JEE (ADVANCED) 2026 PAPER-1

[PAPER ANSWER KEY WITH SOLUTION]

HELD ON SUNDAY 17TH MAY 2026

MATHEMATICS

SECTION 1 (Maximum Marks :12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +3 If **ONLY** the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.

[Q.1] Consider the function $f:(0,\infty)\to(-\infty,\infty)$ given by $f(x)=\sqrt{x}\log_e(x)-x+1$. Then which one of the following statements is TRUE ?

[A] The derivative of the function f is decreasing in the interval (0, 1)

[B] The function f has a local maximum at some point $a\in(0,\infty)$

[C] The function f has a local minimum at some point $b\in(0,\infty)$

[D] the function f has **NEITHER** a point of local maximum **NOR** a point of local minimum in the interval $(0,\infty)$

[ANS] **D**

[SOLN] $f:(0,\infty)\to(-\infty,\infty)$

$f(x)=\sqrt{x}\log_e(x)-x+1$

$f'(x)=\frac{1}{\sqrt{x}}+\frac{\log_e x}{2\sqrt{x}}-1=\frac{2+\log_e x-2\sqrt{x}}{2\sqrt{x}}$

Consider g(x) = $2 + \log_e x - 2\sqrt{x}$

$$g'(x) = \frac{1}{x} - \frac{1}{\sqrt{x}} = \frac{1-\sqrt{x}}{x}$$

g'(x) > 0 $\quad \forall\ 0 < x < 1$

g'(x) = 0 $\quad$ for $x = 1$

& g'(x) < 0 $\quad$ for $1 < x < \infty$

$\Rightarrow \quad g(x) \le g(1) \quad \forall x \in (0, \infty)$

$\Rightarrow \quad g(x) \le 0$ $\quad$ Equality holds only when x = 1

$\Rightarrow \quad f'(x) \le 0 \ \forall x \in (0, \infty)$

equality holds only when x = 1

[Q.2] Let P be the point on the parabola $y = x^2$ such that the slope of the tangent to the parabola at the point P is 4. Let Q be the point in the first quadrant lying on the circle $x^2 + y^2 = 2$ such that the slope of the tangent to the circle at the point Q is –1. Let R be the point in the first quadrant lying on the ellipse $x^2 + 4y^2 = 8$ such that the slope of the tangent to the ellipse at the point R is $-\frac{1}{2}$. Then the radius of the circle passing through the point P, Q and R is

[A] $\sqrt{10}$ [B] $\sqrt{5}$ [C] $\sqrt{\frac{5}{2}}$ [D] $2\sqrt{5}$

[ANS] **C**

[SOLN] $y = x^2$

$$\frac{dy}{dx} = 2x = 4 \Rightarrow x = 2$$

$\therefore \quad P(2, 4)$

$x^2 + y^2 = 2$

$Q = \left(\sqrt{2}\cos\theta, \sqrt{2}\sin\theta\right)$, Q is in 1st quadrant with $\tan\theta = 1$

$\therefore \quad Q \equiv (1,1)$

$x^2 + 4y^2 = 8$

$\frac{x^2}{8} + \frac{y^2}{2} = 1 \qquad R = \left(2\sqrt{2}\cos\theta, \sqrt{2}\sin\theta\right)$.

Slope of tangent = $-\frac{1}{2}\cot\theta$ it is given as $-\frac{1}{2}$

$\therefore$ $\cot\theta = 1$

$\therefore$ R = (2, 1)

$\because$ ΔPRQ is right angled triangle with right angle at R

$\therefore$ required radius = $\frac{PQ}{2} = \frac{\sqrt{10}}{2} = \sqrt{\frac{5}{2}}$

[Q.3] Which one of the following matrices can be obtained by performing elementary row transformations on the 3×3 identity matrix ?

[A] $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ [B] $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$ [C] $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2 & 5 & 8 \end{bmatrix}$ [D] $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$

[ANS] **B**

[SOLN] Elementary raw transformation on an identity matrix will result in a non-singular matrix

Now, the matrix in option (A) clearly has its determinant = 0

So is the case for option (D) as sum of R, and R_2 is R_3

For option (B) Determinant = $(3-8)-(2-4)+4-3=-2\neq 0$.

& for option (C) determinant = $(24-20)-(16-8)+(10-6)=0$.

Hence (B) is the correct answer.

[Q.4] Considering only the principal values of the inverse trigonometric functions, the value of

$\cot^{-1}(\cot(-11))+10\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)+10\sin\left(2\tan^{-1}(2)\right)$ is

[A] $3\pi+7$ [B] 7 [C] $4\pi+7$ [D] $3\pi-5$

[ANS] **C**

[SOLN] $\cot^{-1}(\cot(-11))+10\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)+10\sin\left(2\tan^{-1}2\right)$

$$\cot^{-1}(\cot(4\pi-11))+10\sin\frac{\pi}{2}+10\cdot\frac{2\tan\left(\tan^{-1}2\right)}{1+\tan^2\left(\tan^{-1}2\right)}.$$

$$4\pi-11+10+10\cdot\frac{4}{1+4}$$

$$4\pi+7$$

SECTION 2 (Maximum Marks :16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get −2 marks.

[Q.5] Suppose that Box I contains 6 red balls and 9 green balls, and Box II contains 8 red balls and 12 green balls. All the balls of Box I and Box II are mixed together and a ball is chosen at random from them. Let E_1 be the event that the ball chosen belonged to Box I and let E_2 be the event that the ball chosen belonged to Box II. Let F_1 be the event that the ball chosen is red and let F_2 be the event that the ball chosen is green. Then which of the following statements is (are) TRUE ?

[A] The events E_1 and F_1 are independent

[B] The events E_2 and F_2 are dependent

[C] The conditional probability $P(F_1 \mid E_1)$ is equal to the conditional probability $P(F_1 \mid E_2)$

[D] The conditional probability $P(F_1 \mid E_1)$ is greater than the conditional probability $P(F_2 \mid E_2)$

[ANS] **A,C**

[SOLN] When the balls are mixed, total balls = 35

Now $P(E_1)=\frac{15}{35}=\frac{3}{7}$

$P(E_2)=\frac{20}{35}=\frac{4}{7}$

$P(F_1)=\frac{14}{35}=\frac{2}{5}$

$P(F_2)=\frac{21}{35}=\frac{3}{5}$

& $P\left(\frac{F_1}{E_1}\right)=\frac{6}{15}=\frac{2}{5}=P(F_1)$ $\Rightarrow$ E_1 & F_1 are independent

& $P\left(\frac{F_1}{E_2}\right)=\frac{8}{20}=\frac{2}{5}$

$P\left(\frac{F_2}{E_2}\right)=\frac{12}{20}=\frac{3}{5}=P(F_2)\Rightarrow E_2$ & F_2 are independent

[Q.6] Let P be the plane such that it contains the straight line $\frac{x-1}{2}=\frac{y-3}{3}=\frac{z+2}{1}$ and is perpendicular to the plane $x+2y+3z=4$. Let P_1 be the plane which passes through the point (4, 2, 2) and is parallel to P. Then which of the following statements is (are) TRUE ?

[A] The equation of the plane P is $7x-5y+z=-10$

[B] The distance between the planes P and P_1 is 30

[C] The distance of the plane P from the origin is $2\sqrt{3}$

[D] The acute angle between the plane P and the plane $2x+2y+z=3$ is $\cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

[ANS] **A,D**

[SOLN] line has D.R^S < 2, 3, 1 > & is $\perp^r$ to plane x + 2y + 3z = 4

A vector normal to P will be

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 1 & 2 & 3\end{vmatrix}=7\hat{i}-5\hat{j}+\hat{k}$$

& P passes through the pt. (1, 3, –2)

$\therefore$ Equation to P is 7(x – 1) – 5(y – 3) + (z + 2) = 0.

$7x - 5y + z = -10$

& Eq^n to P_1 : $7x - 5y + z = \lambda$

As P, passes through (4, 2, 2)

$\lambda = 20$.

$\therefore P_1 : 7x - 5y + z = 20$.

Distance between P and P_1 = $\frac{30}{\sqrt{49+25+1}} = 2\sqrt{3}$

Distance of P from origin = $\frac{10}{\sqrt{75}} = \frac{2}{\sqrt{3}}$.

$\cos\theta = \left|\frac{14-10+1}{\sqrt{75}\sqrt{9}}\right| = \frac{1}{3\sqrt{3}}$

[Q.7] Let $\mathbb{R}$ denote the set of all real numbers. Let $f : R \to R$ be an arbitrary function and let $g : R \to R$ be the function defined by $g(x) = xf(x)$, for all $x \in R$. Then which of the following statements is (are) TRUE ?

[A] The function g is always continuous at $x = 0$

[B] If f is continuous at $x = 0$, then g is differentiable at $x = 0$

[C] If g is differentiable at $x = 0$, then f is continuous at $x = 0$

[D] If g is differentiable at $x = 0$, then $\lim_{x\to 0} f(x)$ exists

[ANS] **B,D**

[SOLN] consider $f(x) = \begin{cases} \frac{1}{x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Then g(x) = $\begin{cases} \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Which is discontinuous at x = 0

g'(0) = $\lim_{h\to 0} \frac{hf(x) - 0}{h}$

= $\lim_{h\to 0} f(h)$

So g is diff. at x = 0

iff $\lim_{h\to 0} f(h)$ exists

so only (B), (D) are correct

[Q.8] Consider the matrix $M=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}$. Let p , q , r , s , a , b , c and d be integers such that $M^{26}=\begin{bmatrix}p & q\\ r & s\end{bmatrix}$ and $\sum_{k=1}^{26} M^k=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ Then which of the following statements is (are) TRUE ?

[A] There exists a 2×2 invertible matrix N with real entries such that $MN=N\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

[B] The value of a is 378

[C] For any two given integers m and n, there exist unique integers x and y such that $px + qy = m$ and $rx + sy = n$

[D] For each positive real number t, the system of linear equations $(a + t)x + by = 1$
$cx + (d + t)y = -1$ has a unique solution

[ANS] **A,C,D**

[SOLN] $M=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}$

$M^2 = 2M - I$

$M^3 = 2M^2 - M = 3M - 2I$

$M^4 = 3M^2 - 2M = 4M - 3I$

$\therefore\quad M^K = KM - (K - 1)^I = \begin{bmatrix}k+1 & -k\\ k & 1-k\end{bmatrix}$

$\therefore\quad M^{26}=\begin{bmatrix}27 & -26\\ 26 & -25\end{bmatrix}$

$\Rightarrow\quad p = 27$

$q = -26 = r$

$s = -25$

$$\sum_{k=1}^{26} M^K=\begin{bmatrix}\sum_{k=1}^{26}(k+1) & \sum_{k=1}^{26}-k\\ \sum_{k=1}^{26}k & \sum_{k=1}^{26}(1-k)\end{bmatrix}=\begin{bmatrix}377 & -351\\ 351 & -325\end{bmatrix}$$

a = 377, b = –351, c = 351, d = –325

for option (A) Let $N=\begin{bmatrix}x & y\\ 2 & 6\end{bmatrix}$

Now $MN = \begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x & y\\ 2 & 6\end{bmatrix}=\begin{bmatrix}2x-z & 2y-t\\ x & y\end{bmatrix}$

$\& \; N\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} = \begin{bmatrix}x & y\\z & t\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} = \begin{bmatrix}x & x+y\\z & z+t\end{bmatrix}$

$MN = N\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} \Rightarrow 2x - z = x \Rightarrow x = z$

$2y - t = x + y \Rightarrow y = x + t$

Now $|N| = xt - yz = xt - x^2 - xt = -x^2$

Which can be non zero $\therefore$ A is correct

Option B is wrong as a = 377

For option C $\quad |M^{26}| \neq 0 \quad \therefore$ unique solution

For option D. $\quad$ Determinant = (a + t)(d + t) – bc

$= t^2 + (a + d)t + ad - bc = (t + 26)^2$

SECTION 3 (Maximum Marks :16)

- This section contains **FOUR (04)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

[Q.9] Let $S = \{1, 2, 3, \ldots, 10\}$. Consider the set $X = \{R : R$ is an equivalence relation on the set S such that R has exactly 42 elements$\}$. Then the number of elements in X is ______________.

[ANS] **2520**

[SOLN] Each partitioning of S gives a unique equivalence relation.

So we partition S into sets containing

$x_1, x_2, \ldots\ldots, x_k$ elements

then $x_1 + x_2 + \ldots\ldots + x_k = 10 \qquad \ldots\ldots(1)$

and no. of elements into corresponding relation R will be

$x_1^2 + x_2^2 + \ldots\ldots + x_k^2 = 42 \qquad \ldots\ldots(2)$

Possible solution of (1) & (2) is (1, 4, 5) and (1, 1, 2, 6)

So no. of such relations R

$= \dfrac{\underline{|10}}{\underline{|1}\,\underline{|4}\,\underline{|5}} + \dfrac{\underline{|10}}{\underline{|1}\,\underline{|1}\,\underline{|2}\,\underline{|6}} \times \dfrac{1}{\underline{|2}} = 2520$

[Q.10] Consider the function $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to(-\infty,\infty)$ defined by $f(x) = (|x| + |x-1|)\sin x + [x\sin x]$, where $[x\sin x]$ is the greatest integer less than or equal to $x\sin x$. Let α be the total number of points in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ at which f is NOT continuous, and let β be the total number of points in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ at which f is NOT differentiable. Then the value of $\alpha+\beta$ is ___________.

[ANS] **5**

[SOLN] $\because\ -\frac{\pi}{2} < x\sin x < \frac{\pi}{2} \quad \forall x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore$ doubtfull prints are 0, 1, p, q

Where $p \in \left(-\frac{\pi}{2},0\right)$ and $q \in \left(0,\frac{\pi}{2}\right)$ s.t.

p sinp = 1 and q sinq = 1.

of these f is discontinuous at p and q only

and non-diff. at x = 1, p, q only.

$\therefore\ \alpha = 2$ and $\beta = 3$

$\therefore\ \alpha+\beta = 5$

[Q.11] The number of ways to distribute 10 identical red pens and 14 identical blue pens among four persons such that each person gets 6 pens, is _____________.

[ANS] **206**

[SOLN] Let the four persons are given x_1, x_2, x_3, x_4

Red pens respectively. So they are given

$6 - x_1, 6 - x_2, 6 - x_3, 6 - x_4$ blue pens.

So the no. of ways will be the no. of integral solutions of the equation

$x_1 + x_2 + x_3 + x_4 = 10$

where $0 \le x_1, x_2, x_3, x_4 \le 6$

$\therefore$ no of solution = coeff. of x^{10} in $(1 + x + \ldots\ldots + x^6)^4$

= coeff. of x^{10} in $(1 - x^7)^4 (1 - x)^{-4}$

$= 1\times{}^{13}C_3 - 4\times{}^{6}C_3 = 206$

[Q.12] Let $\alpha=\left(1-2\cos\left(\frac{\pi}{11}\right)\right)\left(1-2\cos\left(\frac{3\pi}{11}\right)\right)\left(1-2\cos\left(\frac{9\pi}{11}\right)\right)\left(1-2\cos\left(\frac{27\pi}{11}\right)\right)\left(1-2\cos\left(\frac{81\pi}{11}\right)\right)$.

Then the value of $5-\alpha^2$ is ______.

[ANS] 4

[SOLN]
$$1-2\cos\theta=\frac{(1-2\cos\theta)\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}=\frac{\cos\frac{\theta}{2}-2\cos\theta\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$=\frac{\cancel{\cos\frac{\theta}{2}}-\left(\cos\frac{3\theta}{2}+\cancel{\cos\frac{\theta}{2}}\right)}{\cos\frac{\theta}{2}}=-\frac{\cos\frac{3\theta}{2}}{\cos\frac{\theta}{2}}$$

$$\therefore\quad \alpha=\frac{-\cos\frac{3\pi}{22}}{\cos\frac{\pi}{22}}\cdot\frac{-\cos\frac{9\pi}{22}}{\cos\frac{3\pi}{22}}\cdot\frac{-\cos\frac{27\pi}{22}}{\cos\frac{9\pi}{22}}\cdot\frac{-\cos\frac{81\pi}{22}}{\cos\frac{27\pi}{22}}\cdot\frac{-\cos\frac{243\pi}{22}}{\cos\frac{81\pi}{22}}$$

$$=-\frac{\cos\frac{243\pi}{22}}{\cos\frac{\pi}{22}}=-\frac{\cos\left(11\pi+\frac{\pi}{22}\right)}{\cos\frac{\pi}{22}}$$

$= 1$

$\therefore\quad 5-\alpha^2=4$.

SECTION 4 (Maximum Marks :16)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II.**
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

[Q.13] Match each entry in List-I to the correct entry in List-II and choose the correct option.

List – I	**List – II**
(P) If α and β are the distinct roots of the equation $x^2 + x + 1 = 0$, then the quadratic equation with roots $\frac{1}{(\alpha+1)^{2027}}$ and $\frac{1}{(\beta+1)^{2027}}$ is	(1) $x^2 + x + 1 = 0$
(Q) If α and β are the distinct roots of the equation $x^2 + x + 1 = 0$, then the quadratic equation with roots $\frac{1}{(\alpha+1)^{2027}}$ and $\frac{1}{(\beta+1)^{2027}}$ is	(2) $x^2 - x + 1 = 0$
(R) If γ and δ are the distinct roots of the equation $x^2 - x + 1 = 0$, then the value of $\frac{1}{(\gamma-1)^{2026}} + \frac{1}{(\delta-1)^{2026}}$ is	(3) $x^2 + x - 1 = 0$
(S) If p and r are the distinct roots of the Equation $x^2 + x - 1 = 0$, then the value of $\frac{1}{(p+1)^3} + \frac{1}{(r+1)^3}$ is	(4) –1
	(5) –1

[A] (P) → (1), (Q) → (2), (R) → (5), (S) → (4)

[B] (P) → (3), (Q) → (1), (R) → (4), (S) → (5)

[C] (P) → (1), (Q) → (2), (R) → (4), (S) → (5)

[D] (P) → (2), (Q) → (3), (R) → (5), (S) → (4)

[ANS] **C**

[SOLN] (P) roots of $x^2 + x + 1 = 0$ are w and w^2.

$\therefore \quad \frac{1}{(w+1)^{2026}} = \left(\frac{1}{-w^2}\right)^{2026} = w^{2026} = w$

and $\frac{1}{(w^2+1)^{2026}} = w^2$

$\therefore$ quad. Eqn is $x^2 + x + 1 = 0$

(Q) Let $\alpha = w$, $\beta = w^2$

$\frac{1}{(\alpha+1)^{2027}} = \frac{1}{(w+1)^{2027}} = \frac{1}{(-w^2)^{2027}} = -w^{2027}$

$= -w^2$

$\frac{1}{(\beta+1)^{2027}} = -w$

$\therefore$ quad. Eqn is $x^2 - x + 1 = 0$

(R) Let $\gamma = -w$, $\delta = -w^2$

$\therefore \quad \frac{1}{(r-1)^{2026}} = \frac{1}{(-w-1)^{2026}} = \frac{1}{(w^2)^{2026}} = w^{2026} = w$

$\frac{1}{(\delta-1)^{2026}} = w^2$

$\therefore \quad \frac{1}{(\gamma-1)^{2026}} + \frac{1}{(\delta-1)^{2026}} = w + w^2 = -1$

(S) $x^2 + x - 1 = 0$

$\Rightarrow$ $x(x + 1) = 1$

$\Rightarrow$ $x + 1 = \frac{1}{x}$

$\therefore$ $p + 1 = \frac{1}{p}$, $q + 1 = \frac{1}{q}$

$\therefore \quad \frac{1}{(p+1)^3} + \frac{1}{(q+1)^3}$

$= p^3 + q^3 = (p + q)^3 - 3pq(p + q)$

$= (-1)^3 - 3(-1)(-1) = -4.$

[Q.14] Match each entry in List-I to the correct entry in List-II and choose the correct option.

List – I	List – II
(P) The number of elements in the set $\left\{x \in [-\pi, \pi] : \sin^6 x + \cos^4 x = 1\right\}$	(1) is 1
(Q) The number of elements in the set $\left\{x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] : \sin^2 x + \cos^6 x = 1\right\}$	(2) is 2
(R) The number of elements in the set $\left\{x \in [-\pi, \pi] : \cos^2\left(\frac{x}{2}\right) - \sin^2 x = \frac{1}{2}\right\}$	(3) is 3
(S) The number of elements in the set $\left\{x \in [-2\pi, 2\pi] : 6\sin^2\left(\frac{x}{2}\right) - \cos 3x = 3\right\}$	(4) is 4
	(5) is 5

[A] (P) → (2), (Q) → (5), (R) → (3), (S) → (4)

[B] (P) → (5), (Q) → (3), (R) → (2), (S) → (4)

[C] (P) → (5), (Q) → (4), (R) → (1), (S) → (3)

[D] (P) → (4), (Q) → (3), (R) → (2), (S) → (5)

[ANS] **B**

[SOLN] (P) $\sin^6 x + \cos^4 x \le \sin^2 x + \cos^2 x = 1$

$\therefore \sin^6 x + \cos^4 x = 1$

$\Rightarrow \sin^6 x = \sin^2 x$ and $\cos^4 x = \cos^2 x$

$\Rightarrow \sin x = 0, 1, -1$ and $\cos x = 0, 1, -1$

$\Rightarrow x = -\pi, \frac{-\pi}{2}, 0, \frac{\pi}{2}, \pi$

(Q) $\sin^2 x + \cos^6 x = 1$

$\Rightarrow$ $\cos^6 x = \cos^2 x$

$\Rightarrow$ $\cos x = 0, 1, -1$

$\therefore$ $x = \frac{-\pi}{2}, 0, \frac{\pi}{2}$

(R) $\cos^2 \frac{x}{2} - \sin^2 x = \frac{1}{2}$

$\Rightarrow$ $\frac{1+\cos x}{2} - \left(1 - \cos^2 x\right) = \frac{1}{2}$

$\Rightarrow$ $2\cos^2 x + \cos x - 2 = 0$

$\Rightarrow$ $\cos x = \frac{-1 \pm \sqrt{17}}{4}$

But $-1 \le \cos x \le 1$

$\therefore$ $\cos x = \frac{-1 + \sqrt{17}}{4}$

$\therefore$ 2 sol^n in $[-\pi, \pi]$

(S) $6\sin^2 \frac{x}{2} - \cos 3x = 3$

$\Rightarrow$ $3\left(1 - \cancel{\cos x}\right) - \left(4\cos^3 x - 3\cancel{\cos x}\right) = 3$

$\Rightarrow$ $-4\cos^3 x = 0 \Rightarrow \cos x = 0$

$\Rightarrow$ $x = \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

[Q.15] For real numbers $\alpha, \beta, \gamma, \delta$ and μ, consider the matrix $M = \begin{bmatrix} \alpha & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \beta & \frac{1}{\sqrt{3}} \\ \gamma & \delta & \mu \end{bmatrix}$. Suppose that $MM^T = I$, where M^T is the transpose of the matrix M, and I is the 3×3 identity matrix. Let

$\vec{u} = \alpha\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \gamma\hat{k}$, $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \beta\hat{j} + \delta\hat{k}$ and $\vec{w} = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \mu\hat{k}$.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List – I	List – II
(P) The value of $\gamma^2 + \delta^2$ is	(1) 0
(Q) If $x\vec{u} + y\vec{v} + z\vec{w} = \hat{j}$ for some real numbers x, y and z, then the value of x is	(2) 1
(R) The value of $\left|\vec{u} \cdot (\vec{v} \times \vec{w})\right|$ is	(3) $\frac{1}{\sqrt{2}}$
(S) The value of $\left|\vec{u} \times (\vec{v} \times \vec{w})\right|$ is	(4) $\frac{1}{\sqrt{3}}$
	(5) $\frac{5}{6}$

[A] (P) → (5), (Q) → (4), (R) → (2), (S) → (1)

[B] (P) → (4), (Q) → (5), (R) → (1), (S) → (2)

[C] (P) → (5), (Q) → (3), (R) → (2), (S) → (1)

[D] (P) → (5), (Q) → (4), (R) → (1), (S) → (2)

[ANS] **A**

[SOLN] $MM^T = I$

$\Rightarrow$ M is orthogonal matrix

$\Rightarrow$ Rows as well as columns of M are mutually $\perp^r$ unit vectors.

(P) $\therefore \quad \alpha^2 + \frac{1}{2} + \frac{1}{2} = \frac{1}{3} + \beta^2 + \frac{1}{3} = 1$

$\Rightarrow \quad \alpha^2 = 0, \beta^2 = \frac{1}{3}$

also $\alpha^2+\frac{1}{3}+\gamma^2=\frac{1}{2}+\beta^2+\delta^2=1$

$\Rightarrow \quad \gamma^2=\frac{2}{3},\delta^2=\frac{1}{6}$

$\therefore \quad \gamma^2+\delta^2=\frac{5}{6}$

(Q) $\vec{u},\vec{v},\vec{w}$ are mutually $\perp^r$ unit vectors

$\therefore \quad x\vec{u}+y\vec{v}+2\vec{w}=\hat{j}$

$\Rightarrow \quad x\vec{u}\cdot\vec{u}=\vec{u}\cdot\hat{j}$

$\Rightarrow \quad x=\frac{\vec{u}\cdot\hat{j}}{|\vec{u}|^2}=\frac{\frac{1}{\sqrt{3}}}{1}=\frac{1}{\sqrt{3}}$

(R) $\left|\vec{u}\cdot(\vec{v}\times\vec{w})\right|=|\vec{u}||\vec{v}||\vec{w}|=1$

(S) $\left|\vec{u}\times(\vec{v}\times\vec{w})\right|=0 \qquad (\because\ \vec{u}\,||\,\vec{v}\times\vec{w})$

[Q.16] Match each entry in List-I to the correct entry in List-II and choose the correct option.

List – I	**List – II**
(P) The circle with centre (1, 2) and touching the straight line $3x + 4y = 1$, passes through	(1) the point (1, 1)
(Q) The common tangent to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ with positive slope, passes through	(2) the point (7, 9)
(R) Let M be the end point of the latus rectum of the ellipse $3x^2 + 4y^2 = 48$ such that M lies in the first quadrant. Then the normal to the ellipse drawn at M passes through	(3) the point (3, 2)
(S) Let H be the hyperbola whose centre is at the origin, one of the foci is at (5, 0), and one directrix is $5x + 16 = 0$. Then H passes through	(4) the point (2, 5)
	(5) the point $(8,3\sqrt{3})$

[A] (P) → (3), (Q) → (4), (R) → (1), (S) → (2)
[B] (P) → (3), (Q) → (2), (R) → (1), (S) → (5)
[C] (P) → (3), (Q) → (2), (R) → (4), (S) → (5)
[D] (P) → (4), (Q) → (1), (R) → (2), (S) → (3)

[ANS] B

[SOLN] (P) radius $= \dfrac{|3+8-1|}{\sqrt{3^2+4^2}} = 2$

$\therefore$ eqn of circle is

$(x-1)^2 + (y-2)^2 = 2^2$

Which passes through (3, 2)

(Q) Tangent to circle is

$$y = mx \pm \sqrt{2}\sqrt{m^2+1}$$

And to parabola is

$y = mx + \dfrac{2}{m}$

For common tangent

$\pm\sqrt{2}\sqrt{m^2+1} = \dfrac{2}{m}$

$\Rightarrow$ $m^2 + 1 = \dfrac{2}{m^2} \Rightarrow m^4 + m^2 - 2 = 0$

$\Rightarrow$ $m = \pm 1$

$\therefore$ tangent with the slope is

$y = x + 2$

passes through (7, 9)

(R) $\dfrac{x^2}{16} + \dfrac{y^2}{12} = 1$

$M \equiv \left(ae, \dfrac{b^2}{a}\right) \equiv \left(4\sqrt{1-\dfrac{12}{16}}, \dfrac{12}{4}\right)$

$\equiv (2,3)$

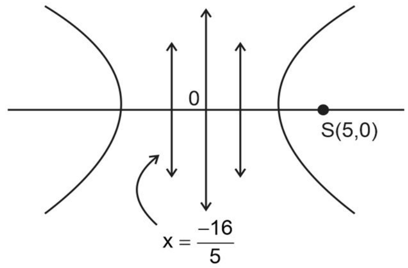
Normal at (2, 3) is

$\dfrac{16x}{2} - \dfrac{12y}{3} = 16 - 12$

$\Rightarrow$ $2x - y = 1$

Passes through (1, 1)

(S) $ae = 5$, $\dfrac{a}{e} = \dfrac{16}{5}$



$\therefore \quad a^2 = 16, e^2 = \frac{25}{16}$

$b^2 = a^2(e^2 - 1)$

$= 9$

$\therefore$ eqn of hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Passes through $\left(8, 3\sqrt{3}\right)$

CHEMISTRY

SECTION 1 (Maximum Marks :12)

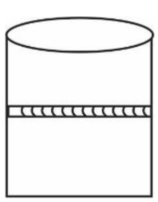
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

[Q.1] An ideal gas (0.5 mol), initially at 2 bar pressure, is compressed at a constant temperature of 600 K in two steps: first, against a constant external pressure of P bar ($2 < P < 8$), and then against constant external pressure of 8 bar. At each step, the compression is stopped only when the pressure of the gas becomes equal to the external pressure. The total work done on the gas in these steps is W. Considering all possible values of P ($2 < P < 8$) and taking the gas constant as R (in J K^{-1} mol^{-1}), the minimum value of $|W|$ (in J) is

[A] 207 R [B] 600 R [C] 630R [D] 900R

[ANS] **B**

[SOLN]



n= 0.5 mol
T_{const} = 600K

1^{st} step:

$P_{ext} = P$

$$\therefore W = -P(V_2 - V_1) = -P\left(\frac{0.5 \times R \times 600}{P} - \frac{0.5R \times 600}{2\,bar}\right)$$

$$\therefore W_1 = -P\left(\frac{300R}{P} - \frac{300R}{2}\right)$$

Single compression Thus, W_1 = +ve

Thus P > 2

2^{nd} step.

$P_{ext} = 8\,bar$

$$\therefore W_2 = -8(v_3 - v_2)$$

$$W_2 = -8\left(\frac{300R}{8} - \frac{300R}{P}\right)$$

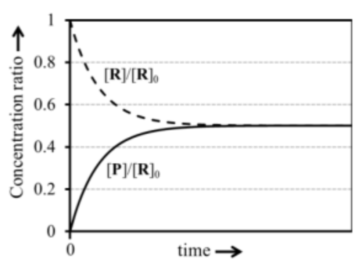
$$\therefore W = W_1 + W_2$$

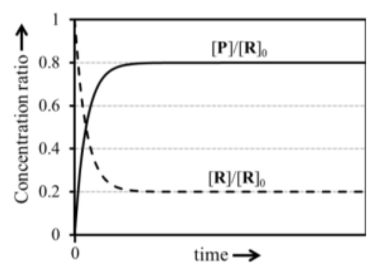
$$W = -300R + \frac{300R}{2} \times P - 300R + \frac{2400R}{P}$$

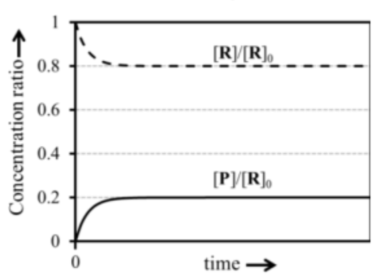
$$= -300R - 300R + \left(150RP + \frac{2400R}{P}\right)$$

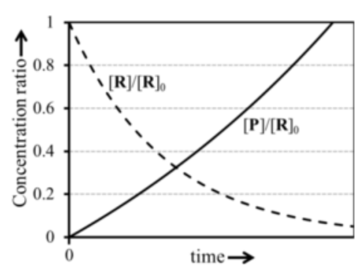
$\therefore P = 4$ Thus 600R

[Q.2] For a reversible reaction R ⇌ P, at constant temperature, both the forward and the backward reactions are first order elementary reactions with rate constants k_f and k_b, respectively. At time zero, the concentration of R is $[R]_0$ and the concentration of P is zero. At any given time, [R] and [P] are the concentrations of R and P, respectively. If $k_b = 4k_f$, the correct graphical representation of the reaction is

[A] 

[B] 

[C] 

[D] 

[ANS] **C**

[SOLN] $R \rightleftharpoons P$ const T.

$t = 0 \quad R_0 \qquad 0$

$t = t_{eq} \quad R_0 - x \quad x$

$\frac{x}{R_0 - x} = \frac{1}{4}$ $\qquad K_{eq} = \frac{K_f}{K_b} = \left(\frac{1}{4}\right)$

$4x = R_0 - x$ $\qquad \therefore \frac{R}{R_0} = \frac{R_0 - \frac{R_0}{3}}{R_0} = 1 - \frac{1}{5} = \frac{4}{5} = 0.8$ at equilibrium.

$5x = R_0$ $\qquad P = \frac{R_0}{5} \quad \therefore \frac{P}{R_0} = \frac{1}{5} = 0.2$

$x = \frac{R_0}{5}$

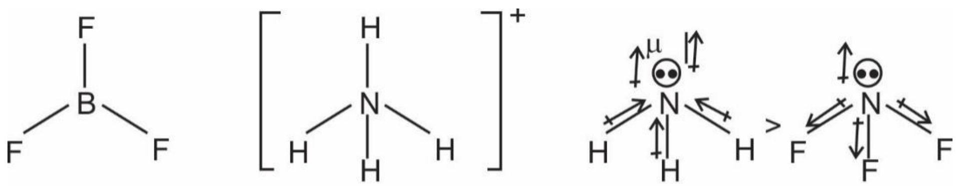
[Q.3] The correct order of dipole moments for the given species is

[A] $BF_3 = NH_4^+ < NF_3 < NH_3$ [B] $BF_3 < NH_4^+ < NF_3 < NH_3$

[C] $NH_4^+ < BF_3 < NH_3 < NF_3$ [D] $BF_3 < NH_4^+ < NH_3 < NF_3$

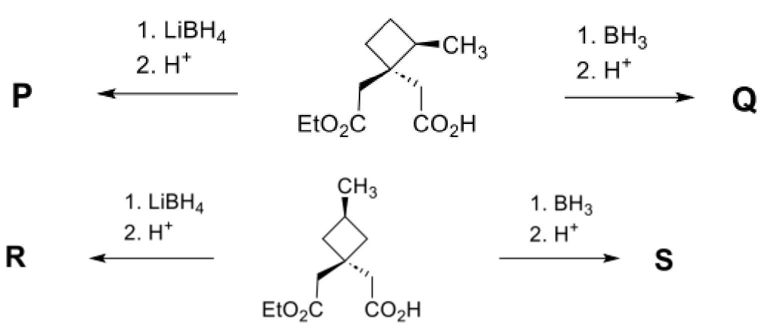
[ANS] **A**

[SOLN] Correct order of dipole moment:



$\mu = 0$ $\qquad \mu = 0$

[Q.4] Considering $LiBH_4$ reduces an ester group to the corresponding alcohol and does not reduce a carboxylic acid group, the correct statement about the major products P, Q, R and S is



[A] P & Q are identical, and R & S are diastereomers.

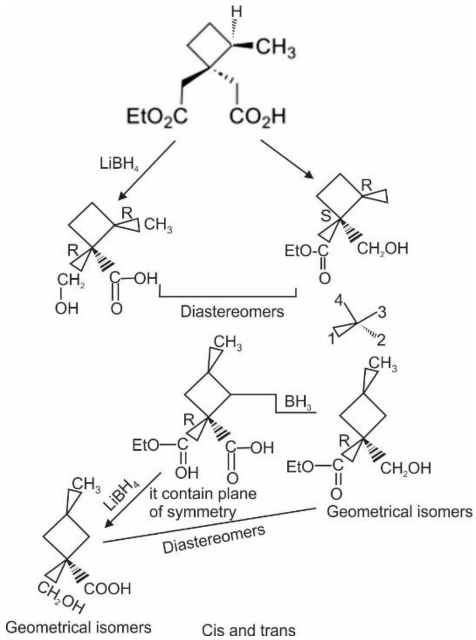
[B] P & Q are diastereomers, and R & S are identical.

[C] P & Q are diastereomers, and R & S are diastereomers

[D] P & Q are identical, and R & S are identical.

[ANS] **C**

[SOLN]



NOT: $LiBH_4$ / H^+ reduces ester while BH_3 / H^+ reduces acid to alchol

SECTION 2 (Maximum Marks :16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get −2 marks.

[Q.5] The 2s and the 2p orbital energies of hydrogen atom are $E_{2s}(H)$ and $E_{2p}(H)$, respectively. The 2s and the 2p orbital energies of lithium atom are $E_{2s}(Li)$ and $E_{2p}(Li)$, respectively. The correct option(s) about the orbital energies is(are)

[A] $E_{2s}(Li) < E_{2p}(Li)$ [B] $E2s(H) = E_{2p}(H)$ [C] $E_{2p}(H) < E_{2s}(Li)$ [D] $E_{2s}(H) > E_{2s}(Li)$

[ANS] **ABD**

[SOLN] For H-atoms $E_{2s} = E_{2p}$ $E_{2SH} > E_{2SLi}$

For Li-atoms $E_{2S} < E_{2P}$ more Zeff.

[Q.6] Correct statement(s) about the compounds X, Y and Z is(are)

$$MnO_2 + \text{Conc. HCl} \longrightarrow MnCl_2 + \underset{\text{(greenish yellow gas)}}{\mathbf{X}} + H_2O$$

$$NH_3 + \underset{\text{(excess)}}{\mathbf{X}} \longrightarrow \mathbf{Y} + HCl$$

$$\mathbf{X} + \underset{\text{(excess)}}{F_2} \xrightarrow{573\text{ K}} \mathbf{Z}$$

[A] **X** is used for sterilizing drinking water.

[B] **Y** has a planar structure.

[C] **Z** is used in the enrichment of ^{235}U.

[D] **Y** is a stronger Lewis base than ammonia.

[ANS]

[SOLN] $MnO_2 + conc.HCl \rightarrow MnCl_2 + \underset{\text{greenish yellow}(x)\text{gas}}{Cl_2} + H_2O$

$NH_3 + \underset{\text{excess}}{Cl_2} \rightarrow \underset{Y}{NCl_3} + HCl$

$Cl_2 + \underset{\text{excess}}{F_2} \xrightarrow{573K} \underset{Z}{ClF_3}$

A. True B. Nonplanar(False)

C. True D. $NCl_3 < NH_3 - L.B.$

$U + 3ClF_3 \rightarrow UF_6 + 3ClF$

[Q.7] Reaction of PtF_6 with oxygen (O_2) gas results in the formation of an ionic compound, X^+Y^-. Correct statement(s) is(are)

[A] The bond order of $\mathbf{X^+}$ is 1.5.

[B] Valence *d*-orbitals of the metal ion in $\mathbf{X^+Y^-}$ has 5 electrons.

[C] PtF_6 acts as an oxidant in this reaction.

[D] PtF_6 acts as a fluorinating agent in this reaction.

[ANS] **BC**

[SOLN] $\overset{+6}{PtF_6} + O_2^0 \rightarrow O_2^+ + \left[\overset{+5}{Pt}F_6\right]^-$ Red colour complex

O.A R.A

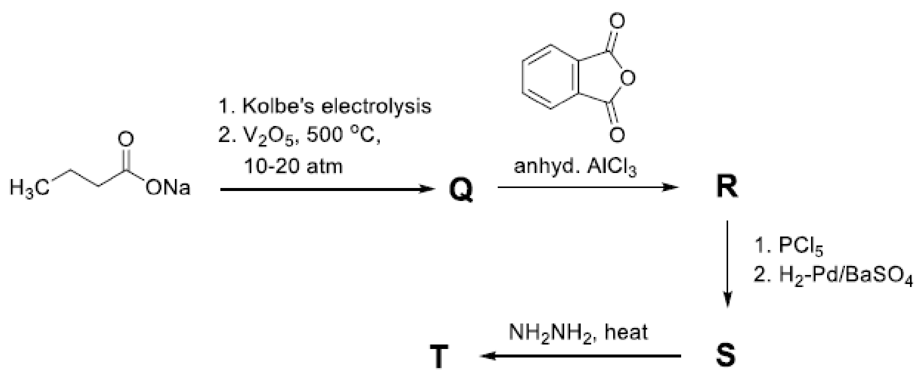
A. O_2^+ $(B.O = 2.5)$

B. $O_2^+\left[\overset{+5}{Pt}F_6\right]^- - Pt^{+5} - 3d^5$

C. $PtF_6 - O.A$

D. PtF_6 (False) not a fluorinating agent in above reaction

[Q.8] In the following reaction sequence, Q, R, S and T are the major products.



The correct statement(s) about **Q**, **R**, **S** and **T** is(are)

[A] **S** on warming with ammonical $AgNO_3$ results in the formation of silver mirror.

[B] **Q** on treatment with Cl_2(excess)/UV gives gammaxane

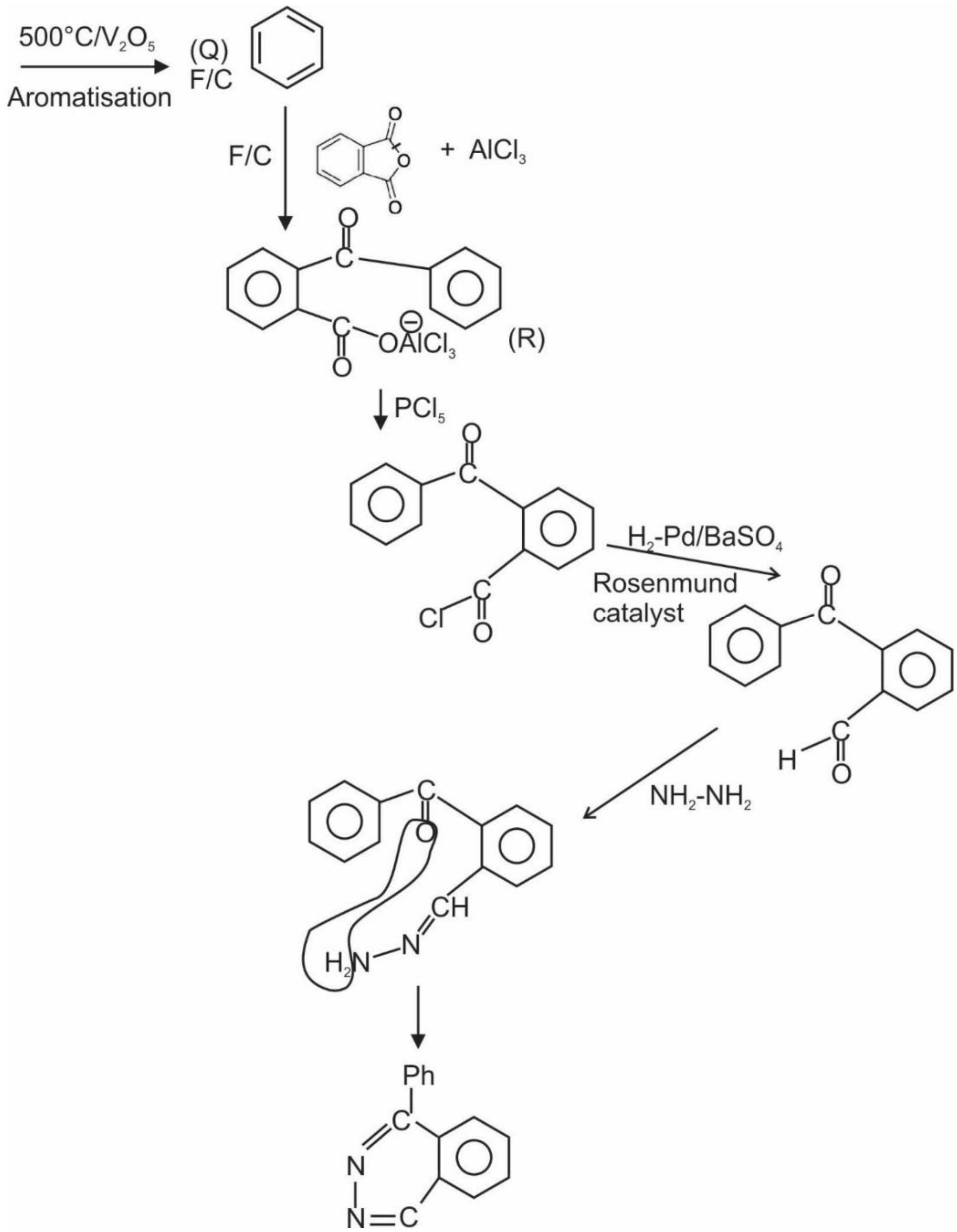
[C] **T** is a heterocyclic compound.

[D] **R** on acid catalyzed intramolecular cyclization followed by treatment with Zn-Hg/HCl gives 9,10-dihydroxyanthracene.

[ANS] **ABC**

[SOLN] $CH_3 - CH_2 - CH_2 - \overset{\overset{O}{||}}{C} - ONa$

$$\xrightarrow{\text{kolbes}} \underset{\text{ethyl radical}}{CH_3CH_2\dot{C}H_2} \xrightarrow{\text{Dimerise}} \underset{\text{n-hexane}}{CH_3CH_2CH_2CH_2CH_2CH_3}$$



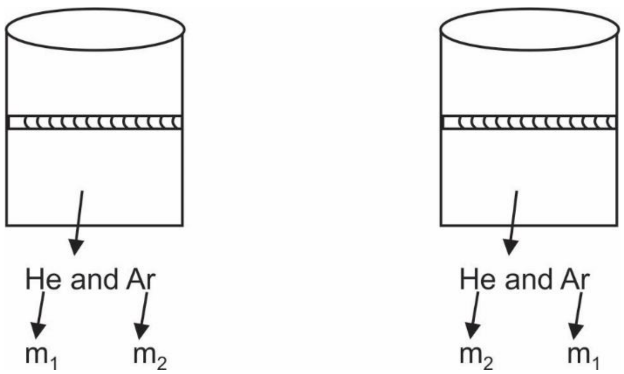
SECTION 3 (Maximum Marks :16)

- This section contains **FOUR (04)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

[Q.9] Two cylinders, both fitted with frictionless pistons, are filled with mixtures of He and Ar gases. In the first cylinder, the masses of He and Ar are m_1 and m_2, respectively. In the second cylinder, the masses of He and Ar are m_2 and m_1, respectively. The molar mass of Ar is 10 times the molar mass of He. The external pressure applied by the piston on the first cylinder needs to be 5 times that on the second cylinder so that the volume of the gas mixtures in both the cylinders are equal at the same temperature. Assuming He and Ar behave like ideal gases, the value of (m_1/m_2) is ___

[ANS] **9.8**

[SOLN]



$n_{total_1} = \frac{m_1}{4} + \frac{m_2}{40}$ $n_{total_2} = \frac{m_2}{4} + \frac{m_1}{40}$

Volume same in both temperature same in both

$\therefore P \propto n$

$\therefore P_1 = 5P_2$

$\therefore \left[\frac{P_1}{P_2} = 5 = \frac{n_{total_1}}{n_{total_2}}\right]$

$$\frac{\frac{10m_1 + m_2}{40}}{\frac{10m_2 + m_1}{40}} = 5$$

$10m_1 + m_2 = 50m_2 + 5m_1$

$5m_1 = 49m_2$

$\frac{m_1}{m_2} = \frac{49}{5} = 9.8 \approx 10$

[Q.10] The total number of all possible isomers for the square planar complex with formula $K[M(NCS)(NO_2)(gly)]$ is ____.

(M = metal ion and gly = $NH_2CH_2COO^-$)

[ANS] **8**

[SOLN] $K[M(NCS)(NO_2)(gly)] = 2$ $\quad\quad$ $[M(AB)_2(a)(b)]$ having 2GI

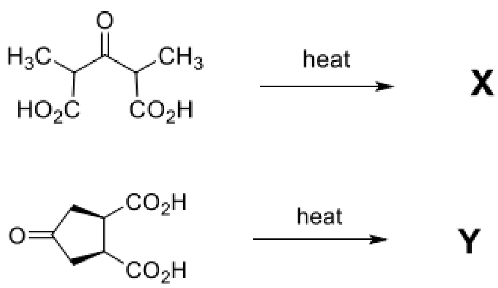
$K[M(SCN)(NO_2)(gly)] = 2$

$K[M(SCN)(ONO)(gly)] = 2$

$K[M(ONO)(NCS)(gly)] = 2$

$\overline{\quad 8 \quad}$

[Q.11] The sum of total number of carbonyl groups (>C=O) present in the major products X and Y in the following reactions is ____.



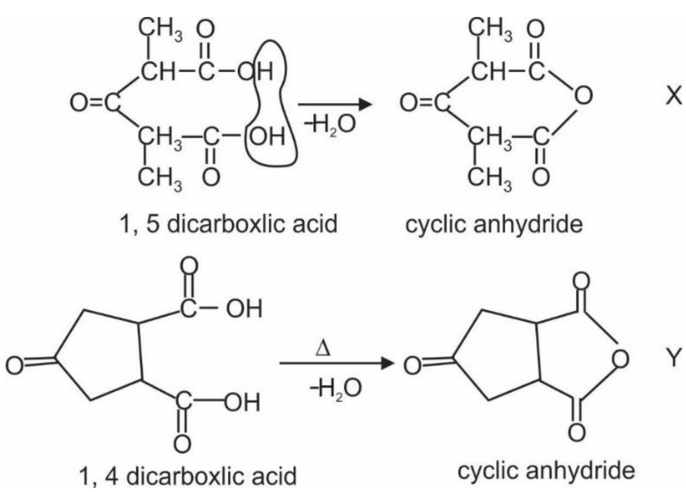
[ANS] **6**

[SOLN] According Blanc rule

A.1,2 & 1,3 on heat CO_2 will remove

B. 1,4 and 1,5 on heat H_2O molecule will remove and gives anhydride

C. 1,5,1,6 on heat H_2O & CO_2 will remove and gives cyclic ketone



X contain 3 carbonyl group

Y contain 3 carbonyl group

Total = 6

[Q.12] Treatment of buta-1,3-diyne with $NaNH_2$ (2 equivalents), followed by reaction with excess of trans-CH_3-CH=CH-CH_2-Br gives X as the major product. The maximum number of carbon atoms that are collinear (in a straight line) in X is ___

[ANS] **6**

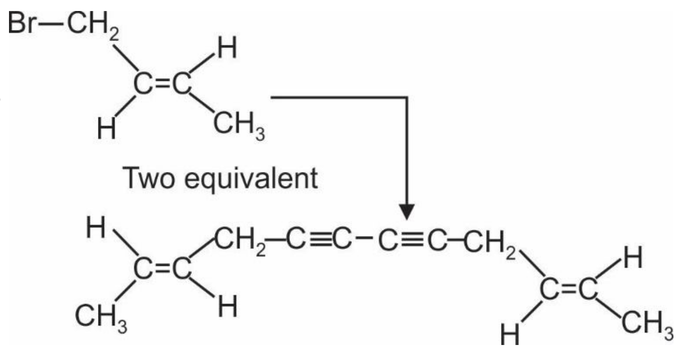
[SOLN]

$\overset{1}{CH} \equiv \overset{2}{C} - \overset{3}{C} \equiv \overset{4}{CH}$ It contain 2 acidic hydrogen

1,3 butadiyne

$\downarrow 2NaNH_2$

$^{-}C \equiv C - C \equiv C^{-}$ +



Total number of colliner hydrogen = Total number of sp hybridise carbon + carbon attached to

sp hybride carbon = 6

SECTION 4 (Maximum Marks :16)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II.**
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.

[Q.13] List-I contains various physical/chemical processes, and List-II contains combinations of changes in enthalpy (ΔH) and entropy (ΔS). Match each entry in List-I to the appropriate entry in List-II, and choose the correct option.

	List I		List II
P.	Physisorption	**1.**	$\Delta H > 0$ and $\Delta S > 0$
Q.	Diamond ⟶ Graphite	**2.**	$\Delta H < 0$ and $\Delta S < 0$
R.	Denaturation of protein	**3.**	$\Delta H < 0$ and $\Delta S = 0$
S.	Propene ⟶ Cyclopropane	**4.**	$\Delta H > 0$ and $\Delta S < 0$
		5.	$\Delta H < 0$ and $\Delta S > 0$

[A] $P \to 2; Q \to 3; R \to 5; S \to 4$ [B] $P \to 4; Q \to 3; R \to 5; S \to 1$

[C] $P \to 2; Q \to 5; R \to 1; S \to 4$ [D] $P \to 2; Q \to 5; R \to 1; S \to 3$

[ANS] **C**

[SOLN] P. Physisorption $\to$ (2) $\Delta H < 0$ $\Delta S < 0$

Q. Diamond $\to$ Graphite $\to$ (5) $\Delta H < 0$ $\Delta S > 0$

R. Denaturant of protein $\to$ (1) $\Delta H > 0$ $\Delta S > 0$

S. Propene $\to$ Cyclopropane $\to$ (4) $\Delta H > 0$ $\Delta S < 0$

[Q.14] Consider the following species:

$SOCl_2$, $XeOF_4$, ClF_3, ClF_5, XeF_5^+, SO_3^{2-}, XeF_3^+, SF_4

List-I contains different molecular shapes and List-II contains total number of species with the same molecular shapes from the given species. Match each entry in List-I with the appropriate entry in List-II and choose the correct option.

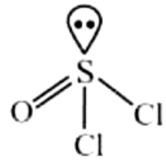
List-I	List-II
(P) See-saw	(1) one
(Q) T-Shaped	(2) two
(R) Trigonal Planar	(3) three
(S) Square Pyramidal	(4) four
	(5) zero

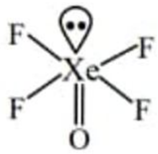
[A] $P \to 1; Q \to 2; R \to 5; S \to 3$ [B] $P \to 5; Q \to 4; R \to 2; S \to 3$

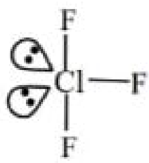
[C] $P \to 3; Q \to 2; R \to 1; S \to 4$ [D] $P \to 1; Q \to 3; R \to 5; S \to 4$

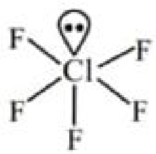
[ANS] A

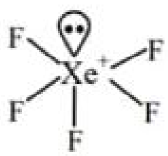
[SOLN]

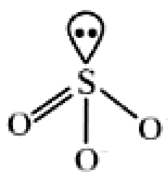
$SOCl_2 \Rightarrow$  (Pyramidal)

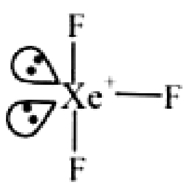
$XeOF_4 \Rightarrow$  (Square Pyramidal)

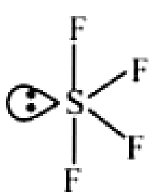
$ClF_3 \Rightarrow$  (T-shape)

$ClF_5 \Rightarrow$  (Square Pyramidal)

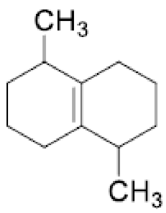
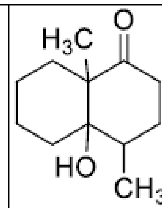
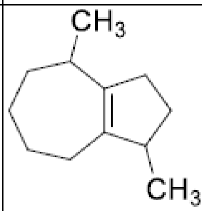
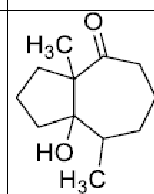
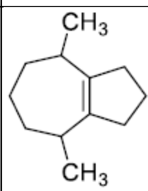
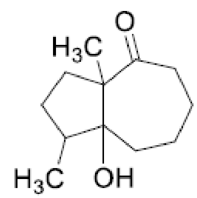
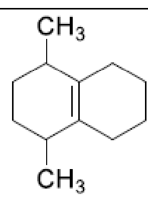
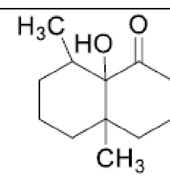
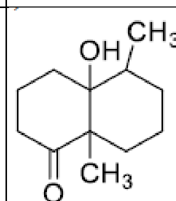
$XeF_5^{\oplus} \Rightarrow$  (Square Pyramidal)

$SO_3^{2-} \Rightarrow$  (Pyramidal)

$XeF_3^{\oplus} \Rightarrow$  (T-shape)

$SF_4 \Rightarrow$  (See-saw)

[Q.15] The List-II contains products obtained from the reaction of compounds in List-I with O_3/Zn-H_2O followed by cyclization (via more stable enolate) in the presence of aqueous NaOH. Match each entry in List-I with appropriate entry in List-II and choose the correct option.

List I		**List II**	
P.		**1.**	
Q.		**2.**	
R.		**3.**	
S.		**4.**	
		5.	

[A] $P \rightarrow 2; Q \rightarrow 4; R \rightarrow 1; S \rightarrow 3$

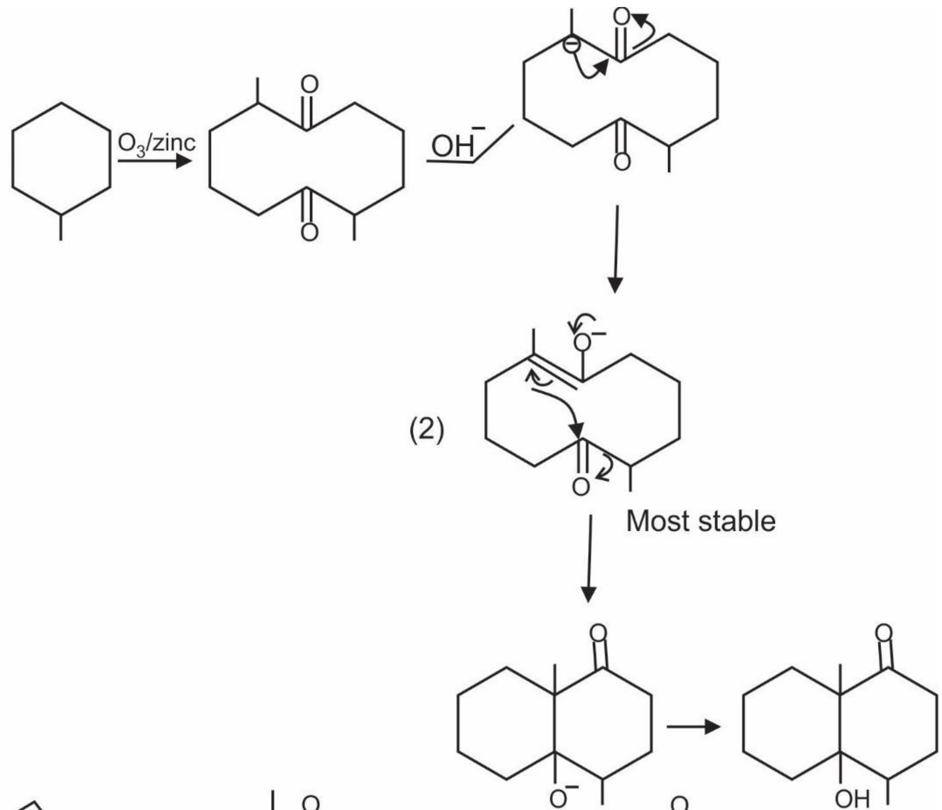
[B] $P \rightarrow 3; Q \rightarrow 4; R \rightarrow 5; S \rightarrow 2$

[C] $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 5; S \rightarrow 3$

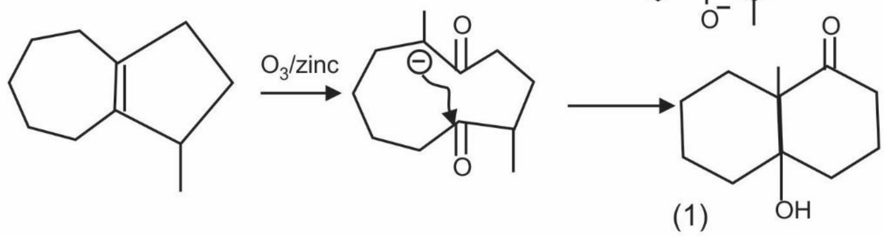
[D] $P \rightarrow 3; Q \rightarrow 5; R \rightarrow 4; S \rightarrow 2$

[ANS] **C**

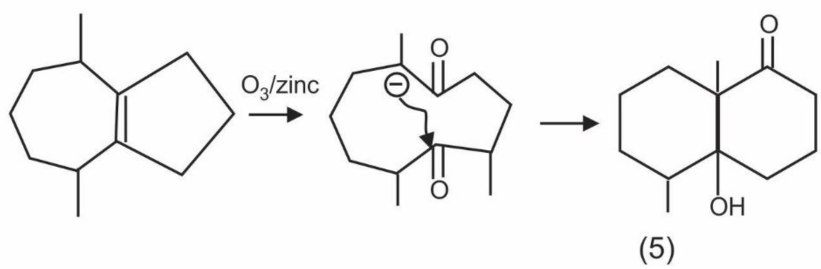
[SOLN] P



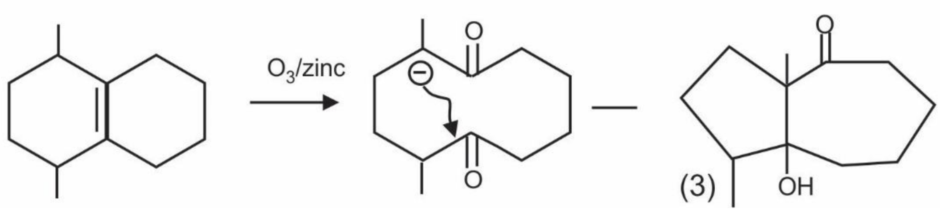
Q



R



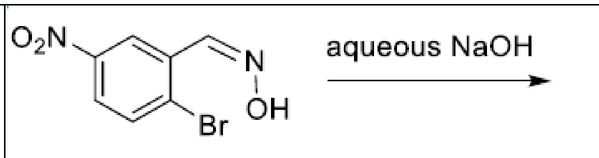
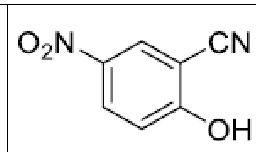
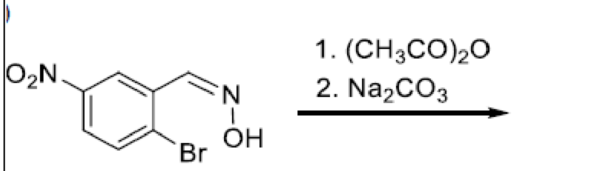
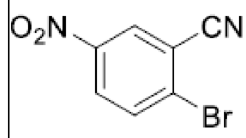
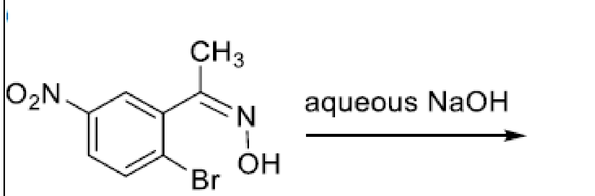
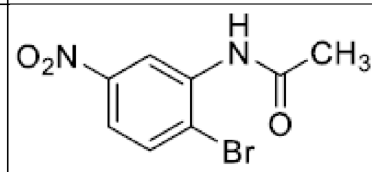
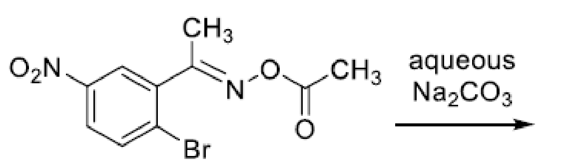
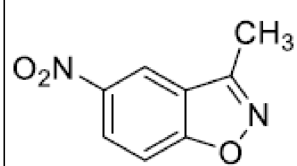
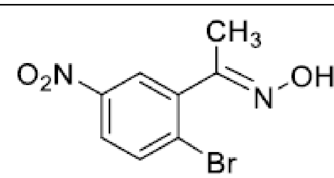
S



In this reaction first will be formed stable enolat ion. It is a example of ALDOL reaction.

P-2, Q = 1, R= 5, S= 3

[Q.16] Match the major products obtained in the reactions given in List-I with the corresponding structures in List-II and choose the correct option.

	List I		List II
P.		**1.**	
Q.		**2.**	
R.		**3.**	
S.		**4.**	
		5.	

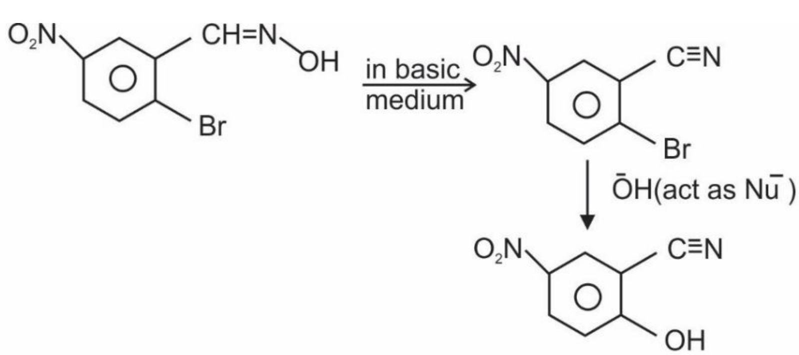
[A] $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 5; S \rightarrow 4$ [B] $P \rightarrow 1; Q \rightarrow 2; R \rightarrow 4; S \rightarrow 5$

[C] $P \rightarrow 1; Q \rightarrow 2; R \rightarrow 3; S \rightarrow 4$ [D] $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 3; S \rightarrow 5$

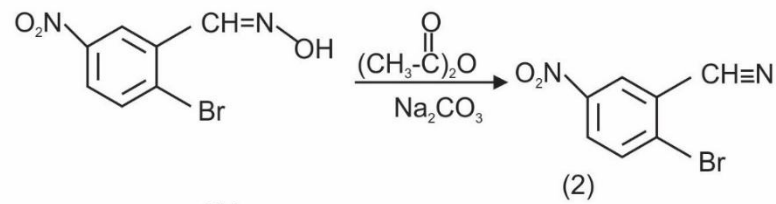
[ANS] **B**

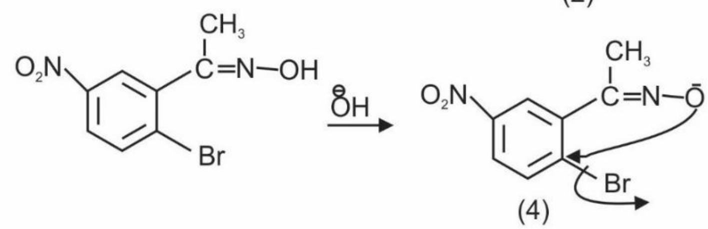
[SOLN]

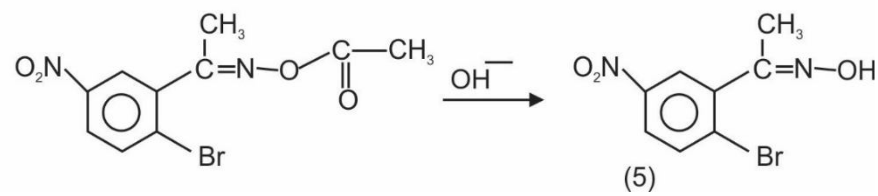
P



(1)

Q. 

R 

S 

P: Normally aldooxime do not gives nitrile compound but in presence of strong electron NO_2 adooxime changes in to nitrile. In this reaction OH- act as nucleophile and gives aromatic nucleophilic substitution reaction because electron withdrawing group i.e. NO_2 present at paraposition w.r.t to Br

B. Normally aldoxime converted into nitrile compound either in the presence acidic medium or anhydride

C. ketooxime neve gives nitrile compound, OH group nitrogen release H^+ so O^- of nitrogen act as nucleophile and gives cyclic nucleophilic aromatic substitution reaction which helps by electron withdrawing group at para position of Bromine.

D. In this reaction simple hydrolysis.

PHYSICS

SECTION 1 (Maximum Marks :12)

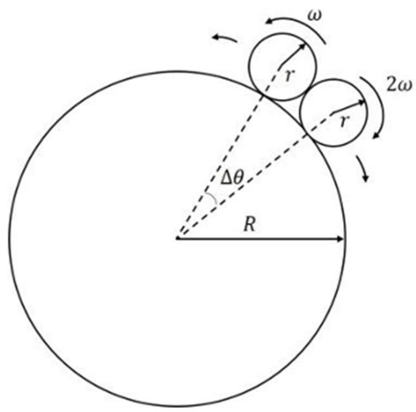
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

[Q.1] Consider a large disk of radius R and two smaller disks, each of radius $r = R/50$, lying on its circumference, as shown in the figure. The smaller disks are initially in contact with each other, with an angular separation $\Delta\theta$ between their centers. They are made to roll without slipping in opposite directions, with constant angular velocities ω and 2ω while the large disk is held stationary. The time τ at which the smaller disks are again in contact is: [Use $\sin(\Delta\theta) = \Delta\theta$ and ignore gravity.]



[A] $\tau = 51\times\left(2\pi - \frac{4}{51}\right)/\omega$

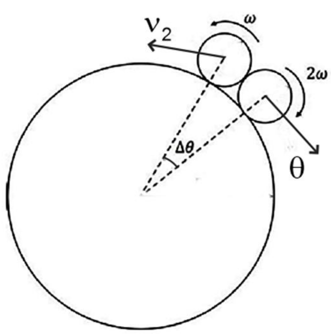
[B] $\tau = 51\times\left(2\pi - \frac{2}{51}\right)/3\omega$

[C] $\tau = 51\times\left(2\pi - \frac{4}{51}\right)/3\omega$

[D] $\tau = 51\times\left(2\pi - \frac{2}{51}\right)/\omega$

[ANS] **B**

[SOLN]



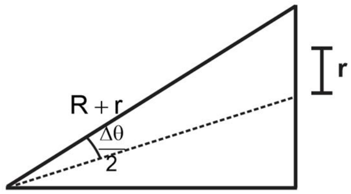
$\left.\begin{array}{l} v_1 = 2\omega r \\ \& v_2 = \omega r \end{array}\right] \rightarrow$ Pure rolling

Assume that centre of both disks moving with angular velocity ω_1 and ω_2 w.r.t. Larger disc.

Then time taken $t = \dfrac{(2\pi - \Delta\theta)}{\omega_1 + \omega_2}$

$\omega_1 (R + r) = v_1$

$\omega_2 (R + r) = v_2$

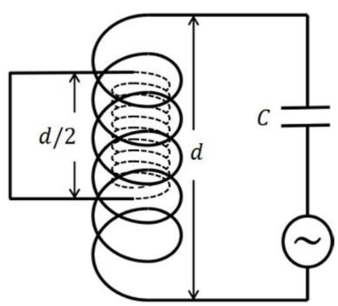


$$\tan\left(\frac{\Delta\theta}{2}\right) = \sin\left(\frac{\Delta\theta}{2}\right) = \frac{\Delta\theta}{2} = \frac{r}{R+r} = \frac{\frac{R}{50}}{\frac{R}{50}+R} = \frac{1}{51}$$

$$\text{So } t = \frac{2\pi - 2\left(\frac{1}{51}\right)}{\frac{\nu_1}{R+r} + \frac{\nu_2}{R+r}} = \frac{(R+r)}{3\omega r}\left(2\pi - \frac{2}{51}\right)$$

$$t = \frac{\left(R + \frac{R}{50}\right)}{3\omega\left(\frac{R}{50}\right)}\left(2\pi - \frac{2}{51}\right) = \frac{51}{3\omega}\left(2\pi - \frac{2}{51}\right)$$

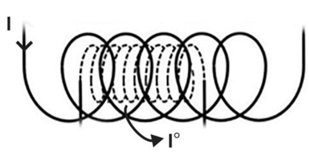
[Q.2] Consider a circuit consisting of a capacitor of capacitance C and a coil with N turns per unit length, cross sectional area N and length d, where $d^2 >> S$.There is another coil of length $d/2$, cross sectional area $s/2$ and 2N turns per unit length completely inside the larger coil, as shown in the figure. The ends of this smaller coil are connected with each other by an insulated conducting wire. The self-inductance of the larger coil is L Neglecting edge effects and all the Ohmic resistances, the resonant frequency of the circuit is:



[A] $\frac{4}{\sqrt{15LC}}$ [B] $\frac{6}{\sqrt{5LC}}$ [C] $\frac{2}{\sqrt{3LC}}$ [D] $\sqrt{\frac{2}{3LC}}$

[ANS] C

[SOLN]



$$\frac{L}{\ell} = \mu_0 n^2 A$$

(Self For, Larger Coil $L = \mu_0 (N)^2 S(d)$

(Self) For, smaller Coil $L' = \mu_0 (2N)^2 \frac{s}{2}\frac{d}{2} = L$

& For mutual inductance $M = \mu_0 (N)(2N)\left(\frac{s}{2}\right)\frac{d}{2} = \frac{L}{2}$

$\Rightarrow \phi_{Larger} = LI + MI'$

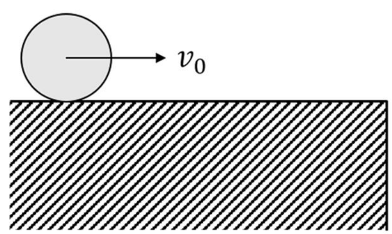
& $\phi_{smaller} = MI + L'I' = 0$ (Short Circular & R is negligible) $\Rightarrow I' = \frac{-MI}{L'} = \left(\frac{-L/2}{L}\right)I = -\frac{I}{2}$

$\Rightarrow \phi_{Larger} = LI + \left(\frac{L}{2}\right)\left(\frac{-I}{2}\right) = \frac{3L}{4}I$

So $Le_{ff} = \frac{3L}{4}$

$w = \frac{1}{\sqrt{Le_{ff}C}} = \frac{2}{\sqrt{3LC}}$

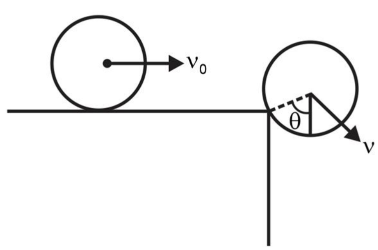
[Q.3] A solid cylinder of radius R rolls without slipping with a center of mass speed $v_0 = \sqrt{\frac{gR}{3}}$ on a horizontal surface with a vertical edge, as shown in the figure. Here, g is the acceleration due to the gravity. At the moment when the cylinder loses contact with the surface due to rotation around the corner, the speed of its center of mass is:



[A] 0 [B] $\sqrt{\frac{5gR}{7}}$ [C] $\sqrt{\frac{gR}{15}}$ [D] $\sqrt{\frac{3gR}{7}}$

[ANS] **B**

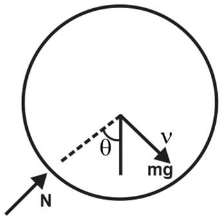
[SOLN]



Apply WET

$$mg R - mg R\cos\theta = \frac{1}{2}mv^2\left(1+\frac{1}{2}\right) - \frac{1}{2}mv_0^2\left(1+\frac{1}{2}\right)$$

Now



$$mg\cos\theta - \cancel{N}^{\,0} = \frac{m\nu^2}{R}$$

$$\nu^2 = gR\cos\theta$$

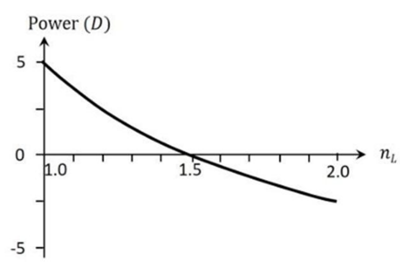
Put in above eq.

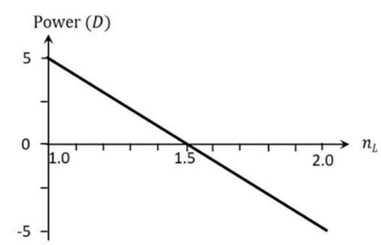
$$mgR - m\nu^2 = \frac{m}{2}\left(\frac{3}{2}\right)\left[\nu^2 - \upsilon_0^2\right]$$

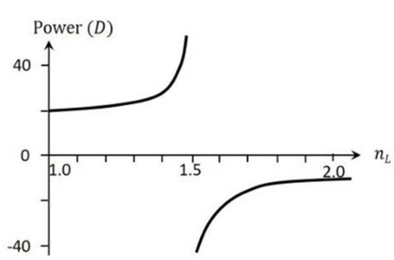
$$gR - \nu^2 = \frac{3}{4}\nu^2 - \frac{3}{4}\times\left[\frac{gR}{3}\right]$$

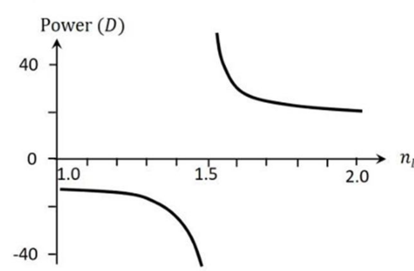
$$gR\left[\frac{5}{4}\right] = \nu^2\left[\frac{7}{4}\right] \Rightarrow \nu = \sqrt{\frac{5gR}{7}}$$

[Q.4] A double convex lens made of glass of refractive index 1.5 and radii of curvature of the curved surfaces 20 cm each is immersed in a liquid of refractive index n_L. The correct plot showing the variation of the power, in the units of diopter (D), as a function of n_L is:

[A] 

[B] 

[C] 

[D] 

[ANS] **A**

[SOLN] $P_{lens} = \frac{1}{f_{lens}} = \left[\frac{n_\ell - n_0}{n_0}\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

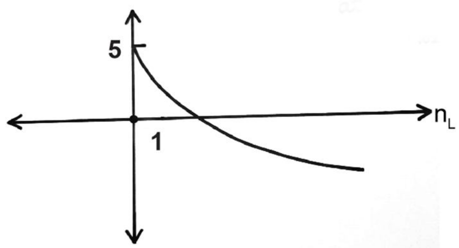
n_0 (n_ℓ) n_0 here n_0 is written n_L

$$P_{lens} = \frac{1.5 - n_L}{n_L}\left[\frac{1}{20} - \left(\frac{-1}{20}\right)\right] \times 100$$

$$P_{lens} = \frac{15}{n_L} - 10 \qquad \Rightarrow P_{lens} \propto \frac{1}{n_L} \text{ (Rectangular hyperbola)}$$

at $n_L = 1$

$P_{lens} = 5D$



SECTION 2 (Maximum Marks :16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get −2 marks.

[Q.5] Consider a hydrogen atom with ν_k, r_k and K_k denoting the velocity, orbital radius and kinetic energy of the electron in the k^{th} orbit, respectively. The electron undergoes a transition from the n^{th} orbit, emitting radiation corresponding to the Lyman series. Considering h to be the Planck's constant and ϵ_0 the permittivity of the free space, the correct statements (s) is/are:

[A] Magnitude of change in kinetic energy of electron can be expressed as $\frac{h}{4\pi}\left|\frac{n\nu_n}{r_n}-\frac{\nu_1}{r_1}\right|$.

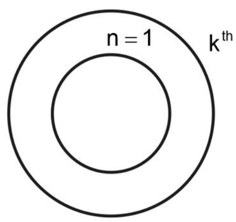
[B] Magnitude of change in de Broglie wavelength of the electron can be expressed as $\frac{e^2}{4\epsilon_0}\left|\frac{1}{k_n}-\frac{1}{k_1}\right|$.

[C] Frequency of the radiation emitted can be expressed as $\frac{e^2}{8\pi\epsilon_0 h}\left(\frac{1}{r_1}-\frac{1}{r_n}\right)$.

[D] Magnitude of change in total energy of the electron can be expressed as $\frac{h}{2\pi}\left|\frac{\nu_1}{r_1}-\frac{n\nu_n}{r_n}\right|$.

[ANS] **A, C**

[SOLN]



$$m\nu_n r_n = \frac{nh}{2\pi}$$

(A) $\Delta k\cdot E = \left|\frac{1}{2}m\nu_n^2 - \frac{1}{2}m\nu_1^2\right|$

$$= \frac{1}{2}\left|\left[(m\nu_n)\nu_n - (m\nu_1)\nu_1\right]\right|$$

$$= \frac{1}{2}\left|\left[\frac{nh}{2\pi r_n}\nu_n - \frac{h}{2\pi r_1}\nu_1\right]\right|$$

$$= \frac{h}{4\pi}\left|\frac{n\nu_n}{r_n}-\frac{\nu_1}{r_1}\right|$$

(B) $|\lambda_n - \lambda_1| = \left|\frac{h}{m\nu_n}-\frac{h}{m\nu_1}\right| =$

$$\frac{m\nu_n^2}{r_n} = \frac{e^2}{r_n^2 4\pi\varepsilon_0}$$

$$\frac{m\nu_1^2}{r_1^2} = \frac{e^2}{r_1^2 4\pi\varepsilon_0}$$

$$k_n = \frac{1}{2}m\nu_n^2$$

$$k_1 = \frac{1}{2}m\nu_1^2$$

$$E_n = \frac{-e^2}{2\pi\varepsilon_0 r_n}$$

$$E_1 = \frac{-e^2}{8\pi\varepsilon_0 r_1}$$

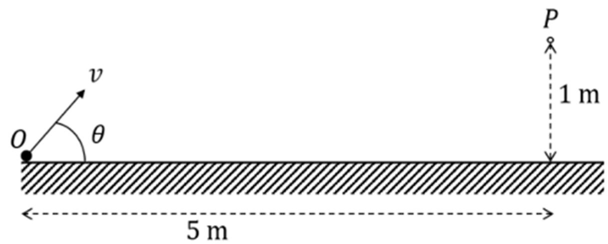
(C) $hf = |E_1 - En|$

$$= \frac{e^2}{8\pi\varepsilon_0}\left|\frac{1}{r_1} - \frac{1}{r_n}\right|$$

$$f = \frac{e^2}{8\pi\varepsilon_0 n}\left|\frac{1}{r_1} - \frac{1}{r_n}\right|$$

(D) $\Delta E = \Delta kE = \frac{h}{4\pi}\left|\frac{n\nu_n}{r_n} - \frac{\nu_1}{r_1}\right|$

[Q.6] A particle is thrown with a speed ν from a point O at an angle θ with the horizontal plane such that it pass through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statements (s) is/are:



[A] If $\theta = 45^o$, than $\nu = \frac{5\sqrt{g}}{2} ms^{-1}$.

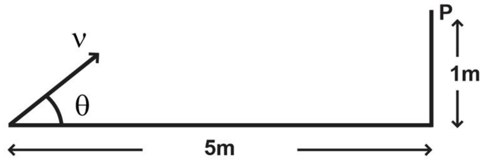
[B] If $\theta = 45^o$, the particle reaches its maximum height before it reaches P.

[C] If $\theta = 30^o$, the particle reaches its maximum height after reaching P.

[D] If $\theta = \tan^{-}\left(\frac{1}{5}\right)$, then $\nu = 125\sqrt{g}\, ms^{-1}$.

[ANS] **A, B**

[SOLN]



$$y = x\tan\theta - \frac{gx^2}{2\upsilon^2\cos^2\theta}$$

$$1 = 5\tan\theta - \frac{g\times(5)^2}{2\times\upsilon^2\cos^2\theta}$$

(i) $\theta = 45^o$, $1 = 5 - \dfrac{g\left((5)^2\right)}{2\times\upsilon^2\times\left(\frac{1}{2}\right)} \Rightarrow \nu^2 = \dfrac{g(5)^2}{4}$

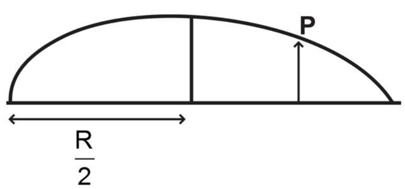
$$\nu = 5\frac{\sqrt{g}}{2}\,m/c$$

(ii) for $\theta = 45^o$

$$h_{max} = \frac{\nu^2\sin^2\theta}{2g} = \frac{25\times g}{4}\times\frac{\frac{1}{2}}{2g} = \frac{25}{8} > 1$$

$$\& \; R = \frac{\nu^2\sin 2\theta}{g} = \frac{25\times g}{4}\times 2\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\times\frac{1}{g} = \frac{25}{4} > 5$$

$$\& \; \frac{R}{2} = \frac{25}{8} < 5$$



(iii)

For $\theta = 30^o$,

$$\tan\theta = \frac{1}{\sqrt{3}}$$

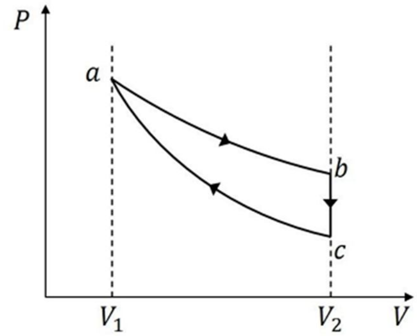
$$x = 3.82 < 5$$

(iv) if $\theta = \tan^{-1}\left(\frac{1}{5}\right)$

then $1 = 5\times\frac{1}{5} - \frac{g(5)^2}{2\nu^2}\left(1+\frac{1}{25}\right)$

$\nu \to \infty$

[Q.7] A quasi-static cycle of a monoatomic ideal gas contains an isothermal process (ab), followed by an isochoric process (bc) and an adiabatic process (ca) as shown in the figure. The volumes of the gas are V_1 and V_2 at a and b, respectively. If the cycle has heat input Q_{in} and output Q_{output}, then the efficiency of the cycle is defined as $\eta = \frac{Q_{in} - Q_{out}}{Q_{in}}$. The correct statement (s) is/are: [Given : $\ln 2 \approx 0.7$]



[A] If $V_2 / V_1 = 8$, the heat released in the process bc is smaller than the heat absorbed in the process ab.

[B] For a given value of V_2 / V_1, η does not depend on the temperature of the isothermal process.

[C] If $V_2 / V_1 = 8$, then the temperature of the gas at a is 4 times the temperature of the gas at c.

[D] If $V_2 / V_1 = 8$, then the pressure of the gas at a is 4 times pressure of the gas at b.

[ANS] **A, B, C**

[SOLN] $\theta_{ab} = nRT_a \ell n\left(\frac{V_2}{V_1}\right) = nRT_a \ell n8$

$\theta_{bc} = nC_v\left(T_c - T_b\right) = n\left(\frac{3}{2}R\right)\left(T_c - T_a\right) < 0 \quad T_c < T_a$

$\left[T_b = T_a\right]$

$C \rightarrow a$ (Adiabatic) $\left(\theta_{ca} = 0\right)$

$T_c V_2^{\gamma-1} = T_a V_1^{\gamma-1}$

$\frac{T_a}{T_c} = \left(\frac{V_2}{V_1}\right)^{5/3-1} = \left(8\right)^{2/3} = 4$

$T_a = 4T_c$

Option (A) $\theta_{ab} = nRT_a \ell n8 = nRT_a\left(3\ell n2\right) = 2.1\,nRT_a$

& $\left(\theta_{bc}\right) = \frac{3}{2}nR \cdot \left|\left(\frac{T_a}{4} - T_a\right)\right| = \frac{9}{8}nRT_a = 1.125\,nRT_a$

$\theta_{ab} > |\theta_{bc}|$

Option (B) $\eta = 1 - \frac{|\theta_{bc}|}{\theta_{ab}} = 1 - \frac{1.125\,nRT_a}{2.1nRT_a} = 1 - \frac{1.125}{2.1}$

Option (D) $P_aV_1 = P_aV_2 \Rightarrow \frac{P_a}{P_b} = \frac{V_2}{V_1} = 8$

$P_a = 8P_b$

[Q.8] The electric field associated with an electromagnetic wave travelling in vacuum is given by $E_0 \sin(3y + 4z + \omega t)\hat{i}$, where ω is the angular frequency. All quantities are in SI units. The correct statement (s) about this wave is/are:

[Given: speed of light in vacuum $c = 3 \times 10^8\ \text{ms}^{-1}$.]

[A] The wave is travelling in $-\frac{1}{5}(3\hat{j} + 4\hat{k})$ direction.

[B] The magnitude of the wave vector is $0.5\,\text{m}^{-1}$.

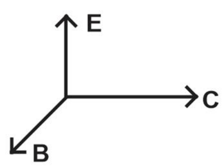
[C] The value of ω is 1.5×10^9 rad s^{-1}.

[D] The magnetic field associated with this wave is given by $\frac{E_0}{c}\sin(3y + 4z + \omega t)(4\hat{j} - 3\hat{k})$.

[ANS] **A, C**

[SOLN] $E = E_0 \sin(3y + 42 + \omega t)\hat{i}$

$\hat{E} \times \hat{B} = \hat{C}$

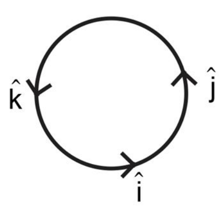


wave moving in $-\frac{(3\hat{j} + 4\hat{k})}{5}$ direction

& $\vec{E}$ is in $\hat{i}$

$\longrightarrow \vec{E}(\hat{i})$

$\hat{B} = (\hat{C} \times \hat{E}) = -\left[\frac{3\hat{j} + 4\hat{k}}{5}\right] \times \hat{i}$



$= -\left(-3\hat{k} + 4\hat{j}\right)$

$B = \dfrac{E}{C} = \dfrac{E_0}{C}\sin\left(3y + 4z + \omega t\right)\left(-4\hat{j} + 3\hat{k}\right)$

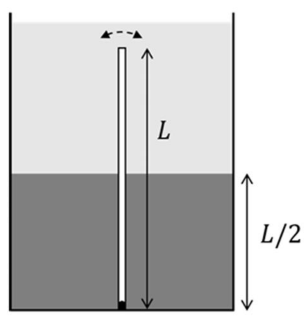
$\Rightarrow$ wave vector $= \sqrt{3^2 + 4^2} = 5\,m^{-1}$

& $\omega = C(k) = 1.5 \times 10^9$ radian s^{-1}

SECTION 3 (Maximum Marks :16)

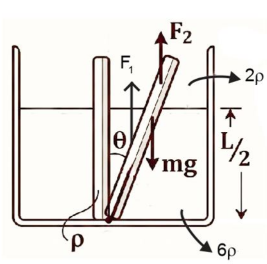
- This section contains **FOUR (04)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

[Q.9] A tank contains two immiscible liquids of densities 6ρ and 2ρ. The higher density liquid is filled up to a height $L/2$ from the bottom. A thin rod of density ρ and length L is fully immersed and hinged at the bottom so that is can oscillate freely, as shown in the figure. If the rod is slightly disturbed from its equilibrium, the time period of small oscillations is $\dfrac{2\pi}{n}\sqrt{\dfrac{L}{g}}$, where g is the acceleration due to gravity. The value of n is:



[ANS] **1.732**

[SOLN]



Cremation area of rod (s)

$$F_1 = 6\rho\left[\frac{5L}{2}\right]g$$

$$F_2 = 2\rho\left[\frac{5L}{2}\right]g$$

$$\tau\,\text{resting} = F_1\frac{1}{4}\sin\theta + F_2\left[\frac{3L}{4}\sin\theta\right] - mg\frac{L}{2}\sin\theta$$

$$\frac{ML^2}{3}\propto = \left[6\rho g\left(\frac{5L}{2}\right)g\cdot\frac{1}{4} + (2\rho)\left(\frac{5L}{2}\right)g\cdot\frac{3L}{4} - (\rho\cdot 5L)g\cdot\frac{L}{2}\right]\theta$$

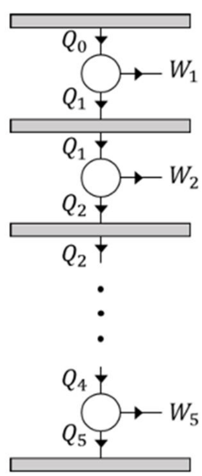
$$\frac{L^2}{3}\propto = -gL\left[\frac{6}{4} - \frac{1}{2}\right]\theta = (-gL\times 1)\theta$$

$$\propto = -\frac{3g}{L}\theta$$

$$T = 2\pi\sqrt{\frac{L}{3g}} = \frac{2\pi}{\sqrt{3}}\sqrt{\frac{L}{g}}$$

$$n = 1.732$$

[Q.10] As shown in the figure, five Carnot engines, each with efficiency η and same number of cycles per unit time, are operating between six heat reservoirs. The amount of heat released per cycle by on engine is completely absorbed by the next engine. Consider Q_0 to be the amount of heat absorbed per cycle by the first engine and W as the amount of total work done by all the engines per cycle, then the net efficiency of the system is found to be $\eta_{net} = \frac{W}{Q_0} = \frac{211}{243}$. The value of η is:



[ANS] **0.33**

[SOLN] $\eta = \frac{W_1}{Q_0} = \frac{W_2}{Q_1} = \frac{W_3}{Q_2} = --- = \frac{W_5}{Q_4}$

$$\eta = \frac{Q_0 - Q_1}{Q_0} = 1 - \frac{Q_1}{Q_0},\quad \eta = 1 - \frac{Q_2}{Q_1}$$

$\frac{Q_1}{Q_0} = (1-n) \qquad \Rightarrow \frac{Q_2}{Q_1} = 1-\eta$

$\Rightarrow Q_2 = Q_1(1-\eta) = Q_0(1-\eta)^2$

$\Rightarrow Q_3 = Q_0(1-\eta)^3$

$\Rightarrow Q_5 = Q_0(1-\eta)^5$

$\eta_{net} = \frac{W}{Q_0} = \frac{Q_0 - Q_5}{Q_0} = 1 - \frac{Q_5}{Q_0} = 1-(1-\eta)^5 = \frac{211}{243}$

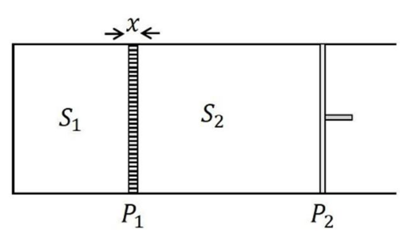
$\Rightarrow \qquad \frac{32}{243} = (1-\eta)^5$

$\Rightarrow \qquad (1-n) = \left(\frac{32}{243}\right)^{1/5}$

$1-\eta = \frac{2}{3}$

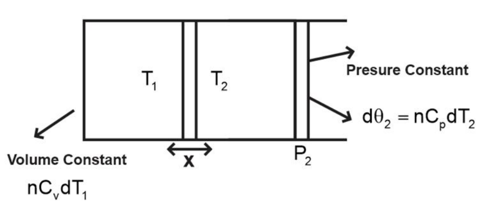
$\eta = \frac{1}{3} = 0.33$

[Q.11] As shown in the figure, an insulated container is fitted with a thermally conducting but immovable partition (P_1) and a freely movable but thermally insulated piston (P_2). The partition P_1 with thermal conductivity K, cross sectional area A and width x divides the container into two sections, S_1 and S_2, each containing one mole of a monoatomic gas. The piston P_2 moves freely such that the gas in S_2 is always at the atmospheric pressure. Initially, the difference between the temperatures of S_1 and S_2 is ΔT_0. The time it takes for the temperature difference to become $\frac{\Delta T_0}{2}$ is n x R/KA, where R is the universal gas constant. The value of n is: [Given: ln $2 \approx 0.7$]



[ANS] **0.66**

[SOLN] at any Instant



At any Instant

$$-\frac{d\theta_1}{dt}=\frac{KA(T_1-T_2)}{x} \quad \text{....(i) \&} \quad \frac{d\theta_2}{dt}=KA\frac{(T_1-T_2)}{x}$$

$$-1\cdot\frac{3}{2}R\cdot dT_1=\frac{kA\Delta T}{x}$$

$$\& \ \frac{5}{2}RdT_2=KA\frac{\Delta T}{x}$$

$$\left.\begin{aligned}-dT_1&=\frac{2KA}{3R}\Delta T\\ dT_2&=\frac{2}{5}\frac{KA}{R}\Delta T\end{aligned}\right]\oplus$$

$$-d(\Delta T)=\frac{16}{15}\frac{KA}{R}\Delta T$$

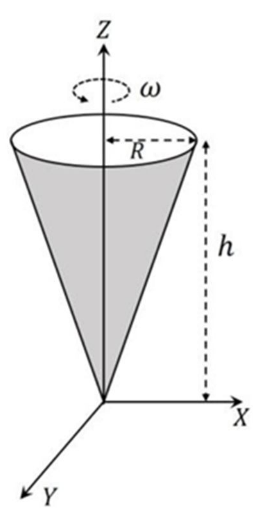
$$\int_{\Delta T_0}^{\frac{\Delta T_0}{2}}\frac{d\Delta T}{\Delta T}=\frac{-16}{15}\frac{KA}{R}\int dt$$

$$\ell n2=\frac{16}{15}\frac{KA}{R}t$$

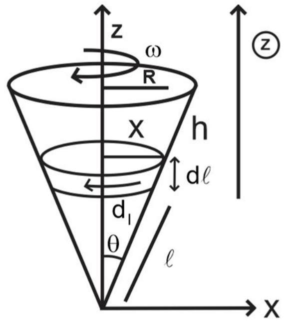
$$t=\frac{15}{16}\frac{R}{KA}\ell n2=\left(\frac{15}{16}\times\ell n2\right)\frac{R}{KA}$$

$$t=0.65625\frac{R}{KA}$$

[Q.12] A hollow, right circular cone of base radius R and height h, with its tip at the origin is rotating about the Z-axis with an angular velocity ω, as shown in the figure. The cone carries a total charge Q uniformly distributed on its curved surface. The magnitude of magnetic field at a point (0, 0, z), where $z >> R$ and $z >> h$, is $\frac{n\mu_0}{4\pi}\frac{QR^2\omega}{z^3}$. The value of n is:



[SOLN]



$$dI = \frac{\omega\left[\dfrac{Q}{\pi R\sqrt{R^2+h^2}}\cdot 2\pi x\cdot d\ell\right]}{2\pi}$$

$$B(z) = \frac{\mu_0 dIx^2}{2\left(x^2+z^2\right)^{3/2}} \qquad [x = \ell\sin\theta] \qquad \sin\theta = \frac{R}{\ell_0}$$

$$B(z) = \frac{\omega\theta\cdot 2\pi}{(\pi R)2\pi\sqrt{R^2+h^2}}\cdot\frac{\mu_0}{2}\int_0^{\ell_0}\frac{x^3 d\ell}{\left(x^2+z^2\right)^{3/2}}$$

$$= \frac{\mu_0\theta\omega}{2\pi R\ell_0 z^3}\int_0^{\ell_0} x^2 d\ell = \frac{\mu_0\theta\omega}{2\pi R\ell z^3}\int_0^{\ell_0}\ell^3\sin\theta^3 d\ell$$

$$= \frac{\mu_0\theta\omega}{2\pi R\ell_0 z^3}\cdot\sin^3\theta\cdot\frac{\ell_0^4}{4} = 6\pi z^3$$

SECTION 4 (Maximum Marks :16)

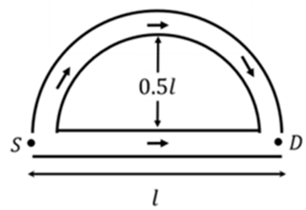
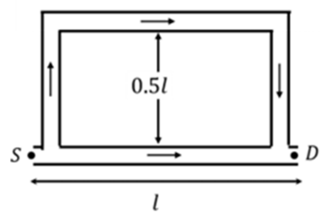
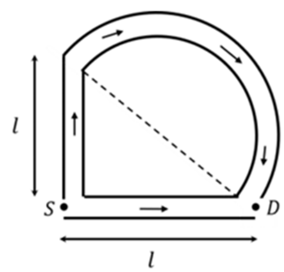
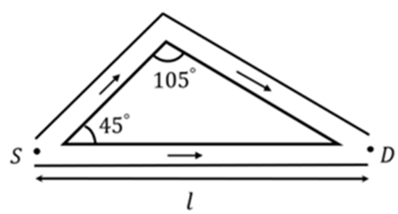
- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II.**
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.

[Q.13] List-I shows four configurations made of straight and semi-circular narrow tubes containing air. A sound wave of wavelength λ = 0.29 m enters these structures at the point S and a sound detector is placed at D. Between the points S and D, the sound travels only through the tubes. List-II contains the possible smallest values of l (refer to the figures) for which the detector D records maximum amplitude. Ignore effects of sharp corners. [Given $\cos(15^\circ) = 0.97$] Choose the option that best describes the match between the entries in List-I to those in List-II.

List-I		List-II	
(P)		(1)	1.32 m
(Q)		(2)	1.19 m
(R)		(3)	0.51 m
(S)		(4)	0.29 m
		(5)	0.13 m

[A] P $\rightarrow$ 4, Q $\rightarrow$ 3, R $\rightarrow$ 5, S $\rightarrow$ 1

[B] P $\rightarrow$ 4, Q $\rightarrow$ 3, R $\rightarrow$ 1, S $\rightarrow$ 5

[C] P $\rightarrow$ 3, Q $\rightarrow$ 4, R $\rightarrow$ 1, S $\rightarrow$ 2

[D] P $\rightarrow$ 3, Q $\rightarrow$ 4, R $\rightarrow$ 5, S $\rightarrow$ 2

[ANS] **D**

[SOLN] (P) $\frac{\pi\ell}{2} - \ell = \lambda$

$\Rightarrow \ell = \frac{2\lambda}{\pi - 2} = 0.51$

(Q) $\ell = \lambda = 0.29$

(R) $\frac{\pi\ell}{\sqrt{2}} = \lambda \Rightarrow \ell = \frac{\sqrt{2}\lambda}{\lambda} = 0.13$

[Q.14] In the List-I, four optical effects are mentioned. The physical phenomena of light which are essential to describe these optical effects are given in List-II. Choose the option which describes the correct match between the entries in List-I to those in List-II.

	List-I		**List-II**
(P)	Colorful sky in north polar region (Aurora Borealis)	(1)	Dispersion and reflection
(Q)	Partially polarized sun light	(2)	Total internal reflection
(R)	Rainbow	(3)	Diffraction
(S)	Dark and bright fringes	(4)	Scattering of light by molecules in the atmosphere
		(5)	Emission of radiation from oxygen and nitrogen atoms excited by charged particles

[A] $P \to 5,\ Q \to 4,\ R \to 1,\ S \to 3$

[B] $P \to 4,\ Q \to 2,\ R \to 1,\ S \to 3$

[C] $P \to 4,\ Q \to 1,\ R \to 2,\ S \to 3$

[D] $P \to 5,\ Q \to 4,\ R \to 1,\ S \to 2$

[ANS] **A**

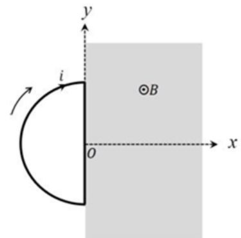
[SOLN] (Q) Partly polarized light – Scattering of light by molecule in the atmosphere

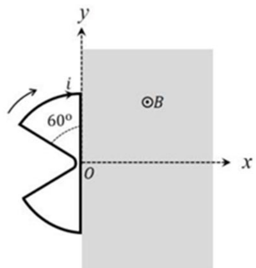
(S) Dark & bright fringes – Diffraction

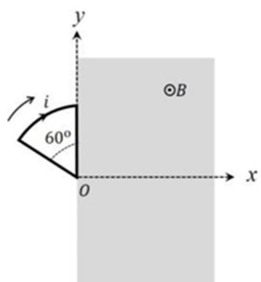
(R) Rainbow – Dispersion & reflection

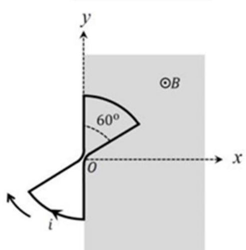
[Q.15] List-I contains four conducting loops lying in the XY plane, as shown in the figures. The loops are rotating about Z axis passing through the point O with time period T in clockwise direction. The region $x > 0$ contains a uniform magnetic field B in the $+z$ direction. List-II contains the qualitative variation of the induced current i(t) for each of these loops. Choose the option which describes the correct match between the entries in List-I to those in List-II.

List-I

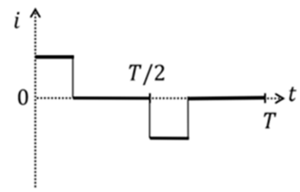
(P) 

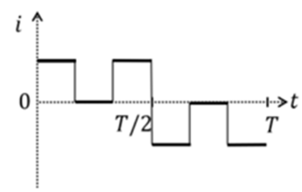
(Q) 

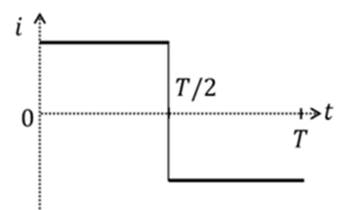
(R) 

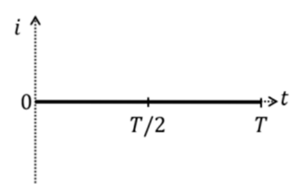
(S) 

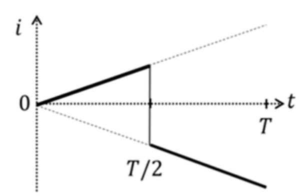
List-II

(1) 

(2) 

(3) 

(4) 

(5) 

[A] $P \to 5, Q \to 4, R \to 1, S \to 3$

[B] $P \to 3, Q \to 2, R \to 5, S \to 4$

[C] $P \to 3, Q \to 2, R \to 1, S \to 4$

[D] $P \to 5, Q \to 1, R \to 2, S \to 3$

[ANS] **C**

[SOLN] for time $\left(0 \text{ to} \frac{T}{2}\right)$

(P) $\phi = B \times \frac{r^2}{2}\omega t$

$$i = \frac{\in}{R} = \frac{1}{R}\frac{d\phi}{d\ell} = \frac{Br^2\omega}{2R} = \text{constant}$$

for $\left(\frac{T}{2} \text{ to } T\right)$

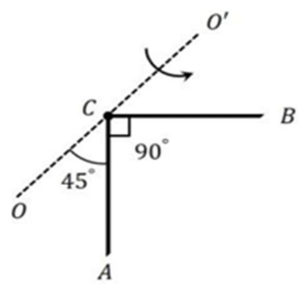
$$i = -\frac{Br^2\omega}{2R}$$

(Q) Similar to P

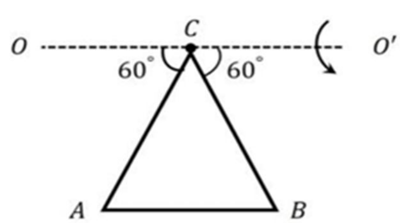
$$\frac{180}{60} = 3$$

[Q.16] List-I shows four planar structures made of uniform solid rods each of mass m and length l. In the List-II the possible moment of inertia of these structures about an axis OCO', which lies in the plane of the structures, are given. Choose the option that describes the correct match between the entries in List-I to those in List-II.

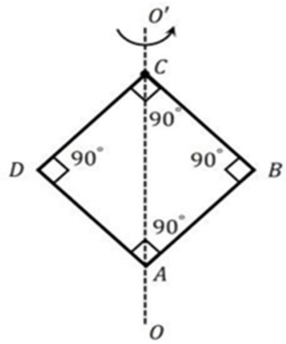
List-I **List-II**

(P) 

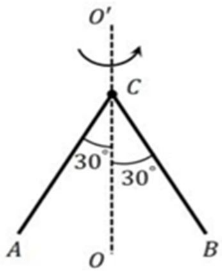
(1) $\frac{5}{4}m\ell^2$

(Q) 

(2) $\frac{1}{6}m\ell^2$

(R) 

(3) $\frac{1}{12}m\ell^2$

(S) 

(4) $\frac{2}{3}m\ell^2$

(5) $\frac{1}{3}m\ell^2$

[A] $P \to 5, Q \to 1, R \to 4, S \to 2$

[B] $P \to 1, Q \to 3, R \to 4, S \to 2$

[C] $P \to 5, Q \to 3, R \to 2, S \to 1$

[D] $P \to 5, Q \to 4, R \to 2, S \to 1$

[ANS] **A**

[SOLN] (P) $I = \frac{m\ell^2}{3} \times \sin^2 45 \times 2 = \frac{m\ell^2}{3}$

(Q) $I = \frac{m\ell^2}{3} \times \sin^2 60 \times 2 + m\left(\ell \sin 60\right)^2 = \frac{5m\ell^2}{4}$

(R) $I = \frac{m\ell^2}{3} \times \sin^2 45 \times 4 = \frac{2}{3}m\ell^2$

(S) $I = \frac{m\ell^2}{3} \times \sin^2 30 \times 2 = \frac{m\ell^2}{6}$