therefore\quad I=\frac{1}{3}$$

SECTION 2 (Maximum Marks : 20)

- This section contains **FIVE (05)** questions.
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chose (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.
- For example, in a question, if [A], [B] and [D] are the ONLY three options corresponding to correct answers, then
  choosing ONLY [A], [B] and [D] will get +4 marks;
  choosing ONLY [A] and [B] will get +2 marks;
  choosing ONLY [A] and [D] will get +2 marks;
  choosing ONLY [B] and [D] will get +2 marks;
  choosing ONLY [A] will get +1 mark;
  choosing ONLY [B] will get +1 mark;
  choosing ONLY [D] will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 marks.

[Q.5] Let $\mathbb{R}$ denote the set of all real numbers. Consider the polynomial function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\frac{d^{10}}{dx^{10}}\left(\left(x^2-10\right)^{10}\right)$, for all $x\in\mathbb{R}$. Here $\frac{d^{10}}{dx^{10}}\left(\left(x^2-1\right)^{10}\right)$ is the 10^{th} order derivative of the function $\left(x^2-1\right)^{10}$. Then which of the following statements is (are) TRUE?

[A] The coefficient of x^8 in the polynomial $f(x)$ is $(-10)\left(\frac{18!}{8!}\right)$

[B] The value of $f(1)+f(-1)$ is equal to $10!2^{11}$

[C] The degree of the polynomial f(x) is 10

[D] The constant term of the polynomial f(x) is $-\left(\frac{10!}{5!}\right)$

[ANS] **A, B, C**

[SOLN] $\left(x^2-1\right)^{10} = 10C_0x^{20} - 10C_1x^{18} + 10C_1x^{16} .... + 10C_{10}$

$$f(x) = \frac{d10}{dx10}\left(x^2-1\right)^{10} = \frac{\lfloor 20x}{\lfloor 1}x^{10} - 10\frac{\lfloor 18}{\lfloor 8}x^8 + 10C_2\frac{\lfloor 16}{\lfloor 6}x^6$$

$$-10C_3\frac{\lfloor 14}{\lfloor 4}x^4 + 10C_4\frac{\lfloor 12}{\lfloor 2}x^2$$

$$-10C_5 \quad \lfloor 10$$

[A] Coefficient of $x^8 = (-10)\frac{\lfloor 18}{\lfloor 8}$

[B] $f(1) + f(-1) = 10!2^{11}$

[C] Degree $\rightarrow 10$

[D] constant term = $= -\frac{(10!)^2}{(5!)^2}$

[Q.6] Let a, b, c be positive integers in arithmetic progression such that the equation $ax^2 + bx + c = 0$ has only integer solutions. Then which of the following statements is (are) TRUE?

[A] $c - b$ is an integer multiple of a

[B] Both the roots of the equation $ax^2 + bx + c = 0$ are odd integers

[C] If c = 15, then ab = 8

[D] If b = 8, then x = 3 is a root of the equation $ax^2 + bx + c = 0$

[ANS] **A, B, C**

[SOLN] Let b = a + d, c = a + 2d

and a divide $a + d \Rightarrow d = ak \left(k \in I\right)$

$$ax^2 + bx + c = 0 \Rightarrow ax^2 + a(1+k)x + a(1+2k) = 0$$

$$\Rightarrow x^2 + (1+k)x + 1 + 2k = 0$$

Now $D = (1+k)^2 - 4(2k+1) = m^2$

$$\Rightarrow k^2 - 6k - 3 = m^2$$

$$\Rightarrow (k-3)^2 - m^2 = 12$$

$\Rightarrow (k-3+m)(k-3-m) = 12 =$

1×12
2×6
3×4

Hene, $(k-3+m)(k-3-m) = 2\times6$ (valid)

$\Rightarrow k-3=4 \Rightarrow k=7$

$\Rightarrow$ Quadratic is $x^2+8k+15=0$

$\Rightarrow$ correct options A, B, C

[Q.7] Let L be the straight line joining the points $P(1,2,-1)$ and $Q(2,3,1)$. Let S be the foot of the perpendicular drawn from the point $R(4,-1,5)$ to the line L. Another line passing through R intersects L at point T such that the point S divides the line segment PT internally in the ratio $|PS|:|ST| = 1:2$, where $|PS|$ and $|ST|$ are the lengths of the line segments PS and ST, respectively. Then which of the following statements is (are) TRUE?

[A] The orthocentre of the triangle PRT is $\left(\frac{23}{5},-4,\frac{31}{5}\right)$

[B] The orthocentre of the triangle PRT is (4, 3, 5)

[C] The area of the triangle PRT is $6\sqrt{5}$

[D] The area of the triangle PRT is $18\sqrt{5}$

[ANS] **A, D**

[SOLN] $\langle PQ\rangle = \langle 1,1,2\rangle$

$PQ = \frac{x-1}{1} = \frac{y-2}{1} = \frac{z+1}{2} \Rightarrow (t+1, t+2, 2t-1)$

Dr of RS $\Rightarrow \langle t-3, t+3, 2t-6\rangle$

$\langle Rs\rangle . \langle PS\rangle = 0$

$\therefore\ t-3+t+3+4t-12=0$

$\therefore\ 6t=12$

$\therefore t=2$

$\therefore S=(3,4,3)$

$\langle RS\rangle = \langle 1,-5,2\rangle$

$\therefore T = (7.8.11)$

Dr of RT $= \langle 3,9,6 \rangle = \langle 1,3,2 \rangle$

Any point on RS $= (t+3, -5t+4, 2t+3)$ Let H

of PH = $(t+2, -5t+2, 2t+4)$

Now, $\langle PH \rangle . \langle RT \rangle = 0 \Rightarrow t+2-15t+6+4t+8=0$

$t = \frac{8}{5}$

$\therefore H = \left(\frac{23}{5}, -4, \frac{31}{5}\right)$

$PT = \sqrt{6^2+6^2+12^2} \Rightarrow 6\sqrt{6}$

RS = $\sqrt{1^2+5^2+2^2} = \sqrt{5}.\sqrt{6}$

$\therefore A = \frac{1}{2} \times 6.6.\sqrt{5} = 18\sqrt{5}$

[Q.8] Let y = f(x) be the real valued function defined on the interval $(0, \infty)$, satisfying y (1) = 0 and the differential equation $x\frac{dy}{dx} = y - x^3$. Then which of the following statements is (are) TRUE?

[A] The function f has a local minimum at $x = \frac{1}{\sqrt{3}}$

[B] The function f has a local maximum at $x = \frac{1}{\sqrt{3}}$

[C] The function f is increasing in the interval (1, 2)

[D] If $g(x) = 4x^3 - 5x^2 + \frac{3}{2}x$ for x > 0, then the number of elements in the set $\{x \in (0, \infty) : f(x) = g(x)\}$ is 2

[ANS] **BD**

[SOLN] $x.\frac{dy}{dx} = y - x^3, y(1) = 0$

$\frac{dy}{dx} = \frac{y}{x} = \frac{-x^3}{x} \Rightarrow \frac{dy}{dx} - \frac{y}{x} = -x^2$

I.F. = $e^{\int -1/x\,dx} = e^{-\ln x} = 1/x$

So : y.IF = $\int Q.IFdx$

$\frac{y}{x} = \int -x dx \Rightarrow \frac{y}{x} = \frac{-x^2}{2} + c$

$y = cx - \frac{x^3}{2}$, $y(1) = 0$

$\therefore \quad c = \frac{1}{2}$

$y = \frac{x - x^3}{2}$

$y_1 = 1 - 3x^2 = 0$

$x = \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$

y' > 0 for $x < -\frac{1}{\sqrt{3}}$ and $x > \frac{1}{\sqrt{3}}$

y' > 0 for $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

[D] $4x^3 - 5x^2 + \frac{3}{2}x = \frac{x}{2} - \frac{x^3}{2}$

$9\frac{x^3}{2} - 5x^2 + x = 0 \Rightarrow 9x^2 - 10x + 2 = 0$

[Q.9] Let $\mathbb{R}$ denote the set of all real numbers and let $i = \sqrt{-1}$. Consider the matrices

$s = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Let a, b, c, d be real numbers such that $ST = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Let $H = \{x + iy : x, y \in \mathbb{R} \text{ and } y > 0\}$. Then which of the following statements is (are) TRUE ?

[A] $\frac{b + ia}{d + ic} = i$

[B] If $\omega = \frac{-1 + i\sqrt{3}}{2}$, then $\frac{a\omega + b}{c\omega + d} = \omega$

[C] If m is an integer greater then 2 such that $(ST)^2 = (ST)^m$, then m is an integer multiple of 8

[D] If $z \in H$, then $\frac{az + b}{cz + d} \in H$

[ANS] **B, D**

[SOLN] $ST = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore\ a = 0,\ b = -1,\ c = 1,\ d = 1$

[A] $\dfrac{-1+0}{1+i} \neq i$

[B] $\dfrac{0+(-1)}{\omega+1} = \dfrac{-1}{-\omega^2} = \dfrac{1}{\omega^2} = \omega$

[C] $(ST)^2 = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$

$$(ST)^3 = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$(ST)^4 = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$$

$$(ST)^5 = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

$$(ST)^7 = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}$$

$$(ST)^8 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$$

[D] Let $Z = x + iy, y > 0.$

$$\frac{az+b}{cz+d} = \frac{ax+aiy+b}{cx+ciy+d},\ \text{Imaginary Part is :}$$

$$= \frac{-1}{x+1+iy} \times \frac{(x+1)-iy}{x+1-iy} = \frac{(x+1)}{(x+1)^2+y^2} + \frac{iy}{(x+1)^2+y^2} \in H$$

SECTION 3 (Maximum Marks : 20)

- This section contains **FIVE (05)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value of to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : **+4** If ONLY the correct numerical value is entered in the designated place;
  Zero Marks **: 0** In all other cases.

[Q.10] Let $\mathbb{N}$ denote the set of all positive integers. Consider the sets $A=\{1,2,3,4,5\}$ and $B=\{1,2,3,4,5,6,7\}$. Let S be the set of all functions $f:A\to B$ such that $f(2)\neq 2$ and $f(4)\neq 4$. Consider the set T = { $f\in S$: there exists a function $g:B\to\mathbb{N}$ such that $g(f(x))=2^x$ for all $x\in A$ }. Then the number of elements in the set T is ___________.

[ANS] **1860**

[SOLN] $g(f(x))=2^x$

$\to$ function is one – one

No. of one – one function in $f:A\to B=\ 7p_5=2520$

If $f(2)=2$

Then no. of one – one function $={}^6P_4\ =360$

If $f(4)=4$

Then no. of one – one function $={}^6P_4\ =360$

If $f(2)=2$ and $f(4)=4$

Then no. of one – one function $={}^5P_3\ =60$

Hence total no. of element $T=2520-(360+360-60)=1860$

[Q.11] A bookshelf contains 6 distinct books of Mathematics and 5 distinct books of Physics. From these 11 books, 6 books are chosen at random. Let X be the absolute value of the difference between the number of Mathematics books chosen and the number of Physics books chosen. If α is the mean of the random variable X, then the value of 77α is ___________.

[ANS] **100**

[SOLN] M $\to$ Math books P-Physic book

$M+P=6$ ways of selection

$1+5\to {}^6C_1\times{}^5C_5=6$

$2+4\to {}^6C_2\times{}^5C_4=75$

$3+3\to {}^6C_3\times{}^5C_3=200$

$4+2\to {}^6C_4\times{}^5C_2=150$

$5+1\to {}^6C_5\times{}^5C_1=30$

$6+0\to {}^6C_6\times{}^5C_0=1$

X	F
0	200
2	225
4	36

$\Rightarrow \alpha = \dfrac{0\times200+2\times225+4\times36+6\times1}{462}$

$= \dfrac{600}{462} = \dfrac{200}{154} = \dfrac{100}{77}$

$\Rightarrow 77\alpha = 100$

[Q.12] Consider a data consisting of 10 observations $x_1, x_2, ..., x_{10}$, whose mean is 5 and variance is 7. If the mean and the variance of the first 8 observations $x_1, x_2, ..., x_8$ are 4 and 3.5, respectively, and $x_9 < x_{10}$, then the value of $3x_9 + 2x_{10}$ is ___________.

[ANS] **44**

[SOLN] $\sum_{x=1}^{8} x_i = 32$, $\frac{1}{8}\sum_{x=1}^{8} x^2 - (4)^2 = 3.5$

$\Rightarrow \frac{1}{8}\sum_{x=1}^{8} x_i^2 = 8\times19.5 = 156$

$32 + x_9 + x_{10} = 10\times5 = 50$

$\Rightarrow x_9 + x_{10} = 18$ .... (1)

$\dfrac{\sum_{x=1}^{8} x_i^2 + x_9^2 + x_{10}^2}{10} - 25 = 7$

$\Rightarrow 156 + x_9^2 + x_{10}^2 = 320$

$x_9^2 + x_{10}^2 = 164$ .... (2)

From (1) and (2) $x_9 = 8, x_{10} = 10$

$\Rightarrow 3x_9 + 2x_{10} = 24 + 20 = 44$

[Q.13] Consider the ellipse E given by $\frac{x^2}{18} + \frac{y^2}{12} = 1$. Let H be the hyperbola whose eccentricity is the reciprocal of the eccentricity of E and whose foci are the same as that of E. Let P and Q be the points of intersection of H and the parabola $\sqrt{5}y = x^2$ in the first quadrant. Let d be the distance between P and Q.

If a and b are the integers such that $d^2 = a + b\sqrt{5}$, then the value of a – b is __________.

[ANS] **18**

[SOLN] $e_1^2 \to 1-\frac{2}{3}=\frac{1}{3} \Rightarrow e_1\frac{1}{\sqrt{3}} \Rightarrow ae_1=\sqrt{18}\times\frac{1}{\sqrt{3}}=\sqrt{6}$

$e_2=\sqrt{3}$

Foci of ellipse $\left(\pm\sqrt{6},0\right)$. Also of Hyperbola

Now, $\Rightarrow A\,e_2=\sqrt{6} \Rightarrow A=\sqrt{2}$

$B^2+A^2=A^2e_2^2 \Rightarrow B^2+2=6$

$\Rightarrow B^2=4$

$\Rightarrow$ Hyperbola in $\frac{x^2}{2}-\frac{y^2}{4}=1$

Solve with $\sqrt{5}y=x^2$

$\Rightarrow 2\sqrt{5}y-y^2=4$

$\Rightarrow y^2-2\sqrt{5}y+4=0$

$y_1+y_2=2\sqrt{5},\ y_1y_2=4$

$x_1=y_1 \Rightarrow y_1-y_2=2$

$\Rightarrow y_1=\sqrt{5}+1, y_2=\sqrt{5}-1$

Hence $x_1^2=\sqrt{5}\left(\sqrt{5}+1\right) \Rightarrow x_1=\sqrt{5+\sqrt{5}}$

$x_1^2=\sqrt{5}\left(\sqrt{5}-1\right) \Rightarrow x_1=\sqrt{5-\sqrt{5}}$

$d^2=\left(\sqrt{5+\sqrt{5}}-\sqrt{5-\sqrt{5}}\right)^2+(2)^2$

$=5+\sqrt{5}+5-\sqrt{5}-2\sqrt{25-5}+4$

$=14-4\sqrt{5}$

a – b = 18

[Q.14] For a real number α, let $[\alpha]$ denote the greatest integer less than or equal to α. For a finite set S, let $|S|$ denote the number of elements in the set S.

Consider the functions $f:(-3,3)\to(-\infty,\infty)$ and $g:(-3,3)\to(-\infty,\infty)$ defined by

$f(x)=\left[x^3\right]\log_e\left(1+\sin^2\left(\pi\left(x-[x]\right)\right)\right)$ and $g(x)=x^3\sin^2\left(\pi\log_e\left(1+x-[x]\right)\right)$.

Let A = $\{x\in(-3,3): f \text{ is discontinuous at } x\}$ and B = $\{x\in(-3,3): g \text{ is discontinuous at } x\}$.

Then the value of $|A|+2|B|-|A\cap B|$ is ____________.

[ANS] **56**

[SOLN] $f(x)=[x^3] \ln(1+\sin^2 \pi\{x\})$

Now, $-3<x<3 \Rightarrow -27<x^3<27 \Rightarrow [x^3]$ has 53 values

Para – I

$\ln(1+\sin^2 \pi\{x\})=0$ at $x=-2,-1,0,1,2$

No. of points f is discontinuous = 53 – 5 = 48

$g(x)=x^3 \sin^2(\pi \ln(1+\{x\}))$

is discontinuous at $x=-2,-1,1,2$

$\Rightarrow$ parts of discontinuity = 4

$\Rightarrow |A|=48,\ |B|=4,\ |A\Lambda B|=0,$

$|A|+2|B|-|A\Lambda B|=48+2\times 4-0=56$

SECTION 4 (Maximum Marks : 8)

- This section contains **TWO (02)** questions stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value of to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : **+2** If ONLY the correct numerical value is entered in the designated place;
  Zero Marks **: 0** In all other cases.

Question Stem for Question Nos. 15 and 16

Consider the curve C_1 given by $y=e^{-x}$ for $x\in[0,10\pi]$, and the curve C_2 given by $y=e^{-x}(\sin x+\cos x)$ for $x\in[0,10\pi]$.

Let n be the total number of points of intersection of the curves C_1 and C_2. Suppose that $\alpha_1,\alpha_2,\ldots,\alpha_n\in[0,10\pi]$ are the x – coordinates of the points of intersection of the curves C_1 and C_2 such that $\alpha_1<\alpha_2<\ldots<\alpha_n$.

[Q.15] The value of n is ___________.

[ANS] **11**

[SOLN] $e^{-x}=e^{-x}(\sin x+\cos x)$

$e^{-x}=0$ or $\sin x+\cos x=1$

$\cos\left(x-\frac{\pi}{4}\right)=\cos\frac{\pi}{4}$

$$x-\frac{\pi}{4}=2n\pi\pm\frac{\pi}{4}$$

$$x=2n\pi+\frac{\pi}{2},2n\pi$$

$$\Rightarrow n=6+5=11$$

[Q.16] Let β be the area of the region enclosed between the curves C_1, C_2, and the lines $\alpha=\alpha_1$ and $\alpha=\alpha_4$. Then the value of $-\frac{1}{\pi}\log_e\left(\beta-2e^{-\frac{\pi}{2}}\right)$ is __________ .

[ANS] **2.5**

[SOLN] $\alpha_1=0, \alpha_2=\frac{\pi}{2}, \alpha_3=2\pi, \alpha_4=2\pi+\frac{\pi}{2}$

$$\beta=\left|\int_0^{\pi/2}e^{-x}(\sin x+\cos x-1)dx\right|+\left|\int_{\frac{\pi}{2}}^{2\pi}e^{-x}(\sin x+\cos x-1)dx\right|$$

$$+\left|\int_{2\pi}^{2\pi+\pi/2}e^{-x}(\sin x+\cos x-1)dx\right|$$

$$\left|\left[-e^{-x}\cos x+e^{-x}\right]_0^{\pi/2}\right|+\left|\left[e^{-x}(1-\cos x)\right]_{\pi/2}^{\pi}\right|+\left|\left[e^{-x}(1-\cos x)\right]_{2\pi}^{2\pi+\pi/2}\right|$$

$$=e^{-\pi/2}+e^{-\pi/2}+e^{-2\pi-\frac{\pi}{2}}$$

$$=2e^{-\frac{\pi}{2}}+e^{-\frac{\pi}{2}}e^{-2\pi}=e^{-\pi/2}\left(e^{-2\pi}+2\right)$$

$$-\frac{1}{\pi}\ln\left(\beta-2e^{-\pi/2}\right)=\frac{-1}{\pi}\ln\left(e^{-2\pi-\frac{\pi}{2}}\right)$$

$$=-\frac{1}{\pi}\left(-2\pi-\frac{\pi}{2}\right)$$

$$=2+\frac{1}{2}=\frac{5}{2}$$

Question Stem for Question Nos. 17 and 18

Consider the ellipses given by $x^2+4y^2=1$ and $4x^2+y^2=1$.

[Q.17] Let P be the point in the first quadrant where the given ellipses intersect. If θ is the acute angle between the tangents to the given ellipses at the point P, then the value of $4\tan\theta$ is ___________.

[ANS] **7.5**

[SOLN] $x^2+4y^2=1$ and $4x^2+y^2=1$

Solviny $16x^2+1-x^2=4=x^2=\frac{1}{5}$

Point of intersection $\left(\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)$

Tangent of $x^2+4y^2=1$ at $\left(\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)$

is $x+4y=\sqrt{5}\Rightarrow m_1=\frac{-1}{4}$

Tangent to $4x^2+y^2=1$ at $\left(\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)$

is $4x+y=\sqrt{5}\Rightarrow m_2=-4$

$$\Rightarrow \tan\theta=\left|\frac{4-\frac{1}{4}}{1+1}\right|=\frac{15}{8}$$

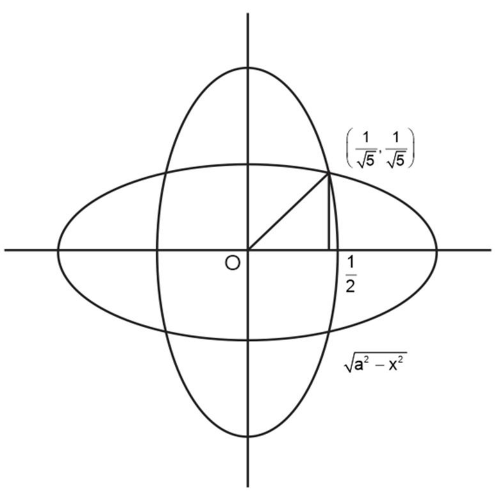
$$\Rightarrow 4\tan\theta=\frac{15}{8}=7.5$$

[Q.18] I f α is the area of the common region that lies inside both the given ellipses, then the value of cot α is ___________.

[ANS] **0.75**

[SOLN] Required area

$$\alpha=8\left(\frac{1}{2}\times\frac{1}{\sqrt{5}}\times\frac{1}{\sqrt{5}}+\int_{\sqrt{5}}^{1/2}\sqrt{1-4x^2}\,dx\right)$$



$$=\frac{4}{5}+\frac{8}{2}\left[x\sqrt{1-4x^2}+\frac{1}{2}\sin^{-1}(2x)\right]_{1/5}^{1/2}$$

$$=\frac{4}{5}+4\left[\frac{1}{2}\sin^{-1}(1)-\frac{1}{\sqrt{5}}\times\frac{1}{\sqrt{5}}-\frac{1}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)\right]$$

$$=\pi-2\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)=2\left(\frac{\pi}{2}-\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)=2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)=2\cot^{-1}(2)$$

$$\rightarrow \alpha=2\cot^{-1}(2)\Rightarrow \cot\frac{\alpha}{2}=2$$

$$\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}=\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}=\frac{4}{3}$$

$$\cot\alpha=\frac{3}{4}$$

CHEMISTRY

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

[Q.1] At 300 K, the molar conductivities of the aqueous solutions of three salts at two different concentrations are given below:

Salt	Concentration (M)	Molar conductivity ($S\ cm^2\ mol^{-1}$)
$NaNO_3$	0.01	111
	0.04	101
NaCl	0.01	117
	0.04	107
$AgNO_3$	0.01	125
	0.04	116

The conductivity of a saturated aqueous solution of AgCl is 1.40×10^{-6} S cm^{-1} at 300 K. If the solubility of AgCl in water at 300 K is X mol L^{-1}, then $\log_{10}(X^{-1})$ is (Assume that AgCl dissolved in water ionizes completely and that the molar conductivity of saturated AgCl solution is equal to its limiting molar conductivity.)

[A] 3 [B] 4 [C] 5 [D] 6

[ANS] **C**

[SOLN] For $NaNO_3$

$\lambda m_1 = \lambda m^o - b\sqrt{c}$

$111 = \lambda m^o - b\sqrt{10^{-2}}$ ----(i)

$101 = \lambda m^o - b\sqrt{4 \times 10^{-2}}$ ----(ii)

$\therefore 10 = -b \times 10^{-1} + b \times 2 \times 10^{-1}$

$10 = 10^{-1} \times b$

$b = 100$

$\therefore \lambda m^o = 111 + 100 \times 0.1 = 121$

For NaCl

$117 = \lambda m^o - b \times 10^{-1}$ ------(i)

$107 = \lambda m^o - b \times 2 \times 10^{-1}$ ----(ii)

$10 = b \times 10^{-1}$

$\therefore b = 100$

$\therefore \lambda m^o = 117 + 100 \times 10^{-1} = 127$

For $AgNO_3$

$125 = \lambda m^o - b \times 10^{-1}$

$116 = \lambda m^o - b \times 2 \times 10^{-1}$

$\therefore 9 = b \times 10^{-1} \quad \therefore b = 90$

$\therefore \lambda m^o = 125 + 8 = 134$

$\therefore$ For AgCl

$\Rightarrow \lambda m^o\ NaCl + \lambda m^o AgNO_3 - \lambda m^o NaNO_3$

= 127+ 134 – 121

$\lambda m^o_{AgCl} = 140$

$\therefore K = 1.4 \times 10^{-6}\ Scm^{-1}$

T = 300 K

$$140 = \frac{1.4 \times 10^{-6} \times 1000}{S}$$

S = 10^{-5}

$\therefore -\log x = 5$

[Q.2] The correct order of ONO bond angle in the given species is

[A] $NO_2^+ < NO_2 < NO_3^- < NO_2^-$ [B] $NO_2^- < NO_3^- < NO_2 < NO_2^+$

[C] $NO_3^- < NO_2^- < NO_2 < NO_2^+$ [D] $NO_2^- < NO_3^- < NO_2^+ < NO_2$

[ANS] **B**

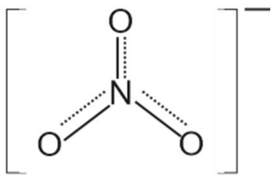
[SOLN] $NO_2^+ \rightarrow S.N = \frac{5-1}{2} = 2$ = 2 Bond pair, 0 Lone pair

Hybridization = sp – linear, Bond angle = 180°

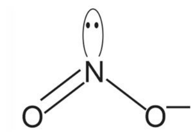
$[O = N = O]^+$

$NO_3^-, S.n = \frac{5+1}{2} = 3$ = 3 Bond pair, 0 lone Pair – sp^2 hybridization

Bond angle – 120°



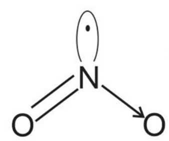
$NO_2^- - S.n = \frac{5+1}{2} = 3 - sp^2$ hybridized, Bent shape



- There is l.p – b.p repultion bond angle less than 120° (117°)

NO_2 = radical, 2 bond pair + 1 radical – sp^2

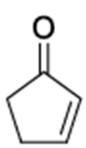
Bent Shape

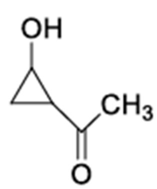


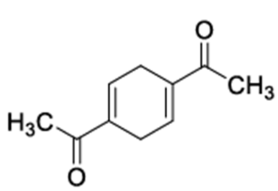
– Since radical does not cause large Repulsion bond angle > 120° (134°)

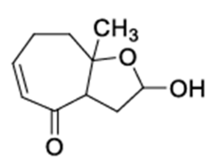
Correct $NO_2^- < NO_3^- < NO_2 < NO_2^+$

[Q.3] Natural rubber on complete ozonolysis (O_3/Zn-H_2O) gives compound X as the major product. X gives positive iodoform and Tollen's tests. X on heating with aqueous NaOH gives Y as the major product. Y is

[A] 

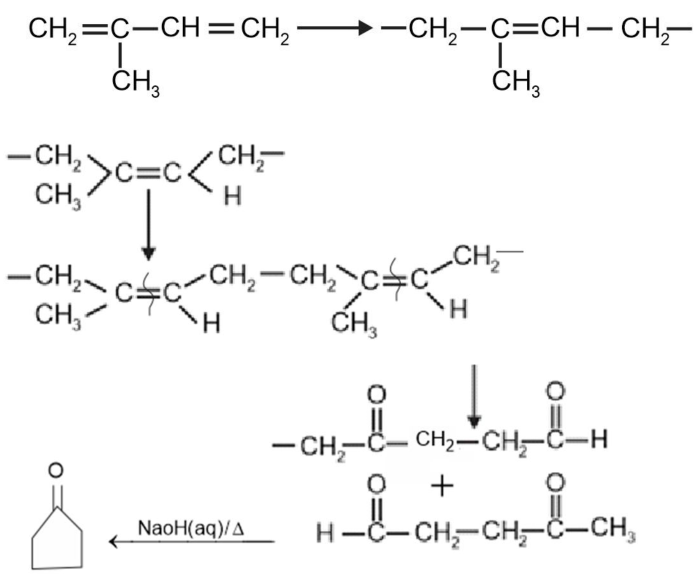
[B] 

[C] 

[D] 

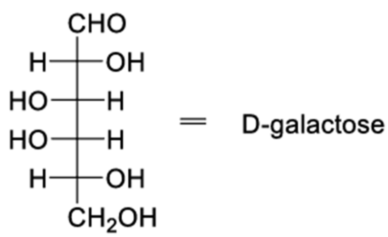
[ANS] **A**

[SOLN]

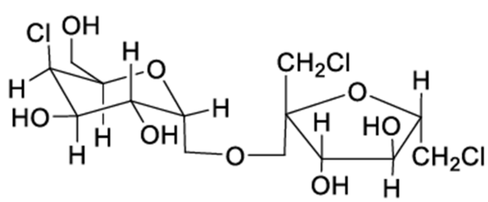


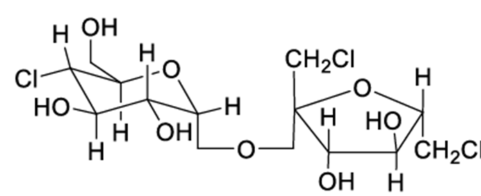
Intramolecular Aldol Condensation

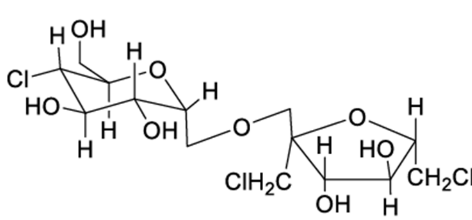
[Q.4] A known artificial sweetener X is composed of 4-chloro-4-deoxy-α-D-galactose and 1,6-dichloro 1,6-dideoxy-β-D-fructose joined by a glycosidic linkage. Structure of D-galactose is given below:

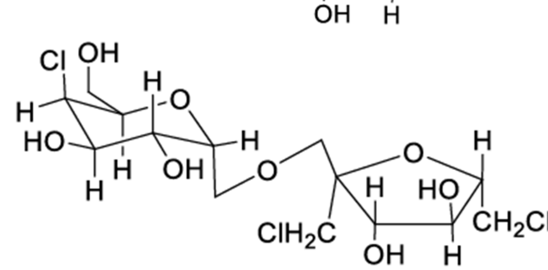


The correct structure of X is

[A] 

[B] 

[C] 

[D] 

[ANS] **A**

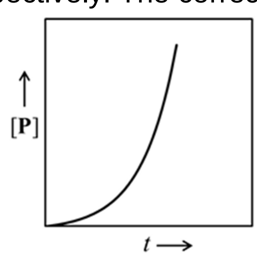
[SOLN] 4- Chloro 4 – deoxy - $\alpha - D - \text{Galactose} + 1{,}6 - \text{dichloro} 1{,}6 - \text{dideoxy} \beta - D - \text{Fructose}$

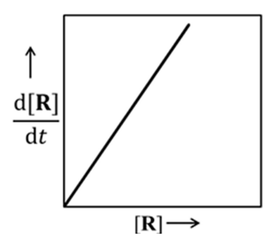
Joined by $C_1 - C_2$

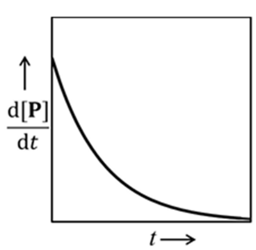
SECTION 2 (Maximum Marks : 20)

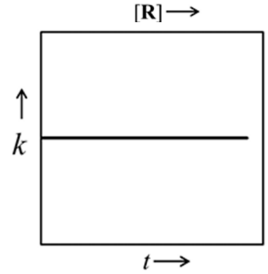
- This section contains **FIVE (05)** questions.
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chose (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.
- For example, in a question, if [A], [B] and [D] are the ONLY three options corresponding to correct answers, then
  choosing ONLY [A], [B] and [D] will get +4 marks;
  choosing ONLY [A] and [B] will get +2 marks;
  choosing ONLY [A] and [D] will get +2 marks;
  choosing ONLY [B] and [D] will get +2 marks;
  choosing ONLY [A] will get +1 mark;
  choosing ONLY [B] will get +1 mark;
  choosing ONLY [D] will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 marks.

[Q.5] For a first-order reaction $R \rightarrow P$ at a given temperature, k is the rate constant. For this reaction, at the given temperature, the concentrations of R and P at a time t are [R] and [P], respectively. The correct graphical representation(s) for this reaction is(are)

[A] 

[B] 

[C] 

[D] 

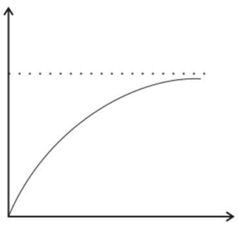
[ANS] **C & D**

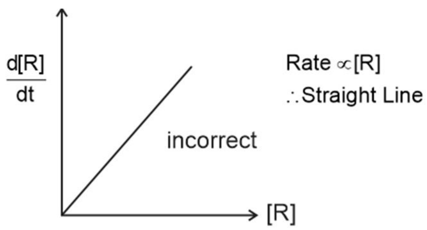
[SOLN] $R \rightarrow P$ 1^{st} order

P vs t

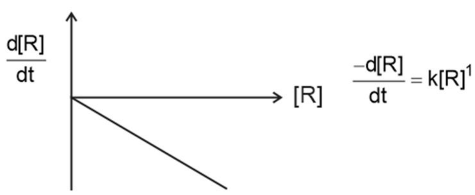
$P = A_o(1 - e^{-kt})$

$\therefore$





or

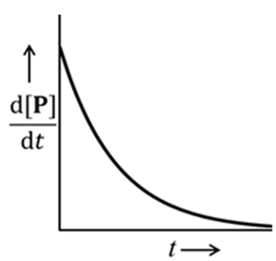


$\frac{dp}{dt}$ vs t

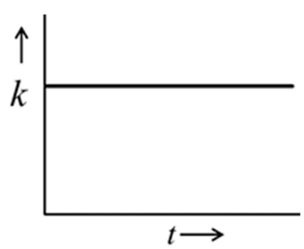
$P = A_o(1 - e^{-kt})$

$$\therefore \frac{dp}{dt} = \frac{d(A_o - A_o e^{-kt})}{dt} = +A_o \times ke^{-kt}$$

[C]



[D]



[Q.6] Correct statement(s) about the compounds P, Q and R is(are)

$$Xe(g) + F_2(g) \xrightarrow{873\text{ K, 7 bar}} P$$
(1 : 5 ratio)

$$P + O_2F_2 \xrightarrow{143\text{ K}} Q + O_2$$

$$Q + H_2O \xrightarrow{\text{complete hydrolysis}} R + HF$$

[A] P has two lone pairs of electrons on the central atom.
[B] Q has a perfect octahedral geometry.
[C] Q can act as a fluorinating agent.
[D] The molecular structure of R is trigonal pyramidal.

[ANS] **ACD**

[SOLN]
$$\underset{1}{Xe(g)} + \underset{5}{F_2(g)} \xrightarrow{873K} XeF_4(P)$$

$$\underset{(P)}{XeF_4} + O_2F_2 \xrightarrow{143K} \underset{(Q)}{XeF_6} + O_2$$

$$\underset{(Q)}{XeF_6} + 3H_2O \xrightarrow[\text{hydrolysis}]{\text{Complete}} \underset{(R)}{XeO_3} + 6HF$$

[A] XeF_4 – 4 bond pair, 2 lone pair
[B] $Q - XeF_6 -$ distorted octahedral
[C] $Q - XeF_6 -$ Fluorinating agent
[D] $R - XeO_3 -$ 3 bond pair, 1 lone pair, Geometry – tetrahedral, shape- Trigonal pyramidal

[Q.7] The correct statement(s) regarding the periodic properties of elements is(are)

[A] Second ionization enthalpy of carbon atom is less than that of boron atom.
[B] Increasing order of ionic radii: $Al^{3+} < Mg^{2+} < Na^+$
[C] Under identical conditions, in solid state, the density of potassium metal is more than density of sodium metal.
[D] The H–H bond is weaker than F–F bond.

[ANS] **AB**

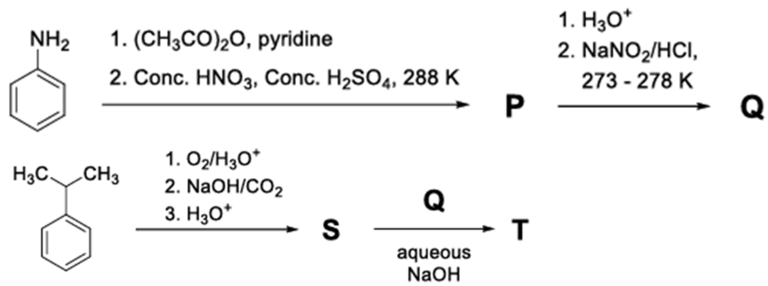
[SOLN] [A] I.E_2 , C – $2s^2\,2p^1$ – Second electron remove from $2p^1$
B = $2s^2$ – Second electron remove from $2s^2$, since penetration power of 2 s greater than 2p hence $IE_2(B > C)$

[B] Ionic radii $\propto \frac{-\text{ve charge}}{+\text{ve charge}}, Al^{3+} < Mg^{2+} < Na^+$

[C] Density $\propto \frac{m}{v}$, $v \propto$ size, $K > Na$, Denisty, $K < Na$

[D] H- H bond stronger than F- F because in F_2 there is into electronic repulsion.

[Q.8] In the following reaction sequence, P, Q, S and T are the major products.



[A] Q on treatment with ethanol generates an aromatic aldehyde.

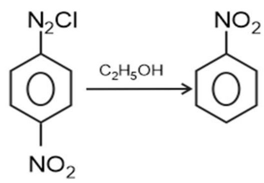
[B] S gives positive phthalein dye test.

[C] P is a dinitro compound.

[D] T is a colored compound.

[ANS] **BD**

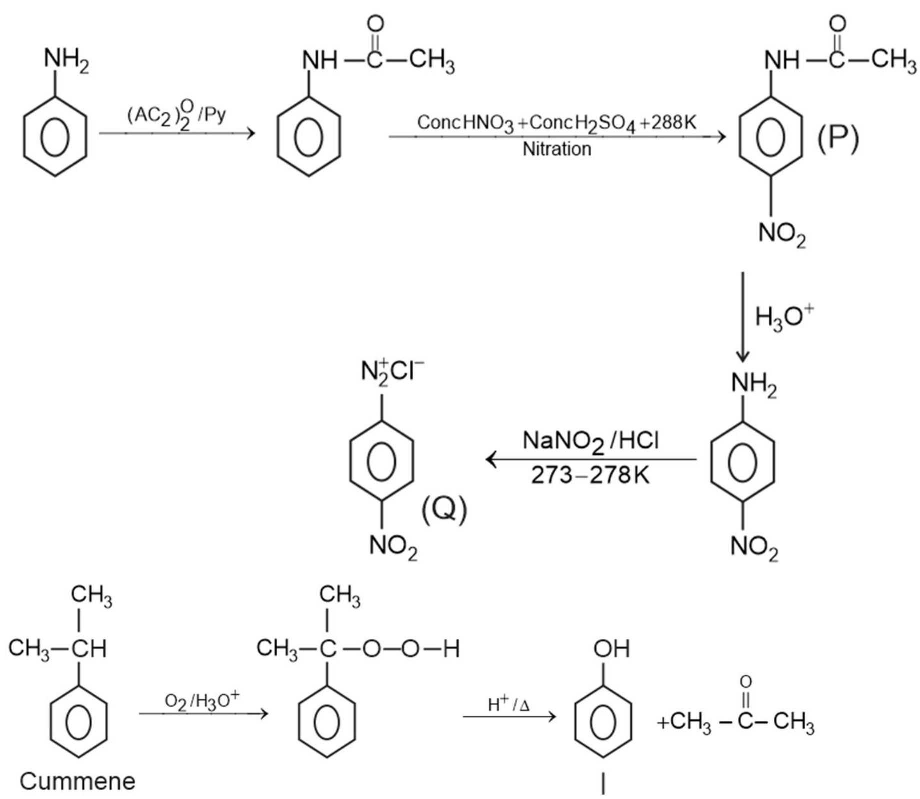
[SOLN] [A]

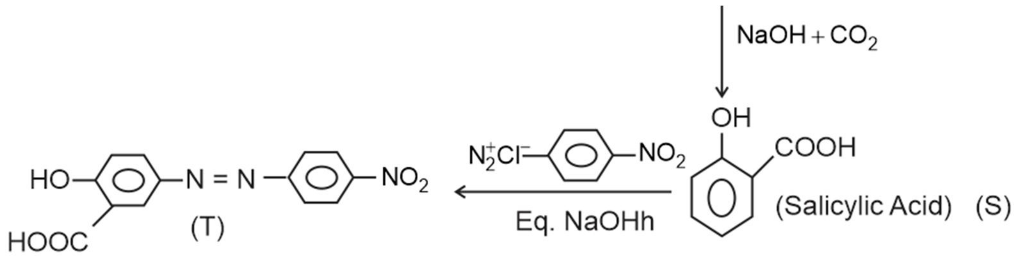


[B] Due to Phenolic give Phthalein test

[C] P 'Having' 1 – nitro

[D] Azo dye





[Q.9] The correct statement(s) regarding sugar is (are)

Given: Specific rotations of L-(−)-glucose and L-(+)-fructose are −52.5° and +92.5°, respectively.

[A] On treatment with HNO_3, gluconic acid is oxidized to saccharic acid, whereas glucose is not oxidized to saccharic acid.

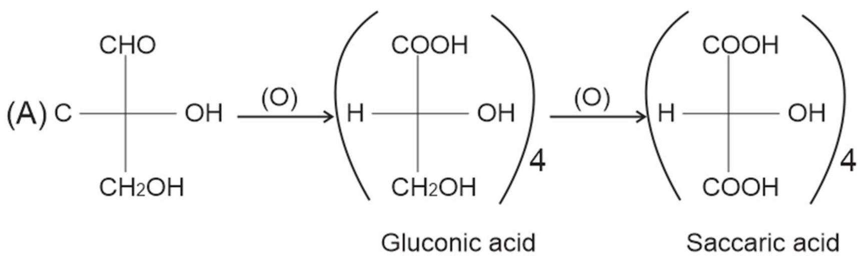
[B] Fructose gives a positive Fehling's test because it isomerises to glucose and another aldohexose in the presence of Fehling's reagent.

[C] Invert sugar is an equimolar mixture of D-glucose and D-fructose formed after hydrolysis of the corresponding disaccharide.

[D] Specific rotation of invert sugar is −40°.

[ANS] **BC**

[SOLN]



B. Fructose + Ve Fehling Test

C. Invert Sugar in Eq. Molar mixture of D-Glucose, D – Fructose

D. Specific average rotation of D- Glucose + Fructose

SECTION 3 (Maximum Marks : 20)

- This section contains **FIVE (05)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value of to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : **+4** If ONLY the correct numerical value is entered in the designated place;
  Zero Marks **: 0** In all other cases.

[Q.10] X^{a+} and Y^{b+} are hydrogen-like species. The wavelength of light absorbed during the transition between the states with principal quantum numbers n = 1 and n = 2 of X^{a+} is λ. The wavelength of light absorbed during the transition between the states with principal quantum numbers n = 2 and n = 4 of Y^{b+} is 9λ. The lowest possible value of (a+b) is ____.

[ANS] **3**

[SOLN] X^{a+} Y^{b+}

n = 1 & n = 2 n = 2 to n = 4

Since λ 9λ

Lowest possible value of a + b

$Z \text{ of } x \rightarrow Z_1$

$\therefore Z_1 - a = 1$

$\Rightarrow \therefore Z_1 = 1 + a$

$Z \text{ of } y = Z_2$

$Z_2 - b = 1$

$\therefore Z_2 = 1 + b$

$$\frac{1}{\lambda} = R_H \times Z_1^2\left(\frac{1}{1} - \frac{1}{Z^2}\right) = \left(R_H Z_2^2 \times \frac{3}{4}\right) = \frac{1}{X}$$

$$\frac{1}{9\lambda} = R_H \times Z^2\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{R_H Z_2^2}{4}\left[\frac{3}{4}\right] = \frac{1}{9X}$$

$$\therefore \frac{4Z_1^2}{Z_2^2} = 9$$

$$\therefore \frac{Z_1^2}{Z_2^2} = \frac{3}{2} = \frac{Z_1}{Z_2} = \frac{3}{2}$$

$$\frac{1+a}{1+b} = \frac{3}{2}$$

$\therefore a = 2, b = 1 \therefore a + b = 3$

[Q.11] At a given temperature, 0.45 g of acetic acid in 50 mL of water is shaken with 1.0 g of charcoal and the pH of the resulting solution is 3.0. Assume, the adsorption of acetic acid from the aqueous solution by charcoal follows Freundlich isotherm,

$$\frac{x}{m} = kC^{1/n}$$

If the plot of log10(x/m) against $\log_{10}C$ gives a straight line with slope 1, the value of k in L mol^{-1} is ____.

Given: The molar mass of acetic acid is 60 g mol^{-1}.

The acid dissociation constant of acetic acid is 1.0×10^{-5} at the given temperature.

x is the mass (in grams) of acetic acid adsorbed. m is the mass (in grams) of charcoal.

C is the equilibrium concentration of acetic acid in the solution after the adsorption is complete.

k and n are constants for acetic acid–charcoal system at the given temperature.

[ANS] **1.5**

[SOLN] 0.45 g CH_3COOH in 50 ml —— $C_{initial} = \frac{0.45/60}{50/1000} = 0.15M$

+

1 g changed , pH = 3

$$\log\frac{x}{m} = \log K + \frac{1}{n}\log C$$

$$\frac{1}{n} = 1 \therefore n = 1$$

$$K_a = 10^{-5}$$

Since $PH = 3 \rightarrow [H^+] = 10^{-3}$

$$\therefore \underset{C}{CH_3COOH} \rightleftharpoons \underset{10^{-3}}{CH_3COO^-} + \underset{10^{-3}}{H^+}$$

$$\therefore \frac{10^{-6}}{10^{-5}} = C = 10^{-1} mol\ L^{-1} effective$$

$$\frac{x}{m} = KC^{1/x} = \frac{0.15}{1} = K(0.1) = K = 1.5$$

[Q.12] In a solvent S, a compound B is partially dissociated into C and D as given below:

$$\mathbf{B} \rightleftharpoons 2\mathbf{C} + 2\mathbf{D}$$

B, C and D are non-volatile in nature. The molar mass of B is 10 times the molar mass of S. The standard boiling point and the standard enthalpy of vaporization of S are 400 K and 10R J mol^{-1}, respectively (R is the gas constant in J K^{-1} mol^{-1}). A solution of B in S with an initial concentration of B as 0.25% (mass/mass) has a boiling point of 408 K at 1 bar pressure. In this solution, the mole percent of B that has been dissociated is ____.

[ANS] **33.167**

[SOLN] $B \rightleftharpoons 2C + 2D$

non – volatile

$10 \times M_S$

$T_b = 400K$

$\Delta H_V = 10K$

$B \rightleftharpoons 2C + 2D$

0.25 %

w/w = 0.25 %

Solution T_b = 408 K, at 1 bar

Find mole of % B dissociated .

$$K_b = \frac{R T_b^2 \times M}{\Delta H_V \times 1000}$$

$$\therefore K_b = \frac{16000 \times Ms \times R}{10R \times 1000}$$

$$\therefore K_b = 16 Ms$$

$$\therefore \Delta T_b = 8$$

$$\therefore 8 = i \times K_b \times m$$

$$8 = i \times 16 \times Ms \times \frac{0.25 \times 1000}{10 \times 99.75 M_s}$$

w/w o/o = 0.25

100 g solution —— 0.25 g of B

$$\therefore n_B \left(\frac{0.25}{10 M_s}\right)$$

100 – 0.25 g = 99.75 g

$$\therefore m = \frac{0.25 / 10 M_s}{99.75 / 1000}$$

$$8 \times i \times 16 \ Ms \times \frac{25}{99.75 Ms}$$

$$i = \left(\frac{99.75}{50}\right)$$

$$\underset{1-\alpha}{B} \rightleftharpoons \underset{2\alpha}{2C} + \underset{2\alpha}{2D}$$

$$\therefore i = 1 + \alpha + 2\alpha = 1 + 3\alpha$$

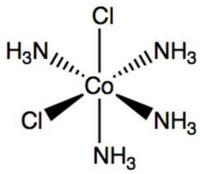
$$\frac{1.995 - 1}{3} = \alpha = 0.331666$$

$$\alpha \text{ of } / o = 33.167$$

[Q.13] Consider that the coordinating atoms of the ligands in cis-$[Co(NH_3)_4Cl_2]Cl$ and mer-$[Co(NH_3)_3Cl_3]$ octahedral complexes are at the vertices of an octahedron. The sum of total number of the triangular faces in both the complexes having one N atom and two Cl atoms at their corners is____.

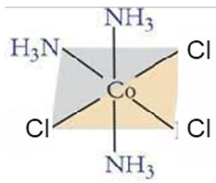
[ANS] **6**

[SOLN] $[Co(NH_3)_4Cl_2]Cl$



Trigonal faces contains 2 Cl and 1 N = 2

mer-$[Co(NH_3)_3Cl_3]$



= Trigunal faces contains 2 Cl and 1 N = 4

[Q.14] In the following reaction sequence, major products X and Y are acyclic monomers.

$$CH_3I \xrightarrow[]{\text{1. KCN; 2. } H_3O^+, \Delta\text{; 3. Red P, } Br_2\text{; 4. } NH_3 \text{ (excess)}} X$$

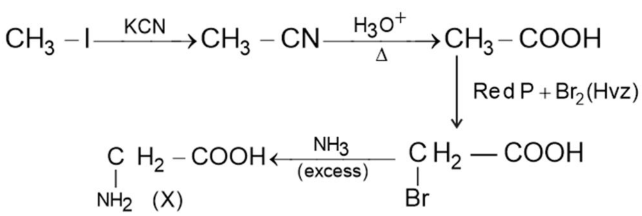
$$\text{Caprolactam} \xrightarrow[\Delta]{H_3O^+} Y$$

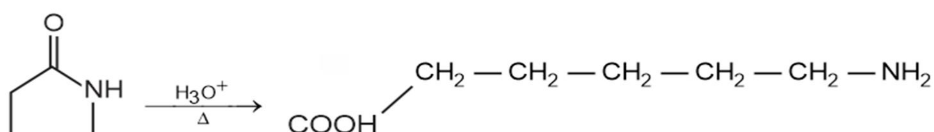
500 mol of X completely reacts with 500 mol of Y to give 1 mol of a single biodegradable acyclic copolymer Z as the only product. The amount of Z formed in grams is ____.

Given: Atomic mass (in amu): H : 1, C : 12, N : 14, O : 16, Br : 80

[ANS] **85018**

[SOLN]





Caprolic- Acid

Moles of X = 500

Moles of Y = 500

Moles of Water = (n – 1) = 1000 – 1 = 999 mole

$$X + Y = \frac{(n-1)H_2O}{\text{Polymerization}} \rightarrow Z$$

Mass of X = 37500

Mass of Y = 65500

Mass of 999 mole of H_2O = 17982

The amount of Z – form = 37500 + 65500 – 17982 = 85018

SECTION 4 (Maximum Marks : 8)

- This section contains **TWO (02)** questions stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value of to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : **+2** If ONLY the correct numerical value is entered in the designated place;
  Zero Marks **: 0** In all other cases.

Question Stem for Question Nos. 15 and 16

Two volatile liquids A and B form an ideal solution. Consider a 5 molal solution of B in A inside a closed container having a total vapour pressure of 100 mm Hg at 300 K. The vapour pressure of pure A at 300K is 105 mm Hg. Assume that A and B behave as ideal gases in the vapour phase.

[Given: The gas constant R = 0.08 L atm K^{-1} mol^{-1}

Molar mass of A is 50 g mol^{-1}

Molar mass of B is 57 g mol^{-1}

Density of liquid B at 300 K is 0.5 g/mL

1 atm = 760 mm Hg]

Solution of 15 & 16

A + B

M_B = 5 molal

P_T = 100 mm of Hg at

$P_A^o = 105$

$\frac{n_B}{W_A} = 5, n_B = 5$, $W_A = 1000\,g$ $n_A = \frac{1000}{50} = 20$ mol

In liquid

$X_B = \frac{5}{25} = \frac{1}{5}$, $X_A = \frac{20}{25} = \frac{4}{5}$

$\therefore 100 = 105 \times \frac{4}{5} + P_B^o \times \frac{1}{5}$

$\therefore 100 = 84 + \frac{P_B^o}{5}$

$\therefore P_B^o = 80$

[Q.15] At 300 K, the ratio of the molar volume of pure B in vapour phase to its molar volume in liquid phase is ____.

[ANS] **2000**

[SOLN] $V_{MB}\text{ in liq} = \dfrac{57g}{0.5\,g/ml} = 114\,ml = 0.114\,L$

$V_{MB}\text{ in vapour} = \text{Ideal gas}$

PV = nRT

$$V_m = \left(\frac{RT}{P}\right) = \frac{0.08\times 300\times 760}{1000} = \frac{912}{4}$$

$$\frac{V_{mv}}{V_{ml}} = \frac{912/4}{114/1000} = \frac{912\times 1000}{4\times 114} = 2000.00$$

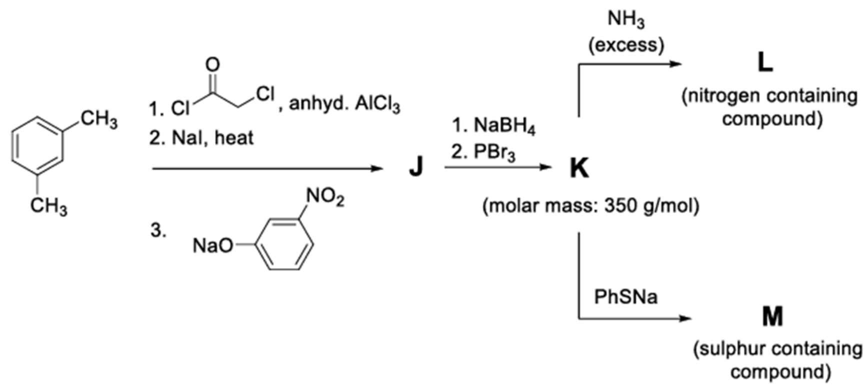
[Q.16] The mole fraction of B in vapour phase which is in equilibrium with this solution is ____.

[ANS] **0.16**

[SOLN] $Y_B = \left[\dfrac{80\times 1/5}{100}\right] = \dfrac{0.8}{5} = 0.16$

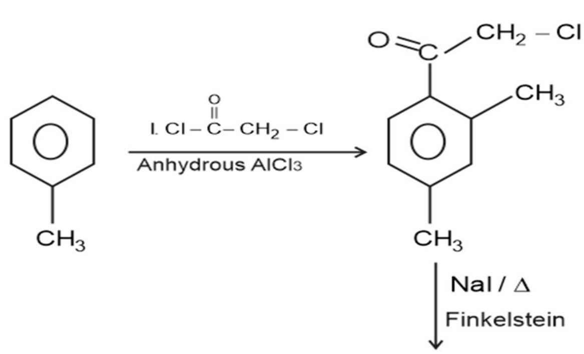
Question Stem for Question Nos. 17 and 18

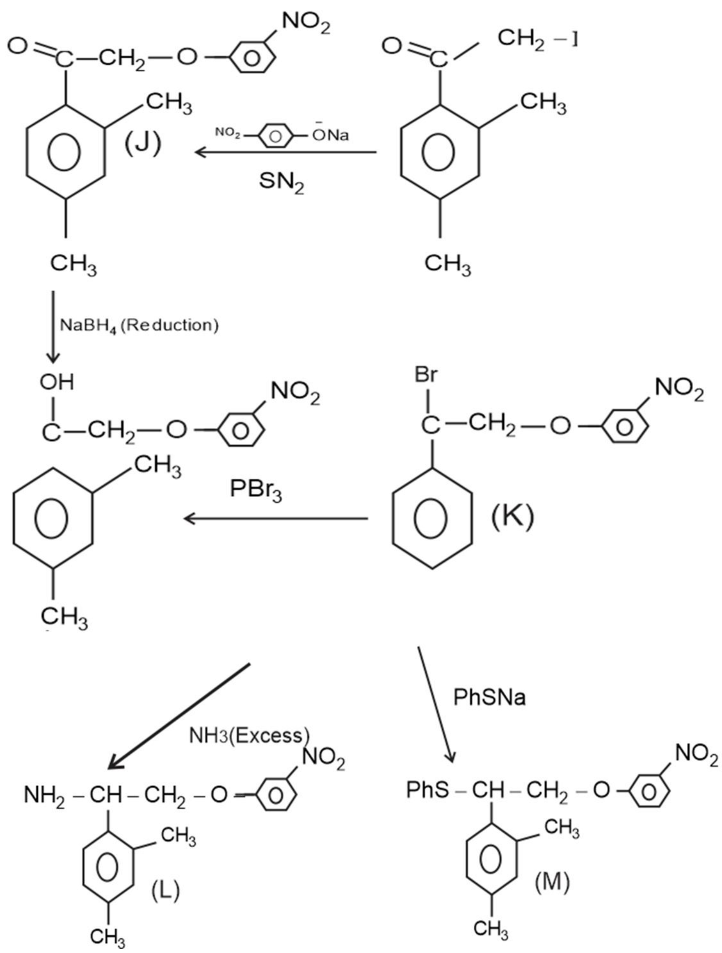
Consider the following reaction sequence in which J, K, L and M are the major products.



Given: Atomic mass (in amu): H : 1, C : 12, N : 14, O : 16, S : 32, Br : 80, Ba : 137

Solution of 17 & 18





[Q.17] The volume of 1 M aqueous H_2SO_4 required to completely neutralize the ammonia evolved from 5.72 g of L in Kjeldahl's method of nitrogen estimation is _____ mL.

[ANS] **10**

[SOLN] Mol. Wt of L = 286

Moles of $L = \frac{wt}{Mol.wt} = \frac{5.72}{286} = 0.02 \text{mole}$

Since each molecule of L yield 1 active moles of NH_3

Since $2NH_3 + H_2SO_4 \rightarrow (NH_4)_2SO_4$

$$1 \times \frac{V}{1000}$$

2 mole of NH_3 required one mole of H_2SO_4

0.02 ———————— $\frac{0.02}{2}$ = 0.01 mole of H_2SO_4

Volume of 0.01 mole H_2SO_4 (1M) = 10

[Q.18] In Sulphur estimation by Carius method, the amount of BaSO4 formed from 3.79 g of M is _____ g.

[ANS] **2.33**

[SOLN] Mole of M = 0.01

All Sulphur connected into $BaSO_4$

Mole of $BaSO_4$ = Mole of S = 0.01

Mass of $BaSO_4$ = 0.01 × 233 = 2.33 gm

PHYSICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

[Q.1] A metal wire of cross-sectional area 0.5 mm^2 and length 100 m is connected across a battery of e.m.f. 2 V and internal resistance 1 Ω. The density, atomic mass and electrical conductivity of the metal are 6.35×10^3 kg m^{-3}, 63.5 gm/mole and 2×10^8 mho m^{-1}, respectively. Assuming one conduction electron per atom of the metal, the drift velocity (in mm s^{-1}) of the electrons in the wire is:

[Take Avogadro's number as 6×10^{23} and charge of the electron as 1.6×10^{-19} C.]

[A] 0.052 [B] 0.104 [C] 0.208 [D] 0.156

[ANS] **[C]**

[SOLN] $A = 0.5 \times 10^{-6}\ m^2$ L = 100 m emf = 2V

$R = 1\Omega$ $\sigma = 2\times10^8$ $\rho = \frac{1}{2\times10^8}$

$$R = \rho\frac{\ell}{A} = \frac{1\times100}{2\times10^8\times0.5\times10^{-6}} = \frac{100}{100} = 1\Omega$$

$$I = \frac{\in}{R+r} = \frac{2}{1+1\Omega} \qquad I = 1\text{ Amp}$$

Now $\eta = \frac{\rho \times N_A}{M} = \frac{6.35\times10^3\ kg/m^3 \times 6.0\times10^{23}}{63.5\times10^{-3}\ kg/mole} = 0.6\times10^{29} = 6\times10^{28}$ electon / m^3

$$V_d = \frac{I}{\eta Ae} = \frac{1}{6\times10^{28}\times0.5\times10^{-6}\times1.6\times10^{-19}} = 0.208\ mm/s$$

[Q.2] A nuclear reactor starts producing a radioactive nuclide X from $t = 0$, at a constant rate of α per second. Each decay of X produces energy E_0, which is utilized to heat a liquid of mass m and specific heat s. Assuming no heat loss from the liquid and taking λ as the decay constant of X, the rate of increase in the temperature of the liquid is:

[A] $\frac{\alpha E_0}{m\,s}(1-e^{-\lambda t})$ [B] $\frac{\alpha E_0}{m\,s}(e^{\lambda t}-1)$ [C] $\frac{\lambda E_0}{m\,s}(1-e^{-\lambda t})$ [D] $\frac{E_0}{m\,s}(\alpha-\lambda e^{-\lambda t})$

[ANS] **[A]**

[SOLN] $\frac{dN}{dt} = \alpha - \lambda N$ $\int_0^N \frac{dN}{\alpha-\lambda N} = \int_0^t dt$ $\ell n(\alpha-\lambda N)_0^N = \lambda t$

$\ell n(\alpha-\lambda N) - \ell n\alpha = \lambda t$ $\ell n\left(\frac{\alpha-\lambda N}{\alpha}\right) = \lambda t$ $\frac{\alpha-\lambda N}{\alpha} = e^{-\lambda t}$

$\alpha - \lambda = \alpha e^{-\lambda t}$ $\alpha - \alpha e^{-\lambda t} = \lambda N$

$\alpha(1-e^{-\lambda t}) = \lambda N$ $N = \frac{\alpha}{\lambda}(1-e^{-\lambda t})$

λ decay rate $= \lambda N = \alpha(1-e^{-\lambda t})$

Power = decay rate x $E_0 = \alpha \in_0 (1-e^{-\lambda t})$

$\Rightarrow Q = ms\Delta T$ $\frac{dQ}{dt} = \frac{msdT}{dt}$ $\frac{msdT}{dt} = \alpha \in_0 (1-e^{-\lambda t})$

$$\frac{dT}{dt} = \frac{\alpha E_0}{ms}(1-e^{-\lambda t})$$

[Q.3] A beam of polychromatic light passes through a thin prism of prism angle 6°. The refractive index of the material of the prism varies with wavelength (λ) as $n(\lambda) = \alpha\lambda + \frac{\beta}{\lambda^2}$, where $\alpha = 3\ \mu m^{-1}$ and $\beta = 0.096\ \mu m^2$. If λ_{min} is the wavelength at which the angle of minimum deviation D_m is smallest, then the correct value of D_m at λ_{min} is

[A] 6.4° [B] 4.8° [C] 3.2° [D] 2.4°

[ANS] **[B]**

[SOLN] Given

Prism angle = 6° $\delta_{min} = (\mu - 1)A = (n-1)A$ $\alpha = 3\mu m^{-1}$ $\beta = 0.096\ \mu m^2$

$n(\lambda) = \alpha\lambda + \frac{B}{\lambda^2}$ $\frac{dn}{d\lambda} = \frac{\alpha - 2\beta}{\lambda^3} = 0$

$\alpha = \frac{2\beta}{\lambda^3}$ $\lambda^3 = \frac{2\beta}{\alpha}$ $\lambda^3 = \frac{2\times 0.096}{3}$ $\lambda^3 = 0.064$ $\lambda_{min} = \sqrt[3]{0.064} = 0.4\ \mu m$

$n = 3(0.4) + \frac{0.096}{(0.4)^2} = 1.2 + \frac{0.096}{0.16} = 1.8$ $\delta_{min} = (n-1)A = (1.8-1)\times 6° = 4.8°A$

[Q.4] A particle of mass m, and angular momentum ℓ is moving in a circular orbit of radius r_0 under the influence of an attractive force $\vec{F}(r) = -\frac{k}{r^2}\hat{r}$. Keeping its angular momentum unchanged, the particle is displaced radially by a small distance $\delta r \ll r_0$, due to which its radial distance varies periodically. The corresponding time period is:

[A] $\frac{2\pi\ell^3}{mk^2}$ [B] $2\pi\sqrt{\frac{m}{k}}$ [C] $\frac{2\pi\ell^3}{3mk^2}$ [D] $\frac{2\pi\ell^3}{5mk^2}$

[ANS] **[A]**

[SOLN] angular momentum $= \ell$(given) $\vec{F}(\hat{r}) = -\frac{-k}{r^2}(\hat{r})$

$u_{net} = \frac{\ell^2}{2I} + u(r)$ $u_{net} = \frac{\ell^2}{2mr^2} - \frac{k}{r}$

for Q.11

$\frac{du_{net}}{dr} = 0 \Rightarrow \frac{-\ell^2}{mr_0^3} + \frac{k}{r_0^2} = 0$ $r_0 = \frac{\ell^2}{mK}$ ...(1)

$keff = \frac{d^2v}{dr^2}|r_0 = \frac{3\ell^2}{3r_0^4} - \frac{2k}{r_0^3}$ $\left\{k = \frac{\ell^2}{mr_0} \text{ from (1)}\right\}$

$= \frac{3\ell^2}{mr_0^4} - \frac{2\ell^2}{mr_0^4} = \frac{\ell^2}{mr_0^4}$ $w = \sqrt{\frac{keff}{m}} = \sqrt{\frac{\ell^2}{mr_0^4.m}} = \sqrt{\frac{\ell^2}{m^2r_0^4}} = \frac{\ell}{mr_0^2}$

$w = \frac{mk^2}{\ell^3}$ $T = \frac{2\pi}{w} = \frac{2\pi\ell^3}{mk^2}$

SECTION 2 (Maximum Marks : 20)

- This section contains **FOUR (05)** questions.
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

  Zero Marks : 0 If none of the options is chose (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.
- For example, in a question, if [A], [B] and [D] are the ONLY three options corresponding to correct answers, then

  choosing ONLY [A], [B] and [D] will get +4 marks;

  choosing ONLY [A] and [B] will get +2 marks;

  choosing ONLY [A] and [D] will get +2 marks;

  choosing ONLY [B] and [D] will get +2 marks;

  choosing ONLY [A] will get +1 mark;

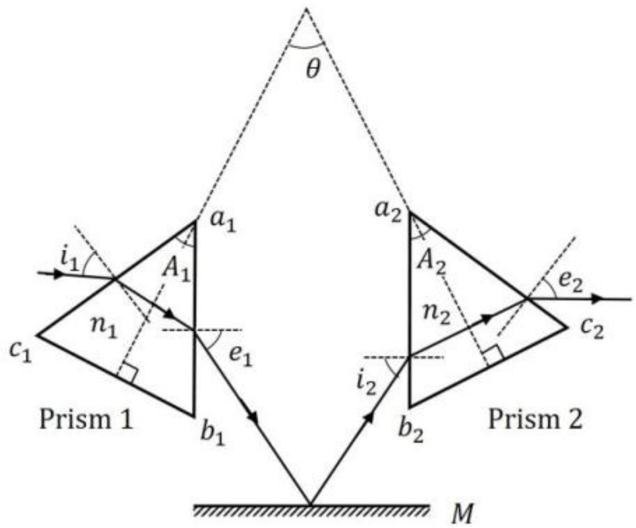
  choosing ONLY [B] will get +1 mark;

  choosing ONLY [D] will get +1 mark;

  choosing no option (i.e. the question is unanswered) will get 0 marks; and

  choosing any other combination of options will get –1 marks.

[Q.5] Consider two isosceles prisms 1 and 2 with prism angles A_1 and A_2 and refractive indices n_1 and n_2, respectively, as shown in the figure. The faces a_1b_1 and a_2b_2 are parallel to each other and perpendicular to the mirror M. If a ray of light is incident on the face a_1c_1 and emerges from the face a_2c_2, then the correct statement(s) is/are:



[A] If both the prisms are at minimum deviation condition, then $\frac{n_2}{n_1} = \sin\left(\frac{A_1}{2}\right) / \sin\left(\frac{A_2}{2}\right)$.

[B] If prism 2 is at minimum deviation condition, then $\sin i_1 = n_2 \sin\left(\frac{A_2}{2}\right)$ is always true.

[C] If both the prisms 1 and 2 are thin and are at minimum deviation condition with angles of deviation δm_1 and δm_2, respectively, then $\theta = \frac{\delta_{m1}}{2(n_1-1)} + \frac{\delta_{m2}}{2(n_2-1)}$.

[D] If prism 1 is at minimum deviation condition, then $\sin i_2 = n_1 \sin\left(\frac{A_1}{2}\right)$ is always true.

[ANS] [A]

[SOLN] Min^{M}deviation $\quad i_1 = e_1 \quad r_1 = \frac{A_1}{2}$ $\quad$ By symmetry $e_1 = i_2$

For prism-1 $\quad n_1 x \sin\frac{A_1}{2} = \sin e_1$

For prism-2 $\quad \sin i_1 = n_2 \sin\frac{A_2}{2}$

$n_1 \sin\frac{A_1}{2} = n_2 \text{sih}\frac{A_2}{2} \qquad \frac{n_2}{n_1} = \frac{\sin\frac{A_2}{2}}{\sin\frac{A_1}{2}}$

Statement B

$\sin i_1 = n_2 \sin\left(\frac{A_2}{2}\right)$ incorrect prism – 2 is at Min^m deviation

no relation i_1 & prism (1)

Statement D prims – 1 at Min^m deviation

$\sin i_1 = n_1 \sin\left(\frac{A_1}{2}\right)$

For surface $\sin e_1 = n_1 \sin(A_1/2)$

$a_1b_1 \parallel a_2b_2 (i_2 = e_1)$ $\sin i_2 = n_1 \sin\left(\frac{A_1}{2}\right)$ correct

$\delta_{m_1} = (n_1 - 1)A_1$ $sm_2(n_2 - 1)A_2$

statement C $\delta_{min} = (\mu - 1)A$

incorrect

$\theta = \frac{A_1}{2} + \frac{A_2}{2}$

$\theta = \frac{\delta_{m_1}}{2(n_1 - 1)} + \frac{\delta m_2}{2(n_2 - 1)}$

[Q.6] In a vacuum chamber, a particle of charge 1 μC and mass 1 mg is projected with a velocity $(\hat{\imath} + 2\hat{\jmath})$ ms^{-1} from the XZ plane at time t = 0 in an electric field of $1\hat{\imath}$ Vm^{-1}. At t = 0.2 s, the electric field is switched off and a magnetic field of $6\hat{\jmath}$ T is switched on. The acceleration due to gravity is $-10\hat{\jmath}$ ms^{-2}. Correct option(s) is/are:

[A] The vertical distance of the particle from the XZ plane at t = 0.3 s is 15 cm.

[B] The v ertical distance of the particle from the XZ plane at t = 0.4 s is 10 cm.

[C] The radius of the trajectory of the particle for t > 0.2 s is 20 cm.

[D] The particle will be in the XZ plane at t = 0.35 s

[ANS] **(AC)**

[SOLN] $m = 10^{-6}$ kg

$2 = 10^{-6}$ C

$\vec{v} = (\hat{i} + 2\hat{j})$ m/s

$E = \hat{i}$

at $t = 0.2$ sec Electric field is switched off q

$\vec{B} = 6\hat{j}$ T is switch on.

$a = \frac{q\vec{E}}{m} + \overrightarrow{geff}$

$\vec{a} = -10\hat{j}$

$a = (\hat{i} - 10\hat{j})$

Now, Fr (0.2) sec

$v = u + at$

$=\left(\hat{i}+2\hat{j}\right)+\left(\hat{i}-10\hat{I}\right)\times 0.2$

$=\hat{i}+2\hat{j}+0.2\hat{i}-2\hat{j}$

$=1.2\hat{i}$

$g=v_g t+\frac{1}{2}a_y t^2=2\times(0.2)+\frac{1}{2}(-10)(0.2)^2=0.2M$

For $T>0.2$ sec

$\vec{v}\perp^{r}\vec{B}$

$\vec{F}_m=q\left(\vec{v}\times\vec{B}\right)=10^{-6}\left(1.2\hat{i}\times 6\hat{i}\right)=7.2\times 10^{-6}\left(1.2\hat{i}\times 6\hat{j}\right)=7.2\times 10^{-6}\hat{k}$

Force $\perp^{r}$ to $\vec{v}$ not affect vertical motion

Check option [A]

$y(0.3)=y(0.2)+v_y.t+\frac{1}{2}a_y t^2=0.2+0-\frac{g}{2}(0.3-0.2)^2=0.2-5\times 0.01$

$=0.2-0.05=0.15M$

$r=\frac{MV}{qB}$ $\qquad$ $r=\frac{10^{-6}}{10^{-6}}\times\frac{1.2}{6}$

$r=0.02M=20M$

$T=\frac{2\pi M}{qB}$ $\qquad$ $T=\frac{2\times\pi x}{6}$ $\qquad$ $T=\frac{\pi}{3}$

[Q.7] Two charges $Q_1 = q$ and $Q_2 = mq$ are placed at the points $P_1(a, b)$ and P_2 (ma, mb), respectively, in the XY plane, where $a, b \neq 0$ and $m \neq 0, 1$. If V_1 is the potential at a point in the XY plane due to charge Q_1 and V_2 is the potential at that point due to charge Q_2. Correct statement(s) for the points at which $|V_1| = |V_2|$ is/are:

[A] For $m = -1$, locus of these points is $ax + by = 0$.

[B] For $m = 2$, the locus of these points is a circle of radius $\frac{2}{3}\sqrt{a^2+b^2}$ centered at $\left(\frac{2}{3}a, \frac{2}{3}b\right)$

[C] For $m = -2$, the locus of these points is a circle of radius $2\sqrt{a^2+b^2}$ centered at $(2a, 2b)$

[D] For $m = -3$, locus of these points is $3bx + 3ay = 0$.

[ANS] ABC

[SOLN] [A] [B] [C] correct

$$|v_1| = |v_2|$$

$$\left|\frac{Kq}{\sqrt{(x-a)^2+(y-b)^2}}\right| = \left|\frac{KMq}{\sqrt{(x-ma)^2+(y-mb)^2}}\right|$$

$$\frac{Kq}{\sqrt{(x-a)^2+(y-b)^2}} = \frac{KMq}{\sqrt{(x-Ma)^2+(y-Mb)^2}}$$

$$(x-Ma)^2+(y-Mb)^2 = M^2\left\{(x-a)^2+(y-b)^2\right\}$$

$$x^2-2Max+m^2a^2+y^2-2mby+m^2b^2 = m^2\left\{x^2-2ax+a^2+y^2-2by+b^2\right\}$$

$$\Rightarrow \frac{(m^2-1)(x^2)+(m^2-1)y^2-2ax(m^2-m)-2by(m^2-m)}{M-1}$$

$$\Rightarrow (m+1)x^2+(m+1)y^2-2\,max-2mby=0$$

$$\Rightarrow (m+1)x^2+(m+1)y^2-2m(ax+by)=0$$

Option [A] $m=-1$

$$2(ax+by)=0$$

$$ax+by=0$$

Option [B] correct

$$m=2$$

$$3x^2+3y^2-6ax-6by=0$$

$$x^2+y^2-2ax-2by=0$$

$$\left(x-\frac{2a}{3}\right)^2+\left(y-\frac{2}{3}b\right)^2=\left(\frac{2}{3}\right)^2(a^2+b^2)$$

Option [C] correct

$$m=-2$$

$$x^2+y^2-4ax-4by=0$$

It is circle (2a, 2b) centre

$R = 2\sqrt{a^2 + b^2}$

Option [D] Not correct

$m = -3$

$x^2 + y^2 - 3ax - 3by = 0$

[Q.8] Consider an electric dipole comprising two charges +q and −q each with mass m, separated by a fixed distance d and initially at rest with its dipole moment pointing along $\hat{i}$. A uniform electric field $E\hat{j}$ is turned on at time t = 0 and it is turned off at t = t_f, when the dipole moment makes an angle θ_f with $\hat{\imath}$. Neglecting any sources of energy loss, correct option(s) is/are:

[A] The center of mass of the dipole is deflected towards $\hat{\jmath}$ in the presence of the field.

[B] If the magnitude of the final angular velocity $\omega_f = \sqrt{\frac{2qE}{md}}$, than $\theta_f = \frac{\pi}{6}$..

[C] If $\theta_f = \pi/3$, then the change in kinetic energy of the dipole is given by $2\sqrt{3}\ qEd$.

[D] For $\theta_f = \pi/4$, the dipole rotates around its center of mass with a constant angular velocity after $t > t_f$.

[ANS] [B]

[SOLN] $F_{net} = 0$

COM = stationary

Now $\Delta KE = \frac{1}{2} I w^2 = \int \tau d\theta$

$$\frac{1}{2}\frac{md^2}{2} w^2 = qdE\left(1 - \frac{1}{2}\right)$$

$$w^2 = \frac{2qE}{Md} \qquad w = \sqrt{\frac{2qE}{Md}}$$

[D] $t > t_{f_{switchoff}}$

W = const

[B] correct

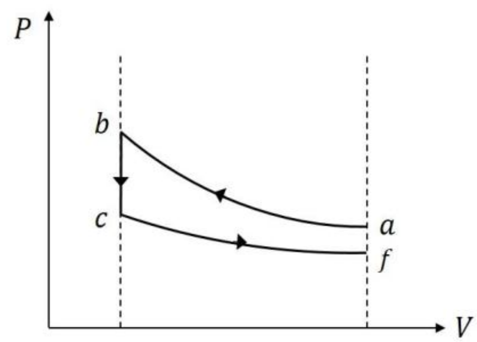
[D] correct

$$\frac{1}{2}\frac{md^2}{2} w^2 = PE(I - \cos\theta)$$

$$\frac{md^2}{4} w^2 = \frac{qdE}{2}$$

[Q.9] Ten moles of an ideal monoatomic gas, initially in state $\boldsymbol{a}$ at atmospheric pressure and temperature T_a = 27°C, is enclosed in a metal cylinder of volume V_0 fitted with a frictionless piston. The gas is suddenly compressed to state $\boldsymbol{b}$ with volume $V_0/3$. Now, keeping the piston stationary, the cylinder is submerged in a water bath of temperature 11℃ until the gas reaches the temperature of the water bath, which is denoted as state $\boldsymbol{c}$. Finally, while still in the water bath, the piston is brought slowly to its initial position, which is denoted as state $\boldsymbol{f}$. If R is universal gas constant, then the correct option(s) is/are: [Given: $9^{1/3} = 2.08$]

[A] The schematic P-V diagram of the processes described above is:



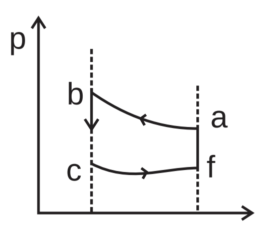
[B] The change in internal energy in going from state $\boldsymbol{a}$ to $\boldsymbol{b}$ is 4860R.

[C] The net change in the internal energy in the whole process is −240R.

[D] The pressure and temperature of the state $\boldsymbol{b}$ are 2.08 times the atmospheric pressure and 624 K, respectively.

[ANS] [A]

[SOLN]



Ta = 27^0

$(a - b)$ adiabatic compression option [A] correct

$(b \to c)$ isochoric

$(c \to f)$ isothermal expression

For a -b

$$\left(\frac{p_a}{p_b}\right)^{\gamma-1} = \left(\frac{v_b}{v_q}\right)^{\frac{\gamma-1}{\gamma}}$$

$$P_a\left(\frac{v_a}{v_b}\right)^{\gamma}=p_b$$

$$p_b=p_a\times(3)^{5/3}$$

$$p_b=(p_b)\times(3)^1\cdot(3)^{2/3}=p_a\times3\times2.08$$

$$p_b=6\cdot24\,p_a$$

Option [D] incorrect

Tc = 11^{0}C

Tf = 284 K

$$(\Delta u)_{net}=nc_v\,(T_f-T_f)$$

$$=10\times\frac{3R}{2}(284-300)$$

$$=15R\times(-16)$$

$$=-240R$$ option [C] correct

$$\left(\frac{v_a}{v_b}\right)^{\frac{\gamma-1}{\gamma}}=\frac{T_b}{T_a}$$

$$\left(\frac{3v_0}{v_0}\right)^{\frac{2}{5}}=\frac{T_b}{300}$$

$$T_b=624K$$

$$\Delta uab=nc_v\,(T_b-T_a)$$

$$=10\times\frac{3R}{2}(624-300)$$ option [B] correct

$$=15R\times324=4860\text{ R}$$

SECTION 3 (Maximum Marks : 20)

- This section contains **FIVE (05)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value of to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : **+4** If ONLY the correct numerical value is entered in the designated place;

  Zero Marks **: 0** In all other cases.

[Q.10] Two thin wires, Wire-1 of diameter 0.650 mm and Wire-2 of unknown diameter d are given. To obtain the value of d, the diameters of the two wires are measured with a screw gauge. The screw gauge has a pitch of 0.5 mm and there are 100 divisions on the circular scale (CS). The smallest division on the linear scale (LS) is 0.5 mm. The table shows the readings of LS and CS for the measurements. The value of d (in μm) is:

	Readings	
	LS (mm)	CS
Wire-1	0.5	42
Wire-2	1.5	95

[ANS] 1915.00 μm

[SOLN] $LC = \frac{\text{pitch}}{n(\text{c.s.})} = \frac{0.5\text{mm}}{100} = 0.005\text{ mm}$

$TR = M.S.R. + (-CSR \times LC) \pm \text{Zero ermor.}$

For wire 1 :- $0.650 = 0.5 + (42 \times 0.005) \pm \text{Zero error}$

$\Rightarrow \text{Zero emor} = -0.06$

For wire 2: - $TR = 1.5 + (95 \times 0.005) - 0.06$

$= 1.915\text{ mm} = 1915.00\ \mu\text{m}$

[Q.11] In a single slit diffraction experiment, a slit of width (0.016 ± 0.002) mm is used to measure the wavelength of a monochromatic light source. In the diffraction pattern, the angular distance between the central maximum and first minimum is measured to be (2° ± 40′). The value of the fractional error in the measurement of wavelength is: [Given: sin(2°) = 0.035]

[ANS] (0.46)

[SOLN] $\sin\theta = \frac{\lambda}{a}$

$\Rightarrow \lambda = a\sin\theta$

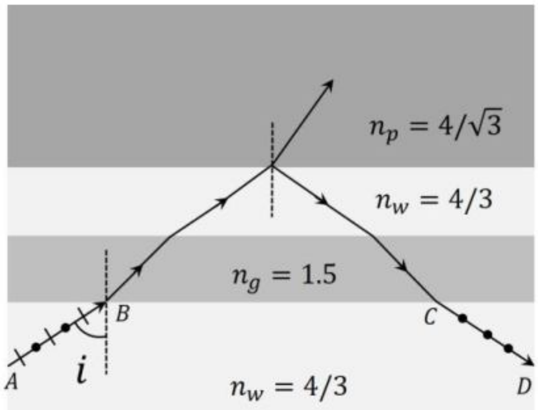
$\Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{\Delta a}{a} + \frac{\cos\theta}{\sin\theta}.\Delta\theta$

$= \frac{0.002}{0.016} + 28.55 \times \frac{\pi}{270}$

$= 0.456$

$= 0.46$

[Q.12] As shown in the figure, a ray AB of unpolarized light enters from water of refractive index n_w = 4/3 into a medium of refractive index $n_p = 4\sqrt{3}$ after passing through a glass plate of refractive index n_g = 1.5 and a layer of water. At a particular incident angle i the reflected ray CD is polarized in the direction as shown in the figure. The value of i (in degrees) is:



[ANS] (60°)

[SOLN] For ray of light CD to be polarized,

$\Rightarrow$ Reflection takes place of Brewster's angle

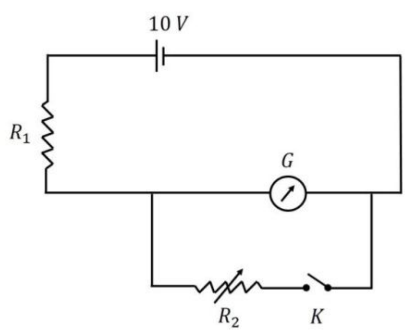
$$\Rightarrow \tan\theta = \frac{n_2}{n_1} = \frac{n_p}{n_w} = \sqrt{3}$$

$$\Rightarrow \theta = 60^\circ$$

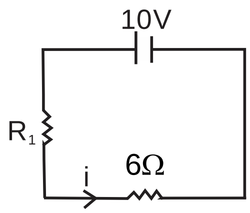
Also, $n_w \sin i = n_w \times \sin 60^\circ$

$$\Rightarrow i = 60^\circ$$

[Q.13] As shown in the figure, the resistance of a galvanometer G can be found by the half-deflection method. Here the resistance R_2 is adjusted such that when the key K is closed the deflection in the galvanometer becomes half of the value as compared to when K is open. Half-deflection is obtained at R_2 = 4Ω and thus the galvanometer resistance is found to be 6Ω. In this half-deflection condition the current (in mA) through the resistor R_1 is:

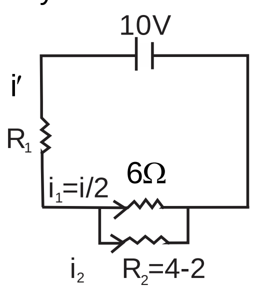


[ANS] Key Open



$i = \frac{10}{6+R_1}$ ...(i)

Key Closed



$i_1, i_2 = 4:6 = 2:3$

$\Rightarrow i_2 = \frac{3i}{4}$

$\Rightarrow i_1 = i_1 + i_2 = \frac{5i}{4}$

$\therefore\ = \frac{5i}{4} = \frac{10}{2.4+R_1}$ ....(ii)

$(1) \div (2)$

$\Rightarrow \frac{4}{5} = \frac{10}{(6+R_1)} \times \frac{(2.4+R_1)}{10} \Rightarrow R_1 = 12\,\Omega$

$\therefore$ Current through

R_1 in half celf

$\Rightarrow \frac{10}{(12+2.4)} = \frac{10}{14.4}$ $= 0.69444$ A $= 694.44$ mA

[Q.14] In a new system of units, the units of mass, length, time and current are 5 kg, 5 m, 5 s and 5 A, respectively. If μ_0 and ϵ_0 are the permeability and permittivity of free space, respectively, then in this new system of units, the magnitude of one SI unit of $\sqrt{\mu_0/\epsilon_0}$, is:

[ANS] (25.00)

[SOLN] $\sqrt{\frac{\mu_0}{\varepsilon_0}} = M^1L^2T^{-3}A^{-2}$

$\Rightarrow \frac{1\text{new}}{1\text{SI}} = \frac{5 \times 5^2 \times 5^{-3} \times 5^{-2}}{1} = \frac{1}{25}$

$\Rightarrow 1\text{SI} = 25$ new

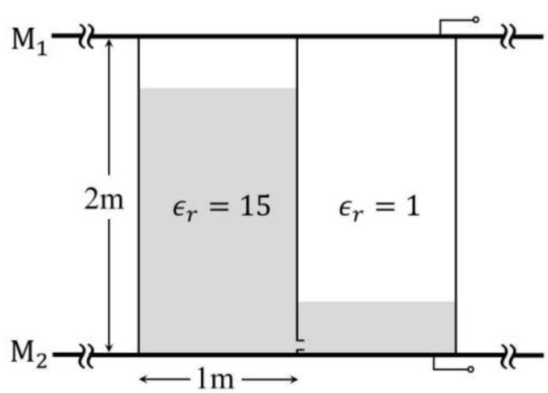
SECTION 4 (Maximum Marks : 8)

- This section contains **TWO (02)** questions stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value of to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : **+2** If ONLY the correct numerical value is entered in the designated place;
  Zero Marks **: 0** In all other cases.

Question Stem for Question Nos. 15 and 16

A container of height 2 m, length 2 m and breadth 1 m is made of insulating vertical walls and two large area horizontal metal plates (M_1 and M_2) which extend far beyond the vertical walls in all directions. The container is partitioned into two equal chambers with a thin insulating vertical wall. The partition wall contains a small hole of cross-sectional area $\sqrt{10}$ cm^2 near its bottom edge. Initially the hole is closed and the left chamber of the container is completely filled with a liquid of dielectric constant ϵ_r = 15 and the right chamber is empty (ϵ_r = 1). At time t = 0, the hole is opened and the liquid flows from the left chamber to the right chamber. In both the chambers, the space above the liquid has ϵ_r = 1 and is maintained at atmospheric pressure. The schematic of the container at a time t > 0 is shown in the figure.

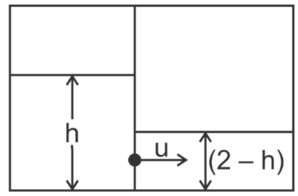
[Given: acceleration due to gravity is 10 ms^{-2}.]



[Q.15] The height (in m) of the liquid in left chamber at t = 500 s is:

[ANS] (1.25)

[SOLN] At any time t



Using Bernonli's theorem

$P_2 = P_R + \frac{1}{2}\rho v^2$

$\rho gh = \rho g(2 - h) + \frac{1}{2}\rho v^2$

$\Rightarrow \frac{1}{2}v^2 = g[h - (2 - h)]$

$\Rightarrow V^2 = 2g[2h - 2]$

$\Rightarrow v = \sqrt{4g(h-1)}$

$\Rightarrow v = 2\sqrt{g(h-1)}$

Now, using volume conservation

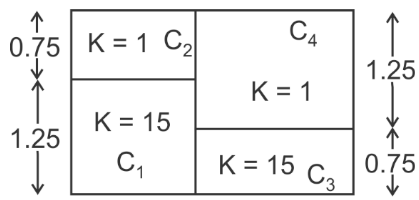
$\Rightarrow -A \cdot dh = a . v . dt$

$\Rightarrow -A . dh = a . 2\sqrt{g(h-1)} . dt$

$\Rightarrow -\int_2^h \frac{dh}{2\sqrt{g(h-1)}} = \int_0^{500} \frac{a}{A} . dt$

$\Rightarrow h = 1.25$ m

[Q.16] The difference in the capacitance (in F) between the metal plates at $t = 0$ and that at $t = 500$ s is $(8 - n)\,\epsilon_0$, where ϵ_0 is the permittivity of free space. The value of n is:



[ANS] (1.97)

[SOLN] At t = 0

$C_{eq} = C_1 + C_2 = \frac{15\varepsilon_0}{2} + \frac{\varepsilon_0}{2} = 8\varepsilon_0$

At t = 500 sec

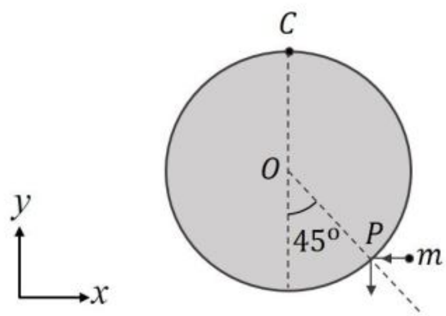
$C_{eq} = \frac{128}{65}\,\varepsilon_0$

$\therefore \quad n = \frac{128}{65} = 1.97$

Question Stem for Question Nos. 17 and 18

A uniform circular disk of radius 0.2 m and mass 1 kg is pivoted at its top point C such that it can rotate freely around C in the XY plane, as shown in the figure. Initially, when the disk is at rest, a particle of mass 20 g, travelling along negative x direction in the XY plane with speed 100 ms^{-1}, hits the circumference of the disk at a point P. After collision the particle moves along negative y direction at a speed of 90 ms^{-1}.

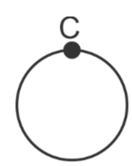
[Given: the acceleration due to gravity (g) = − 10 $\hat{j}$ ms^{-2}]



[Q.17] After the collision the disk starts to rotate around point C in the XY plane. The maximum change in the height (in m) of its center O is :

[ANS] (0.15)

[SOLN] $I_C = MR^2 + \frac{MR^2}{2} = \frac{3}{2}MR^2$



$= 0.06$ Kg m^2

Conservation of Angular Momentum.

Initial $(L_B + L_D) =$ Final $(L_B + L_D)$

$$\Rightarrow 0.02 \times 100 \times \left(0.2 + \frac{0.2}{\sqrt{2}}\right) = L_D + 0.02 \times 90 \times \left(0.2 \times \frac{1}{\sqrt{2}}\right)$$

$\Rightarrow L_D = 0.683 - 0.255$

$= 0.428$ (clockwise)

Just After collision

$I_c\omega = 0.428$

$\Rightarrow \omega = 7.13$ rad/s.

Using conservation of energy

$$\frac{1}{2}I\omega^2 = mgh$$

$\Rightarrow \frac{1}{2} \times (0.06) \times (7.13)^2 = 1 \times 10 \times h$

$\Rightarrow h \times 10 = 1.53$

$\Rightarrow h = 0.15 \text{ m}$

[Q.18] Amount of energy loss (in J) in the collision is:

[ANS] (17.47)

[SOLN] Initial energy $\frac{1}{2} \times 0.02 \times 100 \times 100 = 100 \text{ J (Ball)}$

Final energy $\left[\frac{1}{2} \times 0.02 \times 90 \times 90\right] + \left[\frac{1}{2} \times 0.06 \times 7.13 \times 7.13\right]$

= 82.53 J (Ball + Disc)

$\therefore$ Loss = 100 – 82.53 = 17.47 J